

**S3 Table. Comparison between  $l_{approx}$  and  $l_{mean}$  in binary phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
Base (1,0,0)	r	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
Base (1,0,0)	d	1.00 (0.02)	1.01 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
Base (0,0,1)	r	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
Base (0,0,1)	d	1.01 (0.01)	1.02 (0.02)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1a (1,0,0)	r	1.00 (0.00)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.04)	1.00 (0.04)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.04)	0.99 (0.04)
1a (1,0,0)	d	1.01 (0.01)	1.02 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1a (0,0,1)	r	1.01 (0.01)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1a (0,0,1)	d	1.02 (0.02)	1.02 (0.03)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (0,0,0)	a	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (1,0,0)	a	1.00 (0.01)	1.02 (0.03)	0.98 (0.02)	1.00 (0.00)	1.03 (0.04)	0.97 (0.03)	1.00 (0.00)	1.00 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)
1b (1,0,0)	r	1.00 (0.01)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (1,0,0)	d	1.01 (0.02)	1.02 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.04)	0.99 (0.03)
1b (0,1,0)	a	1.00 (0.01)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1b (0,0,1)	a	1.00 (0.01)	1.01 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.99 (0.03)
1b (0,0,1)	r	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)
1b (0,0,1)	d	1.00 (0.01)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.04)	0.99 (0.03)
1c (0,0,0)	a	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (1,0,0)	a	1.00 (0.01)	1.02 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (1,0,0)	r	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (1,0,0)	d	1.00 (0.01)	1.01 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (0,1,0)	a	1.00 (0.01)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (0,0,1)	a	1.00 (0.01)	1.01 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (0,0,1)	r	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
1c (0,0,1)	d	1.00 (0.01)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.

**S3 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in binary phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
1d	(0,0,0)	a	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(1,0,0)	a	1.00	1.01	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(1,0,0)	r	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(1,0,0)	d	1.00	1.01	0.99	1.00	1.02	0.98	1.00	1.00	1.00	1.00	1.00
			(0.02)	(0.03)	(0.02)	(0.00)	(0.03)	(0.03)	(0.01)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(0,1,0)	a	1.00	1.00	1.00	1.00	0.99	1.01	1.00	1.00	1.00	1.00	1.00
			(0.02)	(0.03)	(0.02)	(0.00)	(0.03)	(0.03)	(0.01)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(0,0,1)	a	1.02	1.03	0.99	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00
			(0.02)	(0.03)	(0.02)	(0.00)	(0.03)	(0.04)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(0,0,1)	r	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
1d	(0,0,1)	d	1.01	1.03	0.99	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00
			(0.02)	(0.03)	(0.02)	(0.00)	(0.03)	(0.03)	(0.01)	(0.02)	(0.02)	(0.00)	(0.03)
2a	(1,0,0)	r	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.01	0.99	1.00	1.03
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2a	(1,0,0)	d	1.00	1.01	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2a	(0,0,1)	r	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2a	(0,0,1)	d	1.00	1.01	1.00	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.01)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(0,0,0)	a	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(1,0,0)	a	1.00	1.02	0.99	1.00	1.03	0.97	1.00	1.00	1.00	1.00	1.00
			(0.02)	(0.03)	(0.02)	(0.00)	(0.04)	(0.04)	(0.01)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(1,0,0)	r	1.00	1.00	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(1,0,0)	d	1.00	1.02	0.99	1.00	1.03	0.98	1.00	1.00	1.00	1.00	1.01
			(0.01)	(0.03)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(0,1,0)	a	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00
			(0.01)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(0,0,1)	a	1.00	1.01	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00	1.00
			(0.01)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(0,0,1)	r	1.00	1.00	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00	1.00
			(0.00)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2b	(0,0,1)	d	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
			(0.01)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2c	(1,0,0)	r	0.89	0.90	0.99	1.00	1.01	0.99	0.85	0.86	1.00	1.00	1.00
			(0.02)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2c	(1,0,0)	d	0.97	0.98	0.99	1.00	1.01	0.99	0.91	0.91	1.00	1.00	1.00
			(0.02)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2c	(0,0,1)	r	0.86	0.86	1.00	1.00	1.00	1.00	0.85	0.85	1.00	1.00	1.00
			(0.01)	(0.02)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.02)	(0.00)	(0.03)
2c	(0,0,1)	d	0.87	0.88	0.99	1.00	1.01	1.00	0.88	0.89	0.99	1.00	1.01
			(0.02)	(0.03)	(0.02)	(0.00)	(0.03)	(0.03)	(0.00)	(0.02)	(0.03)	(0.00)	(0.03)

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**S3 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in binary phenotype simulations**

Scenario			GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M		$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
2d (1,0,0)	r		1.02 (0.32)	1.01 (0.23)	1.00 (0.06)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.02 (0.16)	1.02 (0.12)	1.00 (0.04)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2d (1,0,0)	d		1.03 (0.29)	1.03 (0.21)	0.99 (0.06)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.03 (0.18)	1.02 (0.14)	1.01 (0.04)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2d (0,0,1)	r		1.74 (1.09)	1.52 (0.76)	1.09 (0.12)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	2.00 (0.97)	1.98 (0.95)	1.01 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2d (0,0,1)	d		9.68 (9.66)	5.68 (5.14)	1.35 (0.41)	0.99 (0.01)	0.99 (0.03)	1.00 (0.03)	1.97 (0.95)	1.93 (0.92)	1.02 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,0)	a		1.05 (0.02)	1.05 (0.03)	1.01 (0.02)	1.00 (0.00)	1.00 (0.03)	1.01 (0.03)	1.05 (0.04)	1.04 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (1,0,0)	a		1.06 (0.06)	1.07 (0.06)	0.99 (0.02)	1.00 (0.00)	1.03 (0.03)	0.97 (0.03)	1.04 (0.05)	1.03 (0.05)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (1,0,0)	r		1.06 (0.02)	1.06 (0.03)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.04 (0.03)	1.03 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (1,0,0)	d		1.06 (0.03)	1.06 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.05 (0.07)	1.04 (0.07)	1.00 (0.02)	1.00 (0.00)	1.00 (0.04)	1.00 (0.04)
2e (0,1,0)	a		1.33 (1.91)	1.42 (1.04)	0.87 (0.10)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	11.60 (3.00)	8.88 (1.38)	1.29 (0.15)	0.98 (0.00)	0.98 (0.03)	1.00 (0.03)
2e (0,0,1)	a		1.06 (0.02)	1.06 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	4.20 (2.32)	4.14 (1.92)	0.97 (0.10)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,1)	r		1.06 (0.04)	1.07 (0.04)	0.99 (0.03)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.04 (0.04)	1.04 (0.05)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
2e (0,0,1)	d		1.25 (1.41)	1.32 (0.78)	0.90 (0.08)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	9.76 (2.62)	8.06 (1.62)	1.19 (0.11)	0.99 (0.00)	0.99 (0.03)	1.00 (0.03)
3a (0,0,0)	a		1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (1,0,0)	a		1.01 (0.02)	1.02 (0.03)	0.99 (0.02)	1.01 (0.00)	1.04 (0.03)	0.97 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (1,0,0)	r		1.01 (0.01)	1.01 (0.03)	1.00 (0.03)	1.01 (0.00)	1.02 (0.03)	0.99 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)
3a (1,0,0)	d		1.01 (0.01)	1.02 (0.02)	0.99 (0.02)	1.01 (0.00)	1.02 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.01 (0.03)
3a (0,1,0)	a		1.01 (0.01)	1.01 (0.02)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,1)	a		1.03 (0.02)	1.04 (0.03)	0.99 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,1)	r		1.01 (0.01)	1.01 (0.02)	1.00 (0.02)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3a (0,0,1)	d		1.01 (0.01)	1.02 (0.03)	0.99 (0.03)	1.01 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.

**S3 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in binary phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
3b (0,0,0)	a	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (1,0,0)	a	1.00 (0.01)	1.02 (0.03)	0.98 (0.02)	1.00 (0.00)	1.03 (0.04)	0.97 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (1,0,0)	r	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (1,0,0)	d	1.00 (0.01)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (0,1,0)	a	1.00 (0.04)	1.00 (0.05)	1.00 (0.02)	1.00 (0.01)	1.01 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (0,0,1)	a	1.03 (0.01)	1.02 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.01 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3b (0,0,1)	r	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)
3b (0,0,1)	d	1.01 (0.01)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.04)	1.00 (0.04)	1.01 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (0,0,0)	a	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (1,0,0)	a	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (1,0,0)	r	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (1,0,0)	d	1.00 (0.00)	1.02 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (0,1,0)	a	1.00 (0.00)	1.00 (0.02)	1.00 (0.03)	1.00 (0.00)	1.00 (0.04)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (0,0,1)	a	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3c (0,0,1)	r	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.03 (0.03)	0.98 (0.03)
3c (0,0,1)	d	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.03 (0.04)	0.97 (0.04)
3d (0,0,0)	a	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3d (1,0,0)	a	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3d (1,0,0)	r	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)
3d (1,0,0)	d	1.00 (0.01)	1.02 (0.03)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3d (0,1,0)	a	1.00 (0.03)	1.00 (0.04)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.01)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3d (0,0,1)	a	1.01 (0.01)	1.01 (0.03)	1.00 (0.02)	1.00 (0.00)	1.03 (0.04)	0.97 (0.03)	1.03 (0.01)	1.03 (0.02)	1.00 (0.02)	1.00 (0.00)	0.99 (0.03)	1.00 (0.03)
3d (0,0,1)	r	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)
3d (0,0,1)	d	1.01 (0.01)	1.02 (0.02)	0.99 (0.02)	1.00 (0.00)	1.03 (0.03)	0.97 (0.03)	1.02 (0.01)	1.02 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.

**S3 Table (continued). Comparison between  $l_{approx}$  and  $l_{mean}$  in binary phenotype simulations**

Scenario		GxE ( $n = 1000$ )			Marginal ( $n = 1000$ )			GxE ( $n = 10000$ )			Marginal ( $n = 10000$ )		
$(b_G, b_{GE}, b_Z)$	M	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$	$l_{approx}$	$l_{mean}$	$\frac{l_{approx}}{l_{mean}}$
4a	(0,0,0)	a	1.00 (0.00)	1.00 (0.07)	1.01 (0.07)	1.00 (0.00)	1.00 (0.10)	1.01 (0.09)	1.00 (0.00)	1.01 (0.07)	1.00 (0.07)	1.00 (0.00)	1.01 (0.09)
4a	(1,0,0)	a	1.00 (0.02)	1.19 (0.09)	0.84 (0.06)	1.00 (0.00)	1.39 (0.17)	0.73 (0.09)	1.00 (0.01)	2.25 (0.22)	0.45 (0.05)	1.00 (0.00)	3.58 (0.42)
4a	(1,0,0)	r	1.00 (0.00)	1.00 (0.06)	1.01 (0.06)	1.00 (0.00)	1.01 (0.09)	1.00 (0.08)	1.00 (0.00)	1.01 (0.06)	1.00 (0.06)	1.00 (0.00)	1.01 (0.09)
4a	(1,0,0)	d	1.00 (0.01)	1.05 (0.07)	0.96 (0.07)	1.00 (0.00)	1.10 (0.12)	0.92 (0.10)	1.00 (0.00)	1.59 (0.14)	0.63 (0.06)	1.00 (0.00)	2.17 (0.27)
4a	(0,1,0)	a	1.00 (0.02)	1.00 (0.07)	1.01 (0.07)	1.00 (0.00)	1.00 (0.09)	1.01 (0.09)	1.00 (0.01)	1.01 (0.07)	1.00 (0.07)	1.00 (0.00)	1.01 (0.09)
4a	(0,0,1)	a	1.01 (0.01)	1.12 (0.09)	0.91 (0.07)	1.00 (0.00)	1.00 (0.09)	1.01 (0.09)	1.02 (0.01)	2.28 (0.16)	0.45 (0.03)	1.00 (0.00)	0.99 (0.09)
4a	(0,0,1)	r	1.00 (0.00)	1.02 (0.07)	0.99 (0.06)	1.00 (0.00)	1.01 (0.10)	1.00 (0.09)	1.00 (0.00)	1.17 (0.10)	0.86 (0.07)	1.00 (0.00)	1.01 (0.08)
4a	(0,0,1)	d	1.02 (0.02)	1.09 (0.09)	0.94 (0.07)	1.00 (0.00)	1.01 (0.09)	1.00 (0.09)	1.01 (0.01)	1.85 (0.16)	0.55 (0.05)	1.00 (0.00)	1.01 (0.09)
4b	(0,0,0)	a	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.01 (0.04)	1.00 (0.04)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)
4b	(1,0,0)	a	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.99 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)
4b	(1,0,0)	r	1.00 (0.00)	1.01 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)
4b	(1,0,0)	d	1.00 (0.01)	1.01 (0.02)	0.99 (0.02)	1.00 (0.00)	1.02 (0.03)	0.98 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)
4b	(0,1,0)	a	1.00 (0.02)	1.00 (0.03)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)
4b	(0,0,1)	a	1.02 (0.02)	1.03 (0.03)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.02 (0.01)	1.02 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)
4b	(0,0,1)	r	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.00 (0.02)	1.00 (0.02)	1.00 (0.00)	1.00 (0.03)
4b	(0,0,1)	d	1.01 (0.01)	1.02 (0.02)	0.99 (0.02)	1.00 (0.00)	1.00 (0.03)	1.00 (0.03)	1.00 (0.00)	1.02 (0.02)	0.98 (0.02)	1.00 (0.00)	1.00 (0.03)
4c	(0,0,0)	a	1.00 (0.00)	1.00 (0.05)	1.00 (0.05)	1.00 (0.00)	1.00 (0.07)	1.01 (0.07)	1.00 (0.00)	1.00 (0.06)	1.00 (0.06)	1.00 (0.00)	1.00 (0.09)
4c	(1,0,0)	a	1.00 (0.01)	1.11 (0.08)	0.91 (0.06)	1.00 (0.00)	1.23 (0.13)	0.83 (0.09)	1.00 (0.00)	1.07 (0.06)	0.94 (0.06)	1.00 (0.00)	1.14 (0.11)
4c	(1,0,0)	r	1.00 (0.00)	1.01 (0.05)	1.00 (0.05)	1.00 (0.00)	1.01 (0.07)	1.00 (0.07)	1.00 (0.00)	1.27 (0.09)	0.79 (0.06)	1.00 (0.00)	1.55 (0.17)
4c	(1,0,0)	d	1.00 (0.01)	1.09 (0.06)	0.92 (0.06)	1.00 (0.00)	1.18 (0.11)	0.86 (0.08)	1.00 (0.00)	1.25 (0.10)	0.80 (0.06)	1.00 (0.00)	1.50 (0.17)
4c	(0,1,0)	a	1.00 (0.02)	1.00 (0.05)	1.01 (0.05)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)	1.00 (0.01)	1.00 (0.05)	1.00 (0.05)	1.00 (0.00)	1.01 (0.09)
4c	(0,0,1)	a	1.03 (0.02)	1.14 (0.08)	0.91 (0.06)	1.00 (0.00)	0.99 (0.07)	1.02 (0.07)	1.00 (0.00)	1.06 (0.06)	0.95 (0.05)	1.00 (0.00)	1.01 (0.08)
4c	(0,0,1)	r	1.00 (0.00)	1.01 (0.05)	1.00 (0.05)	1.00 (0.00)	1.00 (0.08)	1.01 (0.08)	1.00 (0.00)	1.02 (0.06)	0.98 (0.06)	1.00 (0.00)	1.01 (0.08)
4c	(0,0,1)	d	1.01 (0.01)	1.06 (0.06)	0.95 (0.06)	1.00 (0.00)	1.01 (0.08)	0.99 (0.08)	1.01 (0.00)	1.69 (0.14)	0.60 (0.05)	1.00 (0.00)	1.00 (0.08)

Mean and standard deviation (SD) for  $l_{approx}$ ,  $l_{mean}$  and the ratio  $l_{approx}/l_{mean}$  are shown over 200 simulation runs for gene-environment (GxE) interaction and marginal association analyses for two sample sizes,  $n = 1000$  and  $10000$ . M, genotype model (a: additive, r: recessive, d: dominant).  $b_G$ ,  $b_{GE}$  and  $b_Z$  are effect size parameters of genotype, gene-environment interaction and covariate, respectively; 0 and 1 denote absence and presence of effect.