Supplementary Materials

A. *Simplified Granule Cell Model Parameters*

TABLE I
PARAMETERS FOR SIMPLIFIED GRANULE CELL MODEL

Mechanism	Soma	GCL	Inner Third	Middle Third	OuterThird
C_m (µm/cm^2)	9.8	17.0	39.96	65.07	44.79
Ra (Ω-cm)	410	293.7	380.1	222.6	90.37
Sodium (S/cm^2)	0.84	0.2186	0.2319	0.2324	0.0
Slow delayed rectifier K^+ (S/cm^2)	6.0e-3	1.041e-2	1.529e-2	2.490e-2	2.285e-2
Fast delayed rectifier K^+ (S/cm^2)	3.6e-2	6.94e-3	1.019e-2	4.150e-3	2.856e-3
A-type K^+ (S/cm^2)	9.0e-3	-	-	-	-
L-type Ca^{2+} (S/cm^2)	2.5e-3	1.301e-2	1.911e-2	2.075e-3	0.0
N-type Ca^{2+} (S/cm^2)	7.35e-4	3.828e-3	1.874e-3	3.051e-3	2.100e-3
T-type Ca^{2+} (S/cm^2)	7.4e-5	1.301e-4	6.370e-4	2.075e-3	2.856e-3
Ca-dependent K^+ (S/cm^2)	6.0e-3	6.940e-4	5.097e-4	0.0	0.0
Ca- and V-dependent K^+ (S/cm^2)	7.0e-4	2.082e-4	5.097e-4	1.992e-3	1.371e-3
Leak (S/cm^2)	2.9e-4	5.034e-4	1.165e-3	1.896e-3	1.305e-3
Tau for decay of intracell. Ca^{2+} (ms)	10.0	10.0	10.0	10.0	10.0
Steady-state intracell. Ca^{2+} (mM)	5.0e-6	5.0e-6	5.0e-6	5.0e-6	5.0e-6

B. *Recursive Form for Legendre Polynomials to Represent Rate Maps*

Recursive formulas for computing the Legendre polynomials and their first and second derivatives were obtained by taking the derivative of the generating function with respect to *t* and then taking the derivatives with respect to *x*. The rate maps were represented by a two-dimensional Legendre polynomial of order 16.

Generating function

$$g(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum_{j=0}^{\infty} P_j(x)t^j,$$

where $P_j(x)$ denotes the Legendre polynomial of order *j* evaluated at *x*.

Recursive form of the Legendre polynomial

$$P_{j+1} = \frac{(2j+1)xP_j - jP_{j-1}}{j+1}$$

Recursive form of the first derivative of the Legendre polynomial

$$P_j' = \frac{jxP_j - jP_{j-1}}{x^2-1}$$

Recursive form of the second derivative of the Legendre polynomial

$$P_j'' = \frac{jP_j + x(j-2)P_j' - jP_j'}{x^2-1}$$

C. *Lower Bound of Mutual Information*

The derivation of the lower bound of mutual information is fully described in [1]. A summary can be found below. The derivation begins with the data processing inequality

$$I[x;r] \geq I[x;\hat{x}(r)] = H[x] - H[x|\hat{x}]$$

where $H[x]$ represents the entropy of the prior stimulus, e.g., rat position, and $H[x|r]$ represents the noise entropy, or the average residual entropy in *x* conditioned on the spiking activity r. Mutual information is represented by $I[x;r]$, and $I[x;\hat{x}(r)]$ denotes the lower bound of the mutual information. Finally, $H[x|\hat{x}]$ is the average residual entropy in *x* conditioned on the estimate for *x*, denoted by $\hat{x}$.

The entropy for the rat position was computed using the formula detailed by Barbieri et al., 2004 [37] as follows:

$$H[x] = \frac{1}{2}\log_2[(2\pi e)^2|W_\varepsilon|]$$

where W_ε is the covariance matrix from the autoregressive model of the rat trajectory. The residual entropy was calculated the formula described by Pillow et al., 2000 [39]:

$$H[x|\hat{x}] = \frac{1}{2}\log_2|E[r\cdot r^T]| + \frac{1}{2}\log_2(2\pi e).$$

In this formulation, *r* represents the residual of the estimator as $r = x - \hat{x}$, and $E[r\cdot r^T]$ represents the covariance of the residuals. The final lower bound for mutual information can then be expressed as

$$I[x;r] \geq \frac{1}{2}[\log_2[(2\pi e)^2|W_\varepsilon|] - (\log_2|E[r\cdot r^T]| + \log_2(2\pi e))].$$

D. References

[1] J. W. Pillow, Y. Ahmadian, and L. Paninski, “Model-Based Decoding, Information Estimation, and Change-Point Detection Techniques for Multineuron Spike Trains,” *Neural Comput.*, vol. 23, no. 1, pp. 1–45, Oct. 2010.