

**S2 Table. Comparison between  $l_{approx}$  and  $l_{mean}$  in quantitative phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
Base (1,0,0)	r	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.05 (0.03)	0.95 (0.02)	1.00 (0.00)	1.10 (0.04)	0.91 (0.03)
Base (1,0,0)	d	1.00 (0.03)	1.04 (0.04)	0.97 (0.02)	1.00 (0.00)	1.07 (0.03)	0.94 (0.03)	1.00 (0.01)	1.21 (0.03)	0.83 (0.02)	1.00 (0.00)	1.42 (0.05)	0.71 (0.02)
Base (0,0,1)	r	1.15 (0.05)	1.18 (0.06)	0.98 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.03 (0.01)	1.05 (0.03)	0.98 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
Base (0,0,1)	d	1.20 (0.05)	1.25 (0.06)	0.96 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.16 (0.02)	1.45 (0.04)	0.80 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1a (1,0,0)	r	1.01 (0.03)	1.02 (0.04)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.01)	1.16 (0.03)	0.86 (0.02)	1.00 (0.00)	1.32 (0.04)	0.76 (0.03)
1a (1,0,0)	d	1.01 (0.03)	1.06 (0.04)	0.95 (0.02)	1.00 (0.00)	1.11 (0.03)	0.90 (0.03)	1.00 (0.01)	1.32 (0.03)	0.76 (0.02)	1.00 (0.00)	1.64 (0.05)	0.61 (0.02)
1a (0,0,1)	r	1.03 (0.04)	1.03 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.02 (0.01)	1.03 (0.03)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1a (0,0,1)	d	1.16 (0.05)	1.19 (0.06)	0.97 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.19 (0.02)	1.70 (0.04)	0.70 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (0,0,0)	a	1.00 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (1,0,0)	a	1.01 (0.03)	1.03 (0.04)	0.98 (0.02)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)	1.01 (0.01)	1.38 (0.03)	0.73 (0.02)	1.00 (0.00)	1.74 (0.06)	0.57 (0.02)
1b (1,0,0)	r	1.00 (0.02)	1.03 (0.03)	0.97 (0.02)	1.00 (0.00)	1.06 (0.04)	0.95 (0.03)	1.00 (0.01)	1.01 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)
1b (1,0,0)	d	1.00 (0.03)	1.04 (0.04)	0.96 (0.02)	1.00 (0.00)	1.09 (0.03)	0.93 (0.03)	0.99 (0.01)	1.46 (0.03)	0.68 (0.01)	1.00 (0.00)	1.93 (0.05)	0.52 (0.01)
1b (0,1,0)	a	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (0,0,1)	a	1.02 (0.03)	1.05 (0.04)	0.97 (0.02)	1.00 (0.00)	1.05 (0.04)	0.95 (0.03)	1.01 (0.01)	1.13 (0.03)	0.90 (0.02)	1.00 (0.00)	1.21 (0.04)	0.83 (0.03)
1b (0,0,1)	r	1.01 (0.03)	1.02 (0.04)	1.00 (0.02)	1.00 (0.00)	1.02 (0.03)	0.99 (0.03)	1.01 (0.01)	1.04 (0.02)	0.97 (0.02)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)
1b (0,0,1)	d	1.00 (0.03)	1.02 (0.04)	0.99 (0.02)	1.00 (0.00)	1.03 (0.03)	0.98 (0.03)	1.00 (0.01)	1.12 (0.02)	0.90 (0.02)	1.00 (0.00)	1.23 (0.04)	0.82 (0.03)
1c (0,0,0)	a	1.00 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (1,0,0)	a	1.01 (0.03)	1.13 (0.04)	0.89 (0.02)	1.00 (0.00)	1.25 (0.04)	0.80 (0.03)	1.00 (0.01)	2.18 (0.05)	0.46 (0.01)	1.00 (0.00)	3.35 (0.09)	0.30 (0.01)
1c (1,0,0)	r	1.00 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.11 (0.03)	0.90 (0.02)	1.00 (0.00)	1.21 (0.04)	0.82 (0.03)
1c (1,0,0)	d	1.00 (0.03)	1.05 (0.04)	0.96 (0.02)	1.00 (0.00)	1.10 (0.03)	0.91 (0.03)	1.00 (0.01)	1.32 (0.03)	0.76 (0.02)	1.00 (0.00)	1.64 (0.05)	0.61 (0.02)
1c (0,1,0)	a	1.00 (0.03)	1.00 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (0,0,1)	a	1.01 (0.03)	1.03 (0.04)	0.98 (0.02)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)	1.00 (0.01)	1.14 (0.03)	0.88 (0.02)	1.00 (0.00)	1.29 (0.04)	0.78 (0.02)
1c (0,0,1)	r	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (0,0,1)	d	1.01 (0.03)	1.02 (0.04)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.01)	1.14 (0.02)	0.87 (0.02)	1.00 (0.00)	1.29 (0.04)	0.78 (0.02)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.

**S2 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in quantitative phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
1d (0,0,0)	a	1.00 (0.03)	1.00 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1d (1,0,0)	a	1.01 (0.03)	1.10 (0.05)	0.92 (0.02)	1.00 (0.00)	1.18 (0.04)	0.85 (0.03)	1.00 (0.01)	2.24 (0.05)	0.45 (0.01)	1.00 (0.00)	3.48 (0.09)	0.29 (0.01)
1d (1,0,0)	r	1.01 (0.03)	1.02 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.01)	1.25 (0.03)	0.80 (0.02)	1.00 (0.00)	1.49 (0.05)	0.67 (0.02)
1d (1,0,0)	d	1.01 (0.03)	1.04 (0.04)	0.96 (0.02)	1.00 (0.00)	1.08 (0.03)	0.93 (0.03)	1.00 (0.01)	1.51 (0.03)	0.66 (0.01)	1.00 (0.00)	2.01 (0.06)	0.50 (0.01)
1d (0,1,0)	a	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1d (0,0,1)	a	1.05 (0.04)	1.07 (0.05)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.33 (0.02)	2.54 (0.06)	0.52 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1d (0,0,1)	r	1.16 (0.05)	1.18 (0.06)	0.98 (0.02)	1.00 (0.00)	1.01 (0.04)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1d (0,0,1)	d	1.21 (0.05)	1.26 (0.06)	0.96 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.20 (0.02)	1.69 (0.04)	0.71 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2a (1,0,0)	r	1.01 (0.03)	1.04 (0.04)	0.97 (0.02)	1.00 (0.00)	1.06 (0.03)	0.95 (0.03)	1.00 (0.01)	1.19 (0.03)	0.84 (0.02)	1.00 (0.00)	1.38 (0.04)	0.73 (0.02)
2a (1,0,0)	d	1.00 (0.03)	1.05 (0.04)	0.96 (0.02)	1.00 (0.00)	1.10 (0.04)	0.92 (0.03)	1.00 (0.01)	1.49 (0.03)	0.67 (0.01)	1.00 (0.00)	1.99 (0.06)	0.50 (0.01)
2a (0,0,1)	r	1.11 (0.05)	1.12 (0.06)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.01 (0.01)	1.02 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2a (0,0,1)	d	1.09 (0.05)	1.12 (0.06)	0.98 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.20 (0.01)	1.71 (0.04)	0.70 (0.01)	1.00 (0.00)	1.00 (0.03)	1.01 (0.03)
2b (0,0,0)	a	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)
2b (1,0,0)	a	1.00 (0.03)	1.03 (0.04)	0.97 (0.02)	1.00 (0.00)	1.05 (0.03)	0.95 (0.03)	1.00 (0.01)	1.11 (0.03)	0.90 (0.02)	1.00 (0.00)	1.23 (0.04)	0.82 (0.02)
2b (1,0,0)	r	1.01 (0.03)	1.01 (0.04)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2b (1,0,0)	d	1.00 (0.03)	1.05 (0.04)	0.96 (0.02)	1.00 (0.00)	1.08 (0.04)	0.93 (0.03)	1.00 (0.01)	1.32 (0.03)	0.76 (0.02)	1.00 (0.00)	1.64 (0.05)	0.61 (0.02)
2b (0,1,0)	a	1.00 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2b (0,0,1)	a	1.00 (0.03)	1.04 (0.03)	0.97 (0.02)	1.00 (0.00)	1.07 (0.03)	0.94 (0.03)	1.00 (0.01)	1.34 (0.03)	0.74 (0.02)	1.00 (0.00)	1.69 (0.05)	0.59 (0.02)
2b (0,0,1)	r	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.05 (0.02)	0.95 (0.02)	1.00 (0.00)	1.09 (0.04)	0.92 (0.03)
2b (0,0,1)	d	1.00 (0.03)	1.02 (0.04)	0.98 (0.02)	1.00 (0.00)	1.04 (0.03)	0.97 (0.03)	1.00 (0.01)	1.11 (0.03)	0.90 (0.02)	1.00 (0.00)	1.22 (0.04)	0.82 (0.03)
2c (1,0,0)	r	2.28 (0.28)	2.28 (0.27)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	2.31 (0.08)	2.33 (0.10)	0.99 (0.02)	1.00 (0.00)	1.04 (0.03)	0.96 (0.03)
2c (1,0,0)	d	2.24 (0.30)	2.26 (0.30)	0.99 (0.03)	1.00 (0.00)	1.03 (0.03)	0.98 (0.03)	2.22 (0.08)	2.39 (0.10)	0.93 (0.02)	1.00 (0.00)	1.34 (0.04)	0.75 (0.02)
2c (0,0,1)	r	2.31 (0.24)	2.32 (0.25)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	2.32 (0.08)	2.33 (0.10)	1.00 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2c (0,0,1)	d	2.33 (0.28)	2.33 (0.27)	1.00 (0.03)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	2.31 (0.10)	2.49 (0.12)	0.93 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)

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**S2 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in quantitative phenotype simulations**

Scenario			GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M		$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
2d (1,0,0)	r		0.94 (0.26)	0.98 (0.18)	0.95 (0.07)	1.00 (0.00)	1.04 (0.03)	0.96 (0.03)	0.99 (0.21)	1.01 (0.16)	0.97 (0.05)	1.00 (0.00)	1.03 (0.03)	0.97 (0.03)
2d (1,0,0)	d		1.00 (0.33)	1.05 (0.24)	0.94 (0.08)	1.00 (0.00)	1.08 (0.03)	0.93 (0.03)	0.99 (0.22)	1.49 (0.16)	0.66 (0.06)	1.00 (0.00)	1.99 (0.06)	0.50 (0.01)
2d (0,0,1)	r		1.45 (0.79)	1.33 (0.56)	1.05 (0.11)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	10.77 (2.81)	8.19 (2.07)	1.31 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2d (0,0,1)	d		13.70 (3.61)	10.08 (2.55)	1.36 (0.03)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	105.30 (8.54)	77.70 (6.50)	1.36 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,0)	a		1.01 (0.21)	1.01 (0.20)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.02 (0.23)	1.02 (0.21)	1.00 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (1,0,0)	a		0.98 (0.22)	1.05 (0.21)	0.93 (0.04)	1.00 (0.00)	1.13 (0.04)	0.89 (0.03)	1.01 (0.20)	2.02 (0.20)	0.50 (0.05)	1.00 (0.00)	3.02 (0.09)	0.33 (0.01)
2e (1,0,0)	r		1.02 (0.25)	1.04 (0.24)	0.97 (0.03)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)	0.99 (0.21)	1.22 (0.19)	0.80 (0.05)	1.00 (0.00)	1.47 (0.04)	0.68 (0.02)
2e (1,0,0)	d		1.03 (0.25)	1.06 (0.23)	0.97 (0.03)	1.00 (0.00)	1.07 (0.03)	0.94 (0.03)	0.99 (0.21)	1.38 (0.19)	0.71 (0.05)	1.00 (0.00)	1.79 (0.05)	0.56 (0.02)
2e (0,1,0)	a		1.12 (0.25)	1.91 (0.27)	0.58 (0.06)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	9.48 (0.43)	16.61 (0.96)	0.57 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,1)	a		1.19 (0.26)	2.59 (0.32)	0.46 (0.06)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	2.65 (0.29)	4.10 (0.47)	0.65 (0.04)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,1)	r		1.00 (0.25)	1.01 (0.23)	0.99 (0.03)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.01 (0.20)	1.02 (0.19)	0.99 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,1)	d		1.04 (0.25)	1.07 (0.23)	0.97 (0.03)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	4.78 (0.37)	7.86 (0.59)	0.61 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,0)	a		1.01 (0.03)	1.01 (0.03)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (1,0,0)	a		1.01 (0.04)	1.09 (0.04)	0.92 (0.02)	1.01 (0.00)	1.17 (0.04)	0.86 (0.03)	1.00 (0.01)	1.92 (0.04)	0.52 (0.01)	1.00 (0.00)	2.83 (0.08)	0.35 (0.01)
3a (1,0,0)	r		1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)
3a (1,0,0)	d		1.01 (0.03)	1.03 (0.04)	0.98 (0.02)	1.01 (0.00)	1.05 (0.03)	0.96 (0.03)	1.00 (0.01)	1.50 (0.03)	0.67 (0.01)	1.00 (0.00)	2.00 (0.06)	0.50 (0.01)
3a (0,1,0)	a		1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,1)	a		1.21 (0.07)	1.27 (0.08)	0.95 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.11 (0.02)	1.41 (0.04)	0.79 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,1)	r		1.04 (0.04)	1.04 (0.04)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.01 (0.01)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,1)	d		1.21 (0.05)	1.26 (0.06)	0.96 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.18 (0.02)	1.56 (0.04)	0.76 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.

**S2 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in quantitative phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
3b (0,0,0)	a	1.00 (0.02)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (1,0,0)	a	1.00 (0.02)	1.12 (0.03)	0.90 (0.02)	1.00 (0.00)	1.23 (0.04)	0.81 (0.03)	1.00 (0.01)	2.22 (0.05)	0.45 (0.01)	1.00 (0.00)	3.44 (0.09)	0.29 (0.01)
3b (1,0,0)	r	1.00 (0.02)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.10 (0.02)	0.91 (0.02)	1.00 (0.00)	1.20 (0.04)	0.83 (0.03)
3b (1,0,0)	d	1.00 (0.02)	1.04 (0.03)	0.96 (0.02)	1.00 (0.00)	1.08 (0.03)	0.93 (0.03)	1.00 (0.01)	1.42 (0.03)	0.70 (0.02)	1.00 (0.00)	1.84 (0.06)	0.54 (0.02)
3b (0,1,0)	a	1.00 (0.02)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (0,0,1)	a	1.08 (0.03)	1.81 (0.08)	0.60 (0.02)	1.00 (0.00)	1.63 (0.06)	0.62 (0.02)	1.04 (0.01)	2.96 (0.11)	0.35 (0.01)	1.00 (0.00)	3.41 (0.14)	0.29 (0.01)
3b (0,0,1)	r	1.01 (0.02)	1.02 (0.03)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.05 (0.01)	2.04 (0.06)	0.52 (0.01)	1.00 (0.00)	2.35 (0.07)	0.43 (0.01)
3b (0,0,1)	d	1.07 (0.02)	1.27 (0.04)	0.84 (0.02)	1.00 (0.00)	1.26 (0.04)	0.80 (0.03)	1.06 (0.01)	3.36 (0.06)	0.32 (0.01)	1.00 (0.00)	3.80 (0.09)	0.26 (0.01)
3c (0,0,0)	a	1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	0.99 (0.03)	1.01 (0.03)
3c (1,0,0)	a	1.01 (0.00)	1.13 (0.02)	0.89 (0.02)	1.00 (0.00)	1.25 (0.04)	0.80 (0.03)	1.00 (0.00)	1.54 (0.04)	0.65 (0.02)	1.00 (0.00)	2.09 (0.06)	0.48 (0.01)
3c (1,0,0)	r	1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (1,0,0)	d	1.01 (0.00)	1.03 (0.02)	0.98 (0.02)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)	1.00 (0.00)	1.48 (0.03)	0.68 (0.01)	1.00 (0.00)	1.96 (0.06)	0.51 (0.02)
3c (0,1,0)	a	1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (0,0,1)	a	1.01 (0.01)	1.07 (0.03)	0.94 (0.02)	1.00 (0.00)	1.06 (0.04)	0.95 (0.03)	1.00 (0.00)	1.62 (0.04)	0.62 (0.01)	1.00 (0.00)	1.55 (0.05)	0.64 (0.02)
3c (0,0,1)	r	1.01 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (0,0,1)	d	1.01 (0.00)	1.03 (0.02)	0.98 (0.02)	1.00 (0.00)	1.03 (0.03)	0.98 (0.03)	1.00 (0.00)	1.23 (0.03)	0.82 (0.02)	1.00 (0.00)	1.22 (0.04)	0.82 (0.03)
3d (0,0,0)	a	1.01 (0.02)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3d (1,0,0)	a	1.01 (0.03)	1.05 (0.04)	0.96 (0.02)	1.00 (0.00)	1.09 (0.03)	0.92 (0.03)	1.00 (0.01)	2.16 (0.05)	0.46 (0.01)	1.00 (0.00)	3.33 (0.09)	0.30 (0.01)
3d (1,0,0)	r	1.00 (0.03)	1.03 (0.04)	0.98 (0.02)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)	1.00 (0.01)	1.15 (0.03)	0.87 (0.02)	1.00 (0.00)	1.29 (0.04)	0.78 (0.02)
3d (1,0,0)	d	1.00 (0.02)	1.03 (0.03)	0.98 (0.02)	1.00 (0.00)	1.05 (0.03)	0.95 (0.03)	1.00 (0.01)	1.50 (0.03)	0.67 (0.01)	1.00 (0.00)	2.00 (0.05)	0.50 (0.01)
3d (0,1,0)	a	1.00 (0.03)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.01 (0.03)
3d (0,0,1)	a	1.10 (0.03)	1.42 (0.06)	0.77 (0.02)	1.00 (0.00)	1.37 (0.05)	0.74 (0.03)	1.09 (0.01)	4.20 (0.10)	0.26 (0.01)	1.00 (0.00)	4.55 (0.12)	0.22 (0.01)
3d (0,0,1)	r	1.06 (0.03)	1.14 (0.04)	0.94 (0.02)	1.00 (0.00)	1.11 (0.03)	0.91 (0.03)	1.01 (0.01)	1.04 (0.02)	0.97 (0.02)	1.00 (0.00)	1.04 (0.03)	0.96 (0.03)
3d (0,0,1)	d	1.07 (0.03)	1.18 (0.04)	0.91 (0.02)	1.00 (0.00)	1.15 (0.04)	0.88 (0.03)	1.07 (0.01)	2.23 (0.05)	0.48 (0.01)	1.00 (0.00)	2.57 (0.08)	0.39 (0.01)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.

**S2 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in quantitative phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )			
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	
4a	(0,0,0)	a	1.01 (0.03)	1.02 (0.07)	0.99 (0.06)	1.00 (0.00)	1.02 (0.09)	0.99 (0.09)	1.00 (0.01)	1.02 (0.06)	0.99 (0.06)	1.00 (0.00)	1.02 (0.09)	0.99 (0.09)
4a	(1,0,0)	a	1.01 (0.03)	1.48 (0.13)	0.69 (0.06)	1.00 (0.00)	1.92 (0.22)	0.53 (0.06)	1.00 (0.01)	5.04 (0.34)	0.20 (0.01)	1.00 (0.00)	9.08 (0.68)	0.11 (0.01)
4a	(1,0,0)	r	1.00 (0.03)	1.08 (0.09)	0.93 (0.07)	1.00 (0.00)	1.16 (0.13)	0.88 (0.10)	1.00 (0.01)	1.02 (0.08)	0.99 (0.07)	1.00 (0.00)	1.05 (0.12)	0.97 (0.10)
4a	(1,0,0)	d	1.01 (0.03)	1.43 (0.12)	0.71 (0.06)	1.00 (0.00)	1.84 (0.23)	0.55 (0.07)	1.00 (0.01)	4.29 (0.32)	0.23 (0.02)	1.00 (0.00)	7.57 (0.63)	0.13 (0.01)
4a	(0,1,0)	a	1.01 (0.03)	1.02 (0.08)	1.00 (0.07)	1.00 (0.00)	1.02 (0.10)	0.99 (0.09)	1.00 (0.01)	1.00 (0.06)	1.01 (0.06)	1.00 (0.00)	1.00 (0.09)	1.01 (0.09)
4a	(0,0,1)	a	1.06 (0.05)	1.13 (0.10)	0.94 (0.07)	1.00 (0.00)	1.00 (0.09)	1.01 (0.09)	1.17 (0.02)	4.42 (0.34)	0.27 (0.02)	1.00 (0.00)	1.00 (0.09)	1.01 (0.09)
4a	(0,0,1)	r	1.12 (0.05)	1.28 (0.12)	0.88 (0.06)	1.00 (0.00)	1.02 (0.09)	1.00 (0.09)	1.05 (0.01)	1.46 (0.13)	0.72 (0.07)	1.00 (0.00)	1.01 (0.10)	1.00 (0.09)
4a	(0,0,1)	d	1.16 (0.05)	1.48 (0.15)	0.80 (0.07)	1.00 (0.00)	1.01 (0.09)	1.00 (0.09)	1.18 (0.02)	4.57 (0.34)	0.26 (0.02)	1.00 (0.00)	1.00 (0.09)	1.00 (0.09)
4b	(0,0,0)	a	1.00 (0.03)	1.00 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
4b	(1,0,0)	a	1.00 (0.03)	1.03 (0.04)	0.98 (0.02)	1.00 (0.00)	1.05 (0.03)	0.96 (0.03)	1.00 (0.01)	2.24 (0.05)	0.45 (0.01)	1.00 (0.00)	3.49 (0.08)	0.29 (0.01)
4b	(1,0,0)	r	1.00 (0.03)	1.00 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
4b	(1,0,0)	d	1.00 (0.03)	1.04 (0.04)	0.97 (0.02)	1.00 (0.00)	1.06 (0.03)	0.94 (0.03)	1.00 (0.01)	1.10 (0.03)	0.91 (0.02)	1.00 (0.00)	1.18 (0.04)	0.85 (0.03)
4b	(0,1,0)	a	1.01 (0.03)	1.01 (0.04)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
4b	(0,0,1)	a	1.04 (0.04)	1.05 (0.05)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.26 (0.02)	2.12 (0.06)	0.59 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
4b	(0,0,1)	r	1.10 (0.05)	1.11 (0.06)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.01 (0.01)	1.02 (0.03)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
4b	(0,0,1)	d	1.04 (0.04)	1.05 (0.04)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.12 (0.02)	1.44 (0.04)	0.78 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
4c	(0,0,0)	a	1.00 (0.03)	1.00 (0.07)	1.01 (0.06)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)	1.00 (0.01)	1.00 (0.06)	1.00 (0.05)	1.00 (0.00)	1.00 (0.08)	1.00 (0.08)
4c	(1,0,0)	a	1.01 (0.03)	1.47 (0.12)	0.69 (0.05)	1.00 (0.00)	1.92 (0.21)	0.53 (0.06)	1.00 (0.01)	2.09 (0.17)	0.48 (0.04)	1.00 (0.00)	3.18 (0.33)	0.32 (0.03)
4c	(1,0,0)	r	1.01 (0.03)	1.20 (0.09)	0.84 (0.05)	1.00 (0.00)	1.40 (0.14)	0.73 (0.07)	1.00 (0.01)	1.00 (0.06)	1.00 (0.06)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)
4c	(1,0,0)	d	1.01 (0.03)	1.21 (0.09)	0.83 (0.06)	1.00 (0.00)	1.42 (0.15)	0.72 (0.07)	1.00 (0.01)	3.66 (0.24)	0.27 (0.02)	1.00 (0.00)	6.32 (0.48)	0.16 (0.01)
4c	(0,1,0)	a	1.00 (0.03)	1.00 (0.07)	1.00 (0.06)	1.00 (0.00)	1.01 (0.08)	1.00 (0.08)	1.00 (0.01)	1.00 (0.06)	1.00 (0.06)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)
4c	(0,0,1)	a	1.04 (0.04)	1.09 (0.08)	0.96 (0.06)	1.00 (0.00)	1.02 (0.08)	0.99 (0.07)	1.33 (0.02)	8.01 (0.46)	0.17 (0.01)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)
4c	(0,0,1)	r	1.03 (0.04)	1.05 (0.08)	0.99 (0.06)	1.00 (0.00)	1.01 (0.08)	1.00 (0.08)	1.00 (0.01)	1.00 (0.06)	1.00 (0.06)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)
4c	(0,0,1)	d	1.09 (0.05)	1.19 (0.10)	0.92 (0.06)	1.00 (0.00)	1.02 (0.08)	1.00 (0.08)	1.07 (0.02)	1.80 (0.15)	0.60 (0.05)	1.00 (0.00)	1.01 (0.08)	1.00 (0.07)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.