# Supplementary Materials

### Note S1. The relationship between the electrode-skin interface impedance and resistance

The impedance of the electrode-skin interface is usually determined by using an RC circuit model. As the capacitor charges, the voltage across the electrode-skin interface will eventually reach a steady state value. The impedance of the electrode-skin interface is defined as the steady state voltage divided by the applied current amplitude (Fig. 1B) [1,9]. However, pulse durations used in electrotactile stimulation are typically not long enough for the voltage to reach steady state [1,11]. As a result, researchers use pre-steady state voltage measurements and divide them by the applied current amplitude to obtain a resistance.

### Note S2. The modeling experiment results are validated by previous studies

In order to validate that the recorded current amplitudes for a given pulse duration during the modeling experiments match the recorded values in previous studies, we compared $I$ vs. $T$ across all subjects, sessions, magnitudes of sensation, stimulation locations, and electrode sizes. They follow the logarithmic trend mentioned in [1,10,18] (Supplementary Fig. S1A). If we take the average of the slopes of the best fit lines of the log of the data, we obtain a value of -0.497, which very closely matches the -0.5 slope that Tachi et al. reports in [10]. In Supplementary Fig. S1B, we show a subset of the data plotted with their best fit lines that have slopes constrained to -0.5. The average $R^2$ value of the constrained best fit lines for all subjects and conditions is 0.956, indicating a strong fit. Therefore, our results when comparing $I$ and $T$ are validated by their consistency with previous studies.

### Note S3. Only keeping $I^2T$ constant does not keep perceived sensation intensity constant

Tachi et al. [10] similarly found that $I^2T$ is constant at constant perceived sensation intensity. However, an erroneous assumption led them to a different conclusion. As in our study, Tachi et al. adjusted pulse durations of monophasic square pulses and found the amount of current needed to reach stimulation threshold for three subjects. They reported that for pulse durations under 1 ms there was a logarithmic trend between $I$ and $T$, that when plotted on a log-log scale resulted in a linear curve with slope -0.5. This is validated by our data as well, which when linearly fit with slopes of -0.5 achieved an $R^2$ of 0.956 (Supplementary Fig. S1B). For a specific waveform at a constant sensation intensity, the relationship between $I$ and $T$ can be written as $\log I = -0.5 \log T + c$, where $c$ is a constant term. It follows that $I = c'T^{-0.5}$, and after squaring both sides and rearranging terms, we obtain $I^2T = c''$. This means that for a given magnitude of sensation, the value of $I^2T$ always remains constant. Under the assumption that the impedance of the electrode-skin interface was constant over the course of an experimental trial, Tachi et al. reached the conclusion that the energy, $E = ZI^2T$, where $Z$ is the impedance of the electrode-skin interface, is also constant for a constant sensation intensity. This assumption fails because it is well-studied that changing $I$, as Tachi et al. did, affects the impedance significantly [1,13]. Therefore, for two different values of $I$, regardless of whether $I^2T$ is constant, $Z$ would not be the same. In his implementation of the constant energy controller, when the value of $Z$ changed over time, $I$ would be adjusted to maintain a constant $E$ for a fixed $T$. Consequently, the value of $I^2T$ will always be changing while trying to maintain a constant level of sensation. This contradicts the notion that $I^2T$ must be held constant in order to maintain a constant level of sensation, as evidenced by the data of Tachi et al. as well as ours.

### Note S4. The point of convergence was computed using a grid search and gradient ascent

Since the linear relationships for $Q$ vs $R_p$ appear to originate from a common point, we sought to solve for the point of convergence which maximizes the average $R^2$ across all subjects and conditions when the linear regression is constrained to go through this point. The problem can be modeled using the linear equation,

$$y_{ki} - y^* = m_k^*(x_{ki} - x^*) \ ,$$

where $k$ is the trial (line) number, $i$ is the $i$-th point within that trial, $x_{ki}$ is the $R_p$ value for a specific point in a trial, $y_{ki}$ is the value of $Q$ for a specific point in a trial, ($x^*$, $y^*$) is the ordered pair representing the point of convergence of all the lines, and $m_k^*$ is the slope of the line for a specified trial. We solved for $m_k^*$, $y^*$, and $x^*$, such that the $R^2$ value for each trial is maximized,

$$\max_{m_k^*, y^*, x^*} \sum_k R_k^2 \ ,$$

where

$$R_k^2 = 1 - \frac{\sum_i (y_{ki} - y^* - m_k^*(x_{ki} - x^*))^2}{\sum_i (y_{ki} - \bar{y}_{ki})^2} \ .$$

The numerator in the second term of the equation above is the sum squared error determined by the linear model we are using, while the denominator is the total sum of squares. The $m_k^* x^*$ term makes the objective function nonlinear, and it can be shown that the Hessian of the objective function is not positive semidefinite, which means that the function is nonconvex. As a result, we used gradient ascent to find a locally optimal solution to the maximization problem. For our initial guess of $x^*$, a grid search was performed between $\pm 3$ kΩ and $\pm 3$ μC in steps of 0.25. These limits were chosen since in our previous work [17], the locally optimal point was found to be (-0.81 kΩ, 0.21 μC). Step sizes for updating $x^*$, $y^*$ and $m_k^*$ on successive iterations were chosen to be $10^{-5}$, $10^{-7}$, and $10^{-6}$, respectively. The algorithm ran for $10^6$ iterations before stopping. Each initial guess converged to (-1.67 kΩ, 0.186 μC) for ($x^*$, $y^*$). Using this result, linear regression was applied to each of the trials with the constraint that the line must go through this new point of convergence. The $R^2$ values from the constrained linear regression are reported in Fig. 2I.

**Note S5. Use of a vibrotactile reference sensation is validated by consistent results**

The large values of $R^2$ from the experiments using a vibrotactile reference sensation suggest that vibrotactile stimulation can perform well as a reference for electrotactile stimulation, since poor comparisons between electrotactile and vibrotactile sensation intensities would have resulted in poor average $R^2$ values. The results after stair ascent and descent using vibrotactile stimulation as a reference produced $R^2$ values greater than 0.9 (Fig. 4E), similar to the results when using an electrotactile reference. As a result, we can assume that the change in skin impedance after exercise did not have a large impact in the perceived vibrotactile sensation intensity, since changes in the reference sensation intensity would have resulted in poor $R^2$ values. However, further study is necessary to directly test how well subjects can compare vibrotactile and electrotactile sensation intensities, as well as the effect of skin impedance changes on vibrotactile sensation intensity.

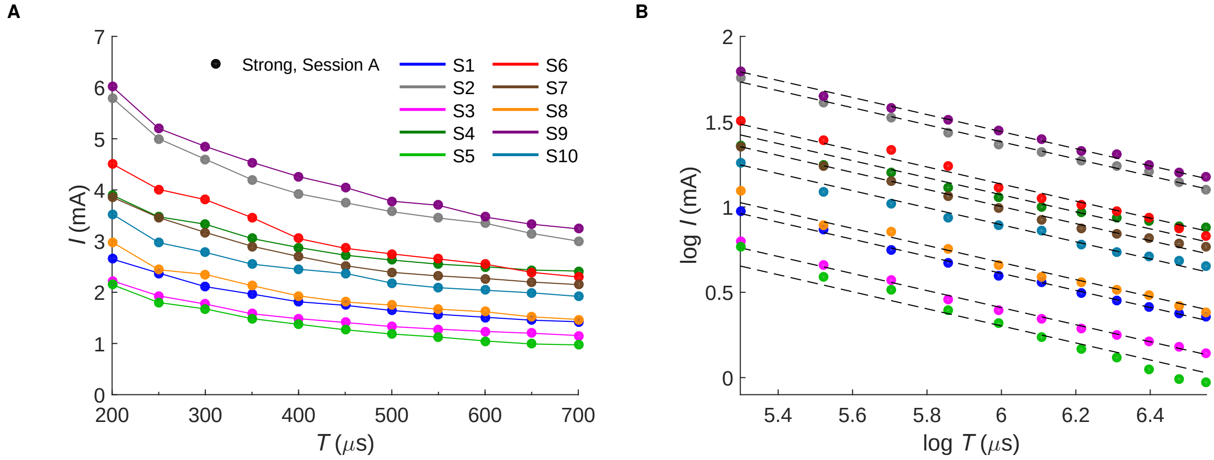


**Fig. S1 Validation of Exp. 1 modeling results.** **(A)** Current ($I$) vs. pulse duration ($T$) for ten subjects without arm impairment. The data points for each subject represent stimulation parameters having the same perceived sensation intensity. For clarity, only the data from Session A at the strong magnitude of sensation are shown. **(B)** Linear relationships from the same data when plotted on a log-log scale. The best fit lines shown are constrained to a slope of -0.5.

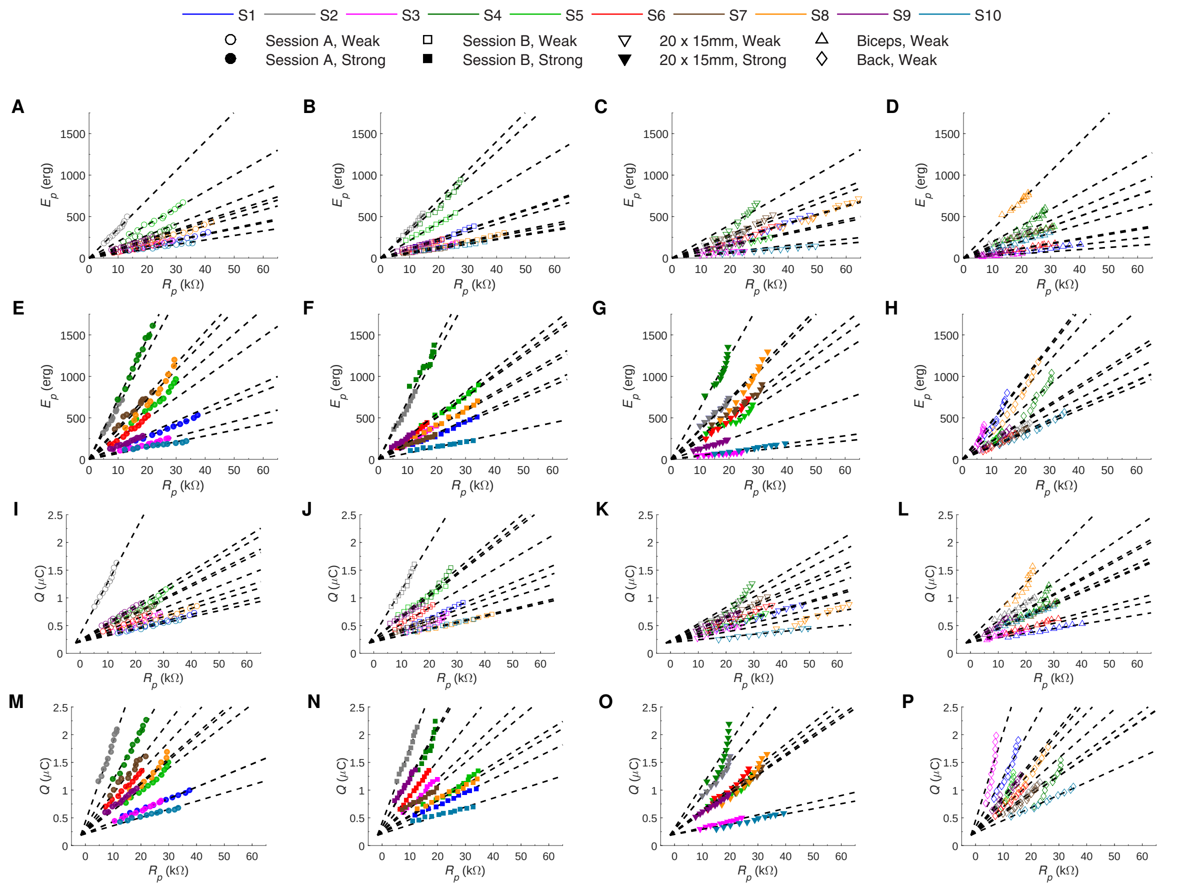


**Fig. S2 Experiment 1 modeling results.** Data is shown for ten subjects without arm impairment for peak pulse energy ($E_p$) and phase charge ($Q$) vs. peak resistance ($R_p$) showing linear trends across each of the eight conditions constrained to the origin for $E_p$ and a point of optimal convergence (-1.67 kΩ, 0.186 μC) for $Q$. Each sensation felt at each data point of the same color and marker was equivalent in subjective intensity.

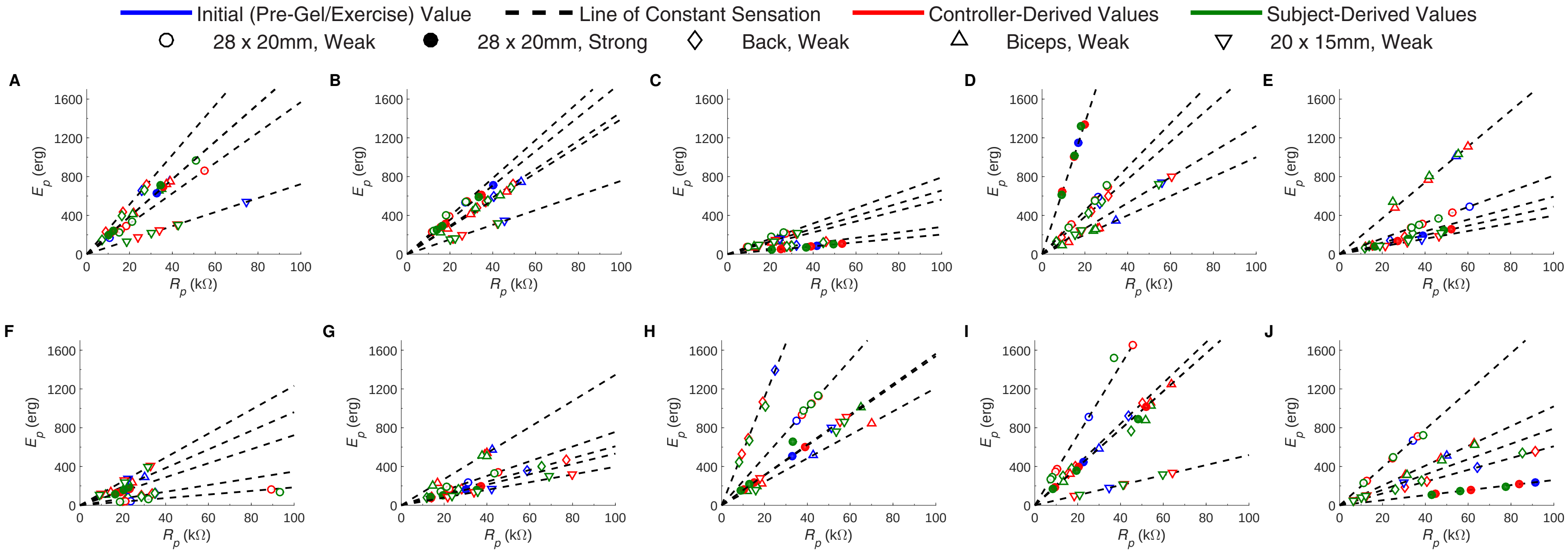


**Fig. S3 Experiment 2A controller results ($E_p$ vs. $R_p$).** Data is shown for ten subjects without arm impairment verifying the linear relationships at constant sensation for peak pulse energy ($E_p$) in response to changes in peak resistance ($R_p$). Each plot shows data from one of the ten subjects across all five conditions. Electroconductive gel was applied or removed to change $R_p$. For each of the five conditions, the subjects were asked to match an electrotactile reference sensation intensity over three trials with differing values of $R_p$.

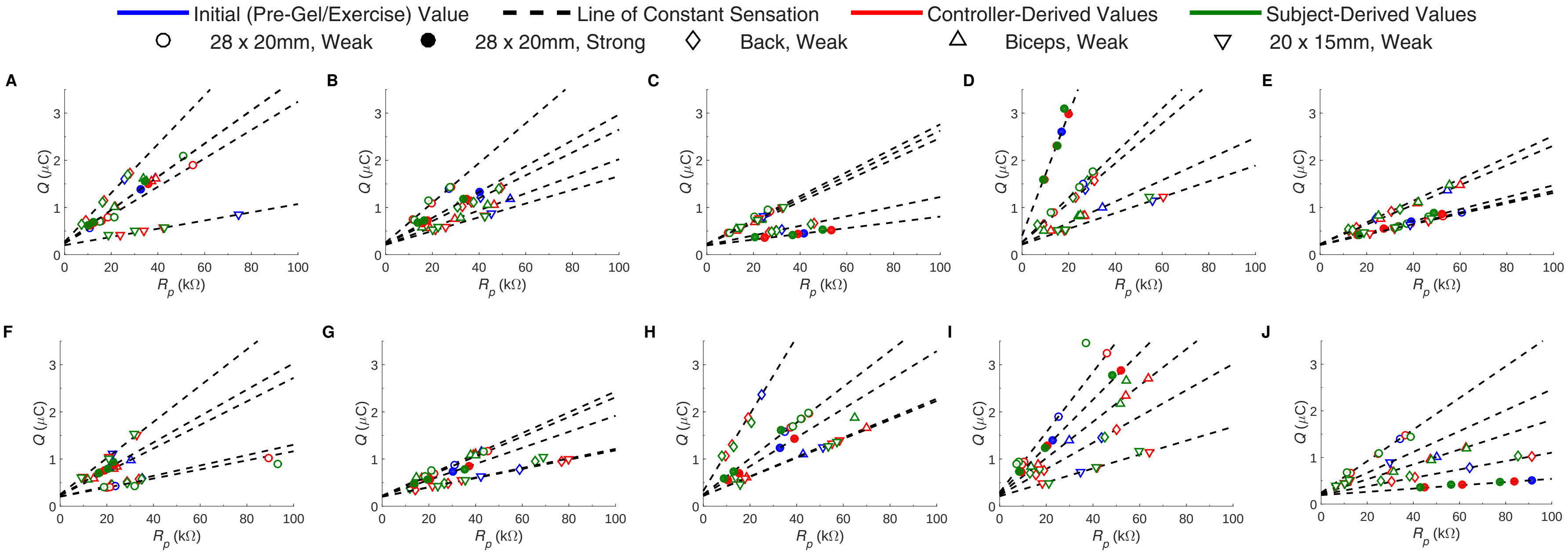


**Fig. S4 Experiment 2A results ($Q$ vs. $R_p$).** Data is shown for ten subjects without arm impairment verifying the linear relationships at constant sensation for phase charge ($Q$) in response to changes in peak resistance ($R_p$). Each plot shows data from one of the ten subjects across all five conditions. Electroconductive gel was applied or removed to change $R_p$. For each of the five conditions, the subjects were asked to match an electrotactile reference sensation intensity over three trials with differing values of $R_p$.

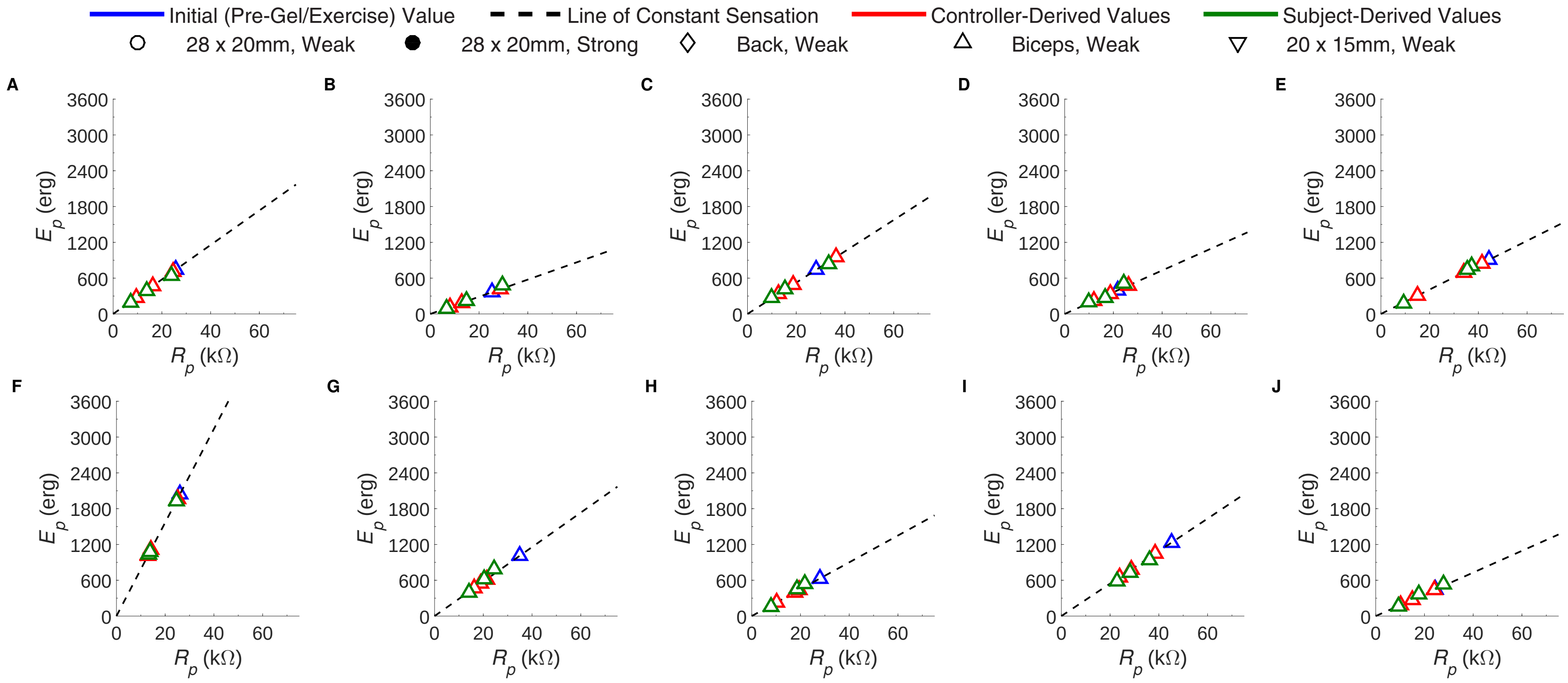


**Fig. S5 Experiment 2B controller results ($E_p$ vs. $R_p$, vibrotactile reference).** Data is shown for nine subjects without arm impairment and one subject with a right below-elbow amputation verifying the linear relationships at constant sensation for peak pulse energy ($E_p$) in response to changes in peak resistance ($R_p$). **(A)-(I)** each show data from one of the nine subjects without arm impairment. **(J)** shows data from a subject with a right below-elbow amputation. Electroconductive gel was applied or removed to change $R_p$. For each of the five conditions, the subjects were asked to match a vibrotactile reference sensation intensity over three trials with differing values of $R_p$.

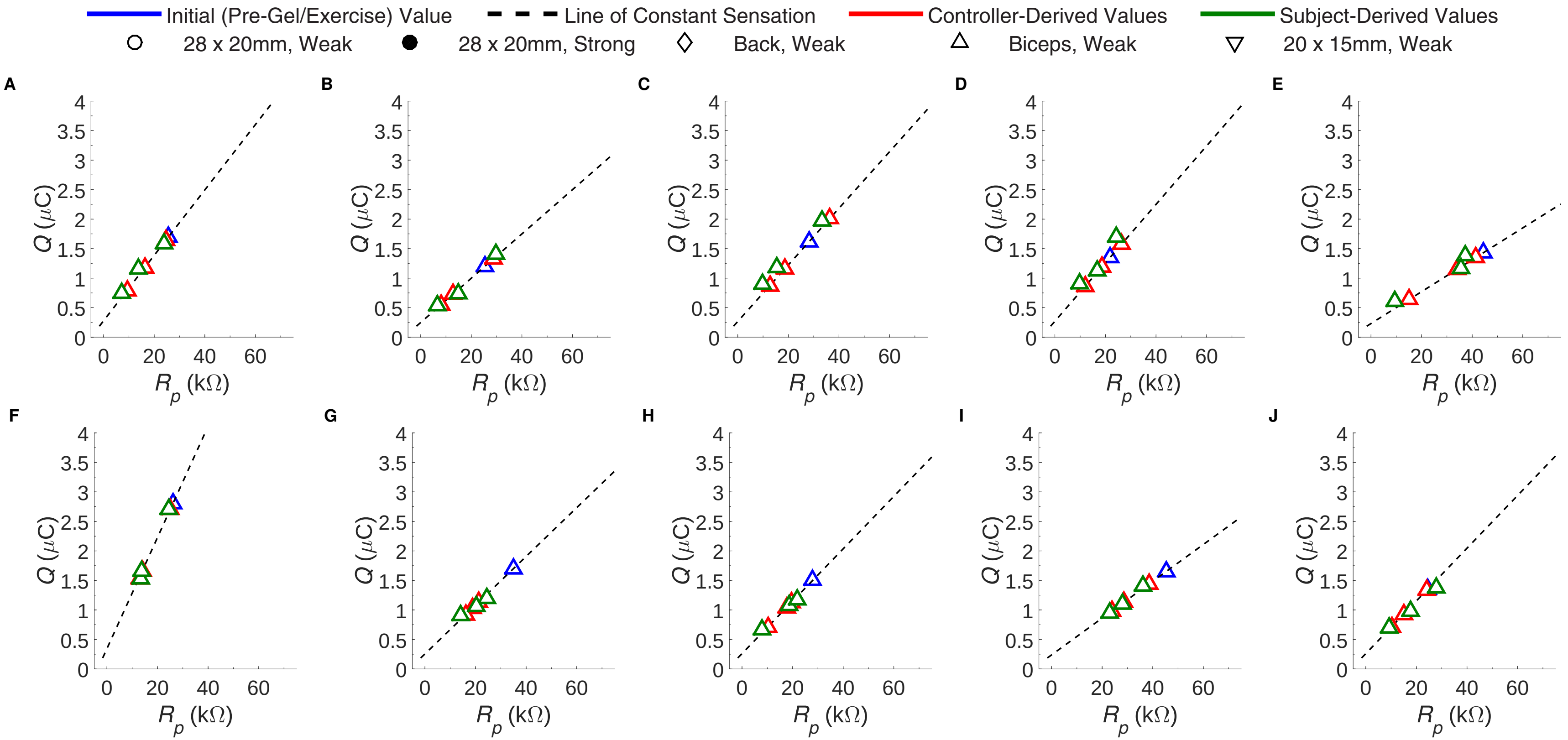


**Fig. S6 Experiment 2B controller results** (*Q* **vs.** $R_p$, **vibrotactile reference).** Data is shown for nine subjects without arm impairment and one subject with a right below-elbow amputation verifying the linear relationships at constant sensation for phase charge (*Q*) in response to changes in peak resistance ($R_p$). **(A)**-**(I)** each show data from one of the nine subjects without arm impairment. **(J)** shows data from a subject with a right below-elbow amputation. Electroconductive gel was applied or removed to change $R_p$. For each of the five conditions, the subjects were asked to match a vibrotactile reference sensation intensity over three trials with differing values of $R_p$.

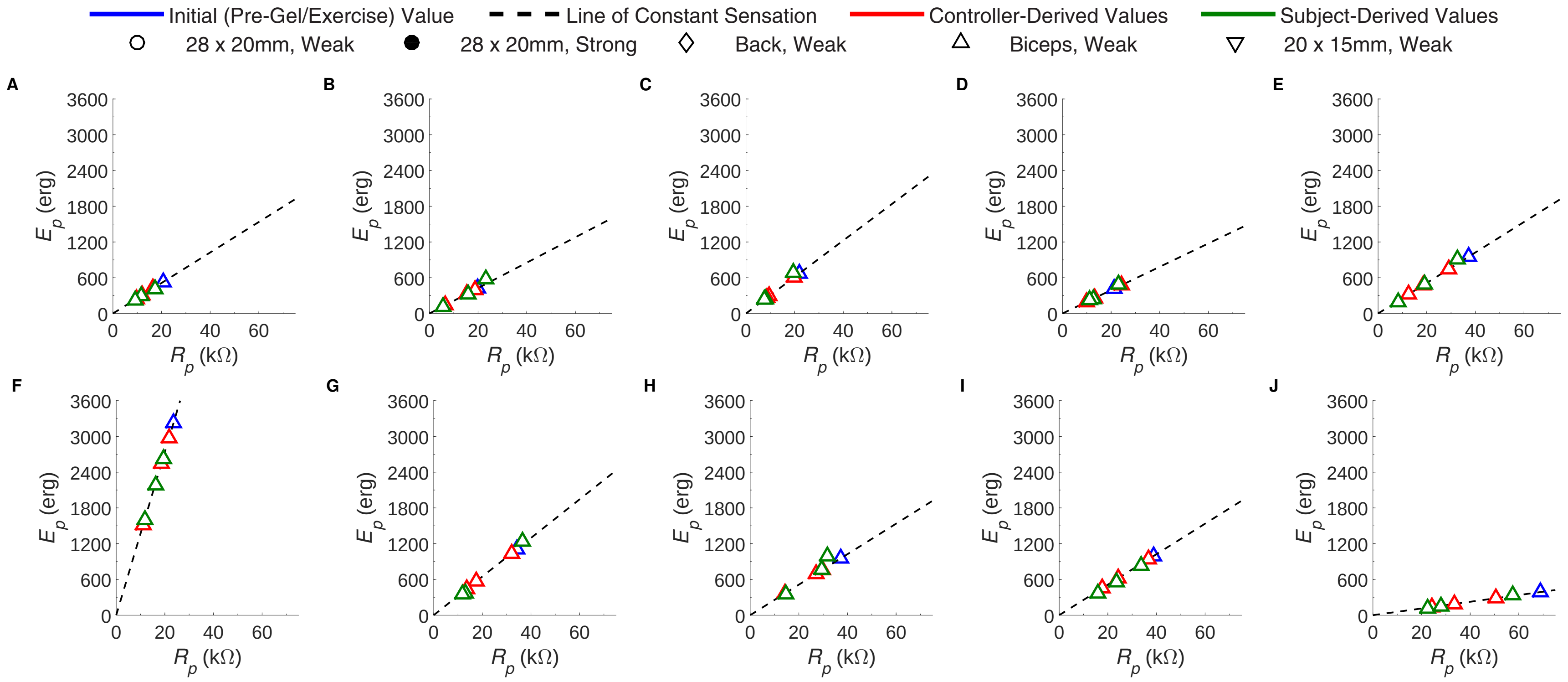


**Fig. S7 Experiment 2C controller results ($E_p$ vs. $R_p$, stair ascent/descent).** Data is shown for nine subjects without arm impairment and one subject with a right below-elbow amputation verifying the linear relationships at constant sensation for peak pulse energy ($E_p$) in response to changes in peak resistance ($R_p$). **(A)-(I)** each show data from one of the nine subjects without arm impairment. **(J)** shows data from a subject with a right below-elbow amputation. Subjects ascended and descended stairs to reduce $R_p$ and subsequently rested for 10 minutes to raise $R_p$. For each of the five conditions, the subjects were asked to match a vibrotactile reference sensation intensity over three trials with differing values of $R_p$.

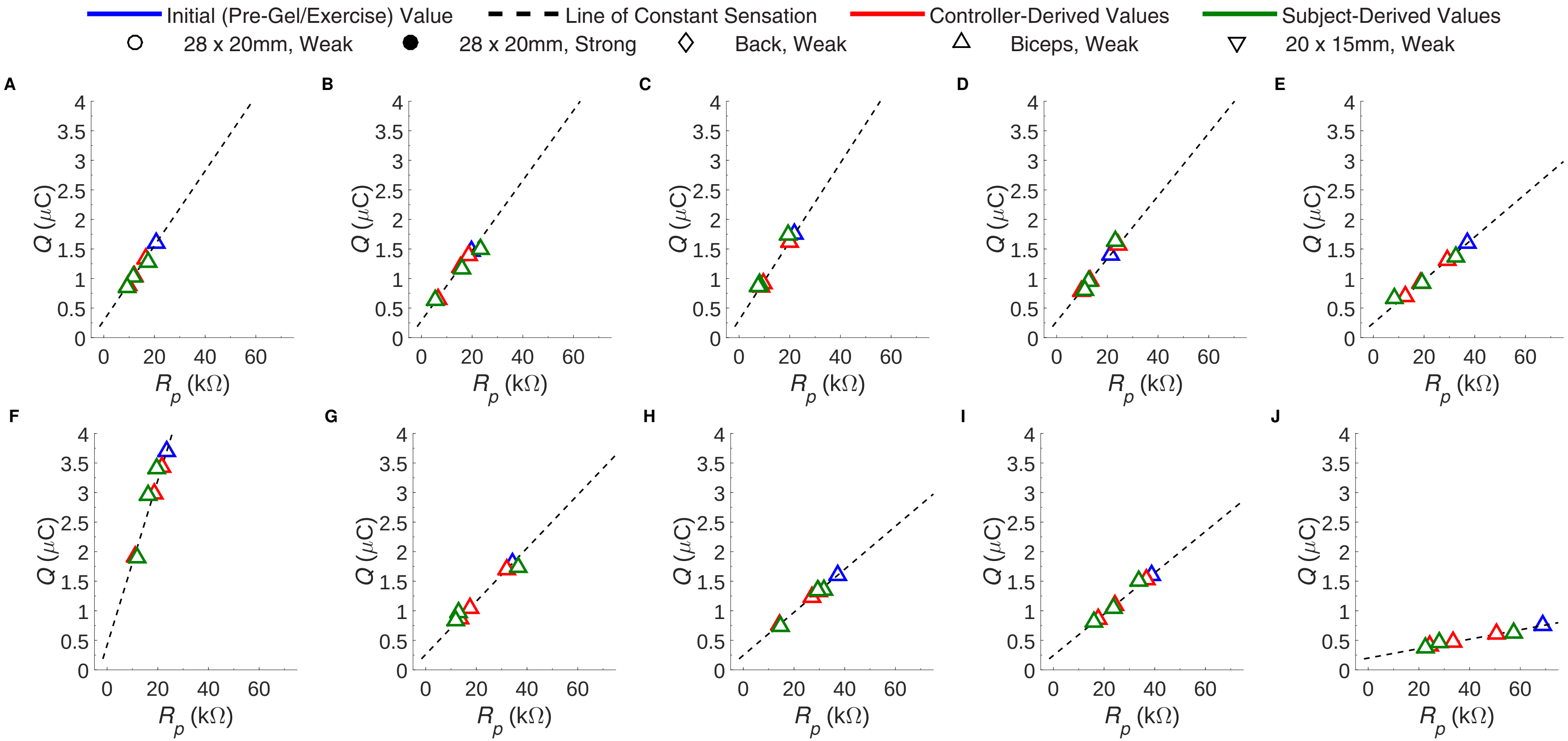


**Fig. S8 Experiment 2C controller results ($Q$ vs. $R_p$, stair ascent/descent).** Data is shown for nine subjects without arm impairment and one subject with a right below-elbow amputation verifying the linear relationships at constant sensation for phase charge ($Q$) in response to changes in peak resistance ($R_p$). **(A)**-**(I)** each show data from one of the nine subjects without arm impairment. **(J)** shows data from a subject with a right below-elbow amputation. Subjects ascended and descended stairs to reduce $R_p$ and subsequently rested for 10 minutes to raise $R_p$. For each of the five conditions, the subjects were asked to match a vibrotactile reference sensation intensity over three trials with differing values of $R_p$.

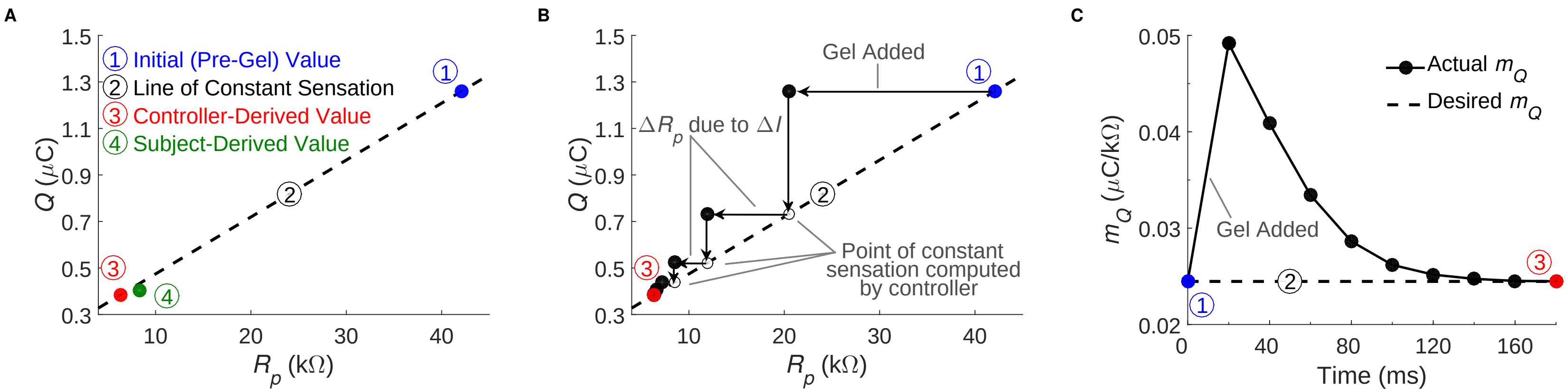


**Fig. S9 Methods for Exp. 2 (controller experiments). (A)** 1. Initial stimulation parameters are chosen by the subject to match a reference sensation on the opposite arm before any gel is added. In this example, $I = 1.8$ mA, $T = 700$ μs, and stimulation was sent at 50 Hz. 2. The computer determines the line of constant sensation intensity to stay on in response to changes in $R_p$. 3. Gel is applied to the electrode-skin interface, reducing $R_p$, and the controller computes new values for $I$ and $T$ to derive a value of $Q$ that is on the line of constant sensation intensity. 4. Fixing $T$ to the controller-computed value, the subject then adjusts $I$ until the sensation intensity matches the reference sensation. The subject's value of $Q$ is derived and compared to the line of constant sensation intensity. **(B)** The controller does not converge in a single iteration. When gel is added, $R_p$ changes and the controller computes and sends new stimulation parameters. However, because changes in $I$ induce changes in $R_p$, the controller requires more iterations in order for both $R_p$ and $Q$ to converge to the line of constant sensation intensity. **(C)** In this example, nine iterations (one iteration per pulse, i.e. every 20 ms) were required before the actual ratio of $Q$ to $R_p$ converged to the desired $m_Q$, representing the slope of the line of constant sensation intensity.

**Movie S1 Experiment 3A response of controller in real-time during electrode peeling/placing.**

Real-time changes in current $I$, pulse duration $T$, peak resistance $R_p$, and the slopes of the lines of constant sensation for peak pulse energy $m_E$ and phase charge $m_Q$ are shown during peeling and placing electrodes every five seconds with and without the controller. Without the controller, $I$ and $T$ remain constant, and the user felt increases in sensation intensity to the point of discomfort when peeling the electrode. With the controller, $I$ and $T$ varied in response to changes in $R_p$, keeping both $m_E$ and $m_Q$ constant, and the user reported no change in sensation intensity throughout the activity.

**Movie S2 Experiment 3B response of controller in real-time during exercise.**

Real-time changes in current $I$, pulse duration $T$, peak resistance $R_p$, and the slopes of the lines of constant sensation for peak pulse energy $m_E$ and phase charge $m_Q$ are shown during 30 seconds of heavy exercise on an elliptical trainer. Without the controller, $I$ and $T$ remain constant, and the user no longer felt sensation by the end of the activity. With the controller, $I$ and $T$ varied in response to changes in $R_p$, keeping both $m_E$ and $m_Q$ constant, and the user was still able to feel the sensation by the end of the activity. Similar results occurred when ascending/descending stairs, hammering a nail, and using the elliptical trainer for five minutes.