Supplementary Information for:

Effects of core formation on the Hf–W isotopic composition of the Earth and dating of the Moon-forming impact

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Equilibration efficiency

The equilibration efficiency of an element *i* can be defined as:

$$E_i = \frac{k}{1+D_i/\Delta} \tag{S1}$$

where *k* is the fraction of incoming metal that equilibrates, D_i is a partition coefficient for element *i* (e.g., $D_{\mathrm{W}} = X_{\mathrm{W}}/X_{\mathrm{WO_3}}$), and Δ is the mass ratio of equilibrated silicate to equilibrated metal, following Deguen et al. (2014). The equilibration efficiency varies between a minimum of zero, in cases of strongly siderophile behavior (high *D*) or a very small mass of equilibrating silicate (low Δ), and a maximum of *k*, in cases of lower siderophility or a large amount of equilibrating silicate relative to the incoming metal. To determine the equilibration efficiency, Δ may be approximated as:

$$\Delta \approx \frac{\rho_{sil}}{\rho_{met}}\left[\left(1+\frac{\alpha}{\gamma^{1/3}}\right)^3 - 1\right] \tag{S2}$$

where ρ_{sil}/ρ_{met} is the ratio of silicate and metal densities, taken here to be 0.5; α is the entrainment coefficient, which varies between 0.2 and 0.3, with a value of 0.26 being used here (Deguen et al., 2014); and γ is the impactor:target mass ratio. The full range of allowable values for the equilibration efficiency is seen in these simulations. For example, for $k = 0.4$, E_i always starts at zero and in some cases ends as high as 0.38, with significant stochastic variability between simulations (Figure S2).

References

Andrault, D., Bolfan-Casanova, N., Lo Nigro, G., Bouhifd, M.A., Garbarino, G., Mezouar, M., 2011. Solidus and liquidus profiles of chondritic mantle: Implication for melting of the Earth across its history. Earth Planet. Sci. Lett. 304, 251–259. https://doi.org/10.1016/j.epsl.2011.02.006.

Deguen, R., Landeau, M., Olson, P., 2014. Turbulent metal–silicate mixing, fragmentation, and equilibration in magma oceans. Earth Planet. Sci. Lett. 391, 274–287. https://doi.org/10.1016/j.epsl.2014.02.007.

Supplemental figure captions

Figure S1: Effect of partial equilibration of the silicate on the resulting mantle tungsten anomaly for a fixed fraction of incoming metal equilibrating, $k = 0.55$. Each data point is the median of 73 Earth analogues, with error bars representing a 68% two-sided confidence interval. Shaded region indicates Earth value. The impactor silicate is always assumed to participate in the equilibration, in addition to the amount of target silicate displayed on the horizontal axis.

Figure S2: Estimated equilibration efficiency of tungsten (Eq. S1) throughout accretion for 73 Earth analogues. A value of $k = 0.4$ was used for the fraction of incoming metal equilibrating. The full allowable range of equilibration efficiencies from zero to k is achieved, with values increasing at later times.

Figure S3: Effects of timing of the Moon-forming impact, fraction of metal that equilibrates *k*, and mass of equilibrating silicate in Earth analogues on the resulting tungsten anomaly, for a fixed mass of equilibrating silicate (three times that of the impactor's silicate). Analogous to Figure 5a of the main text, but for less equilibrating silicate. Symbol size is proportional to planetary mass. Open (filled) symbols indicate Earth analogues whose late veneer mass is greater (less) than 0.02 $M_{\oplus}$. Each column of symbols represents one simulation, with the different colored symbols within the column representing different model conditions. Shaded region is Earth value.

Figure S4: Effects of timing of the Moon-forming impact, fraction of metal that equilibrates *k*, and mass of equilibrating silicate in Earth analogues on the resulting $f^{Hf/W}$. a) Effects of varying *k* for whole mantle equilibration. b) Effects of varying *k* for 3× the impactor's silicate mass equilibrating. c) Effects of varying the mass of equilibrating silicate for a fixed fraction of equilibrating metal ($k = 0.55$). Symbol size is proportional to planetary mass. Open (filled) symbols indicate Earth analogues whose late veneer mass is greater (less) than 0.02 $M_{\oplus}$. Each column of symbols represents one simulation, with the different colored symbols within the column representing different model conditions. Shaded region is Earth value.

Supplemental table captions

Table S1: Model conditions and compositional results for all runs performed on the suite of 73 Earth analogues, including model conditions that do not match the Earth. Model parameters are: *k*, the fraction of incoming metal that equilibrates with silicate; *tar:imp ratio*, the mass of target silicate that participates in the equilibration reaction, in units of impactor silicate mass (the impactor silicate is always assumed to participate); *Pfrac*, the pressure of equilibration expressed as a fraction of the evolving core–mantle boundary pressure; and *Initial* fO_2, the oxygen fugacity at which embryos and planetesimals were initially equilibrated. The temperature of equilibration was the liquidus temperature at the relevant pressure from Andrault et al. (2011), except as noted. Compositional results are reported as a median of the 73 Earth analogues with uncertainties as a 68% two-sided confidence interval. Stated confidence intervals are likely overestimates of the true values for a 1 $M_\oplus$ planet, because they include variations due to planetary mass (Section 4.3 of the main text).

Table S2: Model conditions and compositional results for all runs performed on one select simulation (see main text for details), including model conditions that do not reproduce the Earth. Model parameters are: *k*, the fraction of incoming metal that equilibrates with silicate (equilibration always occurred with an amount of silicate equal to 3× the mass of the impactor's silicate); *Initial* fO_2, the oxygen fugacity at which embryos and planetesimals were initially equilibrated (including two runs with a step function of initial oxygen fugacity, with IW–4 inside of 2 AU and IW–2 outside of 2 AU); *diff time*, the time of initial differentiation of embryos and planetesimals (a value of 0 indicates differentiation at the start of the *N*-body simulations, and a value of "impact" indicates differentiation upon first impact); *delta T*, the temperature of equilibration expressed relative to the liquidus of Andrault et al. (2011); and *Pfrac*, the pressure of equilibration expressed as a fraction of the evolving core–mantle boundary pressure.

Table S3: Comparison of results when model parameters are chosen to match Earth's nickel vs. tungsten mantle abundances (a subset of the results shown in Table S1). Model parameters are: *k*, the fraction of incoming metal that equilibrates with silicate; *tar:imp ratio*, the mass of target silicate that participates in the equilibration reaction, in units of impactor silicate mass (the impactor silicate is always assumed to participate); and *Pfrac*, the pressure of equilibration expressed as a fraction of the evolving core–mantle boundary pressure. The initial oxygen fugacity was always IW–3.5 throughout the disk, and the temperature of equilibration was the liquidus temperature at the relevant pressure from Andrault et al. (2011). For each combination of *k* and *tar:imp ratio* tested, results from two different *Pfrac* are shown: one chosen to approximately match Earth's NiO content (2500 ppm) and one chosen to approximately match Earth's WO_3 content (20 ppb). Compositional results are reported as a median of the 73 Earth analogues with uncertainties as a 68% two-sided confidence interval.