

S2 Appendix. Integrating for infinite SOL-neurons. The computational model can also be described as a linear combination of the outputs of the POL-OP units. We can merge the SOL and TCL layers, in one by integrating for infinite SOL-neurons. Therefore, the output of each TCL neuron is given by:

$$r_{\text{TCL}}^k = \sum_{j=1}^{n_{\text{POL}}} W^{j,k} \cdot g^j(\theta_t, \phi_t) \cdot r_{\text{POL}}^j$$

where $W^{j,k}$ is the weight of the synapse connecting the input, j , and the output, k . The weights of the model are given by the equation below:

$$W^{j,k} = -\frac{n_{\text{TCL}}}{2 \cdot n_{\text{POL}}} \cdot g^j(\theta_t, \phi_t) \cdot \sin \left[\left(\phi_{\text{TCL}}^{\text{pref}} \right)^k - \alpha^j \right]$$

where $\left(\phi_{\text{TCL}}^{\text{pref}} \right)^k$ is the preference angle of the k^{th} TCL-neuron. This equation is the result of integrating the two processing steps (SOL and TCL) to one by assuming infinite SOL-neurons. The processing would look like this:

$$r_{\text{TCL}}^k = \sum_{\xi=1}^{n_{\text{SOL}}} W_{\text{TCL}}^{\xi,k} \sum_{j=1}^{n_{\text{POL}}} W_{\text{SOL}}^{j,\xi} \cdot g^j(\theta_t, \phi_t) \cdot r^j$$

where

$$g^j(\theta_t, \phi_t) = \exp \left\{ -\frac{1}{2} \left[\frac{\sin(\theta_g - \theta^j) \cos(\theta_t) + \cos(\theta_g - \theta^j) \sin(\theta_t) \cos(\phi_t - \phi^j)}{\sigma_g} \right]^2 \right\}$$

$$W_{\text{SOL}}^{j,\xi} = \frac{n_{\text{SOL}}}{n_{\text{POL}}} \cdot \sin \left[\alpha^j - \left(\phi_{\text{SOL}}^{\text{pref}} \right)^\xi \right]$$

$$W_{\text{TCL}}^{\xi,k} = \frac{n_{\text{TCL}}}{2 \cdot n_{\text{SOL}}} \cos \left[\left(\phi_{\text{TCL}}^{\text{pref}} \right)^k - \left(\phi_{\text{SOL}}^{\text{pref}} \right)^\xi \right]$$

Integrating for $\delta = \left(\phi_{\text{SOL}}^{\text{pref}} \right)^\xi$ in the limit where $n_{\text{SOL}} \rightarrow +\infty$ we have:

$$\begin{aligned} W^{j,k} &= \lim_{n_{\text{SOL}} \rightarrow +\infty} \sum_{\xi=1}^{n_{\text{SOL}}} W_{\text{TCL}}^{\xi,k} W_{\text{SOL}}^{j,\xi} \\ &= \lim_{n_{\text{SOL}} \rightarrow +\infty} \frac{n_{\text{TCL}}}{2 \cdot n_{\text{POL}}} \sum_{j=1}^{n_{\text{CBL}}} \cos \left[\left(\phi_{\text{TCL}}^{\text{pref}} \right)^k - \delta \right] \sin \left(\alpha^j - \delta \right) \\ &= \frac{n_{\text{TCL}}}{2 \cdot n_{\text{POL}}} \cdot \frac{1}{180^\circ} \int_0^{360^\circ} \cos \left[\left(\phi_{\text{TCL}}^{\text{pref}} \right)^k - \delta \right] \sin \left(\alpha^j - \delta \right) d\delta \\ &= -\frac{n_{\text{TCL}}}{2 \cdot n_{\text{POL}}} \cdot \sin \left[\left(\phi_{\text{TCL}}^{\text{pref}} \right)^k - \alpha^j \right] \end{aligned}$$

Using this model we get almost the same result as the two-layered one.