Appendix A. Stage 2 Analysis Details

Stage two of our analysis on the impact of HMS-detected smoke plumes on air pollution is as follows. We assume the model

$$\hat{\boldsymbol{\beta}}|\tilde{\boldsymbol{\beta}} \sim \mathcal{N}\left(\tilde{\boldsymbol{\beta}}, \Sigma_1\right)$$
$$\tilde{\boldsymbol{\beta}}|\mu \sim \mathcal{N}\left(\mu \mathbf{1}_{\boldsymbol{m}}, \Sigma_2\right),$$

where m is number of stations (from stage one, $s = 1, \ldots, m$), $\hat{\boldsymbol{\beta}}$ is the m-vector of stage-one plume effect estimates, $\tilde{\boldsymbol{\beta}}$ is the m-vector of true plume effects at the m stations, Σ_1 and Σ_2 are m-dimensional spatial covariance matrices and μ is the nation-wide average effect of plume episodes on a given pollutant. We aim to estimate and present μ for all pollutants and stage-two estimates of $\tilde{\boldsymbol{\beta}}$, denoted as $\hat{\tilde{\boldsymbol{\beta}}}$.

We consider four special cases of this general two-stage spatial model by changing settings on the spatial covariance matrices. Define V as the mxm diagonal matrix of standard errors of the stage-one plume effect estimates, ν_s, and let $\Omega(\rho_1)$ and $\Omega(\rho_2)$ be mxm exponential correlation matrices with (i, j) elements $\exp(-d_{ij}/\rho_1)$ and $\exp(-d_{ij}/\rho_2)$, respectively, where d_{ij} is the distance in kilometers between site i and site j, and ρ_1 and ρ_2 are spatial range parameters. We let

$$\Sigma_1 = V\left[(1-r_1)I_m + r_1\Omega(\rho_1)\right]V \quad \text{(covariance of the stage-one errors)}$$
$$\Sigma_2 = \sigma^2\left[(1-r_2)I_m + r_2\Omega(\rho_2)\right] \quad \text{(covariance of true } \beta\text{)}$$

where r_1 and r_2 represent the proportion of variance due to spatial patterns and σ^2 is the variance of the true effect. Changing these parameters allows us to investigate if spatially correlated ($r \neq 0$) or independent ($r = 0$) stage-one plume errors and/or constant ($\sigma^2 = 0$ and thus $\tilde{\beta}_s = \mu$ for all s) or spatially varying ($\sigma^2 > 0$ and thus $\tilde{\beta}_s \neq \mu$ for all s) true plume effects are the best fit for a given pollutant. Appendix Table 1 summarizes these four models.

Table 1: Description of the four models fit for each pollutant.

Model	Settings	Spatial Errors	$\tilde{\beta}_j \neq \mu, \forall j$
1	$\sigma^2 = 0, r_1 = 0$	no	no
2	$\sigma^2 = 0, r_1 \neq 0$	yes	no
3	$\sigma^2 \neq 0, r_1 = r_2 = 0$	no	yes
4	$\sigma^2 \neq 0, r_1 \neq 0, r_2 \neq 0$	yes	yes

For each pollutant, we fit the four models in Appendix Table 1 and determine best fit with BIC (see results in Table 2 in the main text).

To estimate the nation-wide average plume effect, μ, for each pollutant, we computed the GLS estimate, $\hat{\mu}$, and the variance of $\hat{\mu}$ in R. The derivation for the GLS is as follows:

$$\hat{\beta} = \tilde{\beta} + e_1, \quad e_1 \sim N(0, \Sigma_1)$$
$$\tilde{\beta} = \mu \mathbf{1_m} + e_2, \quad e_2 \sim N(0, \Sigma_2)$$

$$\implies \hat{\beta} = \mu \mathbf{1_m} + e^*, \quad e^* \sim N(0, \Sigma_1 + \Sigma_2)$$
$$\implies \hat{\mu} = \left(\mathbf{1_m}^T \left(\hat{\Sigma}_1 + \hat{\Sigma}_2 \right)^{-1} \mathbf{1_m} \right)^{-1} \mathbf{1_m}^T \left(\hat{\Sigma}_1 + \hat{\Sigma}_2 \right)^{-1} \hat{\beta}$$
$$\implies Var(\hat{\mu}) = \left(\mathbf{1_m}^T \left(\hat{\Sigma}_1 + \hat{\Sigma}_2 \right)^{-1} \mathbf{1_m} \right)^{-1}$$

where $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are functions of the maximum likelihood estimates of the spatial covariance parameters $\boldsymbol{\phi}$ calculated using the R package, `optim`. To estimate the stage-two estimates of the true plume effect, $\tilde{\boldsymbol{\beta}}$, we compute:

$$\hat{\tilde{\boldsymbol{\beta}}} = \left(\hat{\Sigma}_1^{-1} + \hat{\Sigma}_2^{-1} \right)^{-1} \left(\hat{\Sigma}_1^{-1} \hat{\boldsymbol{\beta}} + \hat{\Sigma}_2^{-1} \hat{\mu} \mathbf{1}_m \right).$$