

Online Appendix to “BIC and Alternative Bayesian Information Criteria in the Selection of Structural Equation Models”

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1 IBIC vs. Kashyap’s (1982) BIC

An important question with any Bayes Factor approximation is how much complexity should be retained and how much should be omitted. In using $\log(\frac{N}{2\pi})$ in its approximation, IBIC is slightly more complex than Kashyap’s (1982) use of $\log(N)$ (KBIC). The empirical question then becomes whether the added complexity is worthwhile. We re-ran our simulations substituting $\log(N)$ rather than $\log(\frac{N}{2\pi})$. The results, which are displayed in Figure A.1, indicate that the $\log(\frac{N}{2\pi})$ from IBIC consistently outperforms KBIC’s $\log(N)$ with respect to selection of the true model. This is especially evident at smaller sample sizes. For example, at $N = 250$ the IBIC selections of the true model are 10 (SIM1) and 18 (SIM2) percentage points higher than KBIC’s selections. Not surprisingly, as the sample size increases the two approximations become increasingly similar and both select the true model nearly 100% of the time.

2 SIM2 Percentage Selections of the True Model

Figure A.2 presents the percentage selection of the true model for all fit measures in SIM2.

3 Model Selection Results with No True Model

Tables A.1 and A.2 present the median and IQR values for BIC, HBIC, IBIC, and SPBIC in SIM1 with $N = 250$ and 5000.

Tables A.3–A.7 present the median and IQR values for the four IC measures in SIM2 at all five sample sizes.

Figures A.3–A.6 present SIM2 box-and-whiskers plots for each IC measure for individual models for $N = 100, 250, 500,$ and 1000.

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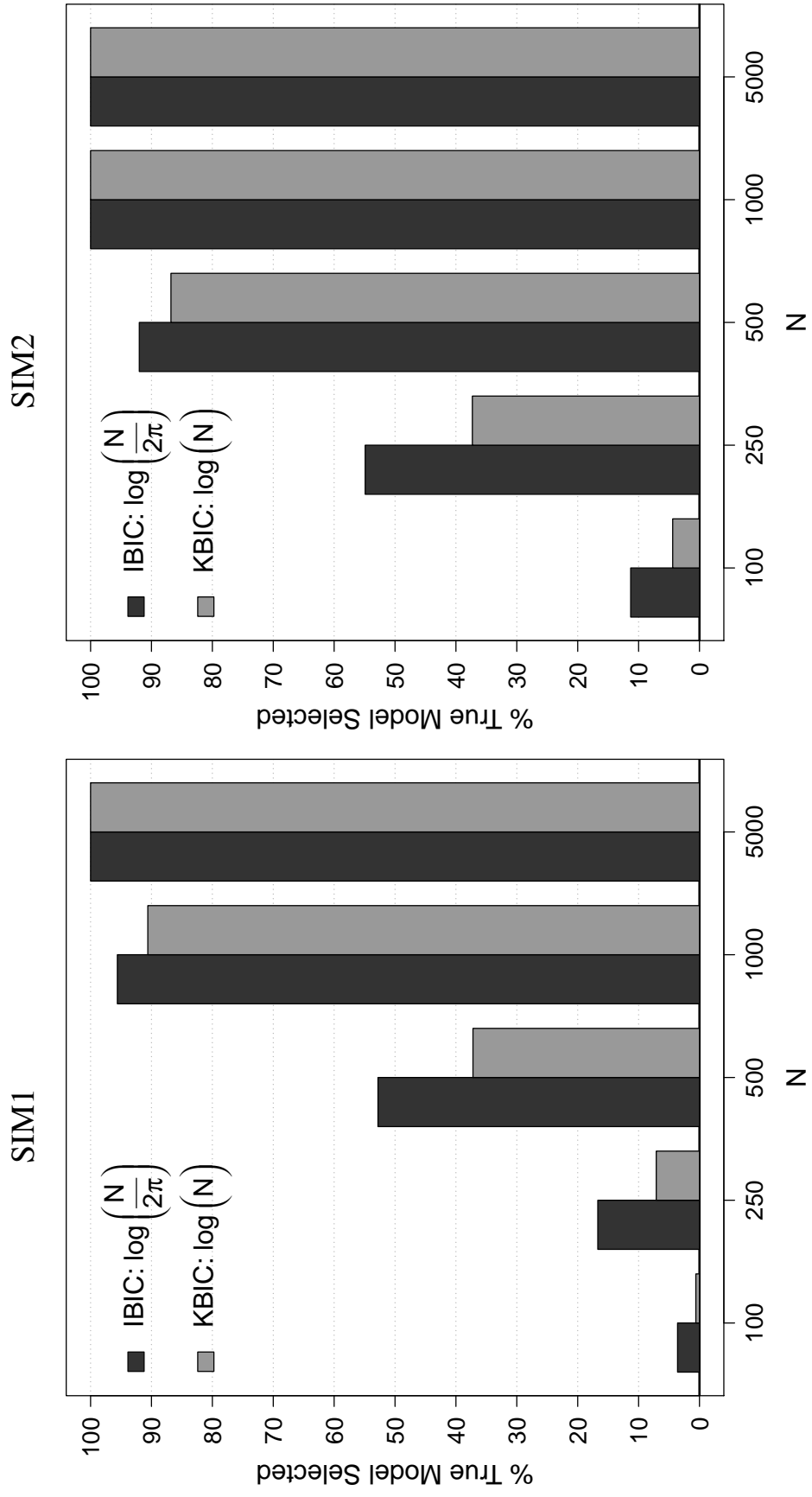


Figure A.1: IBIC vs. KBIC Percentage Selections of the True Model

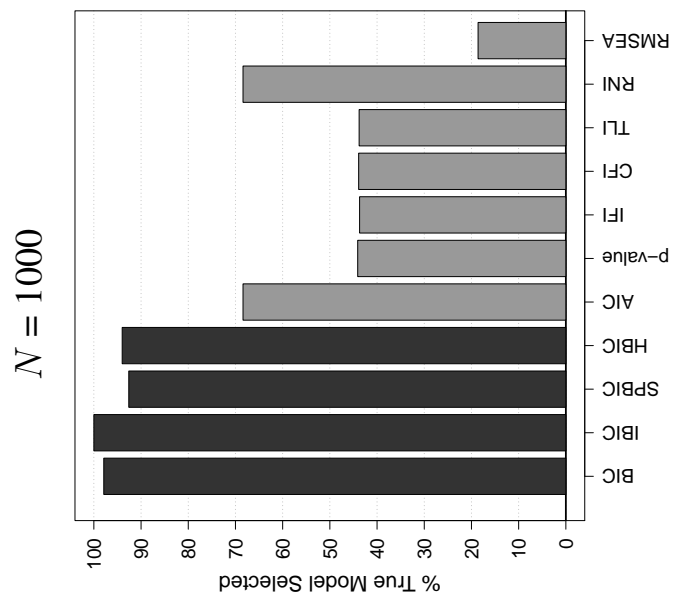
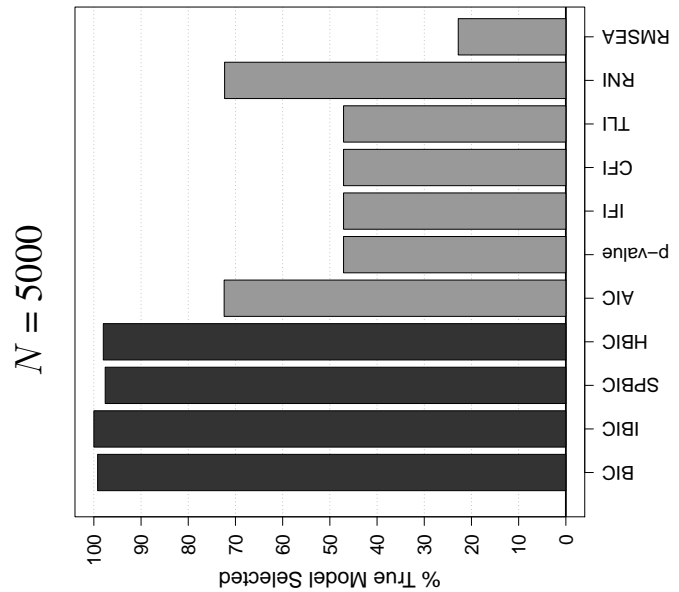
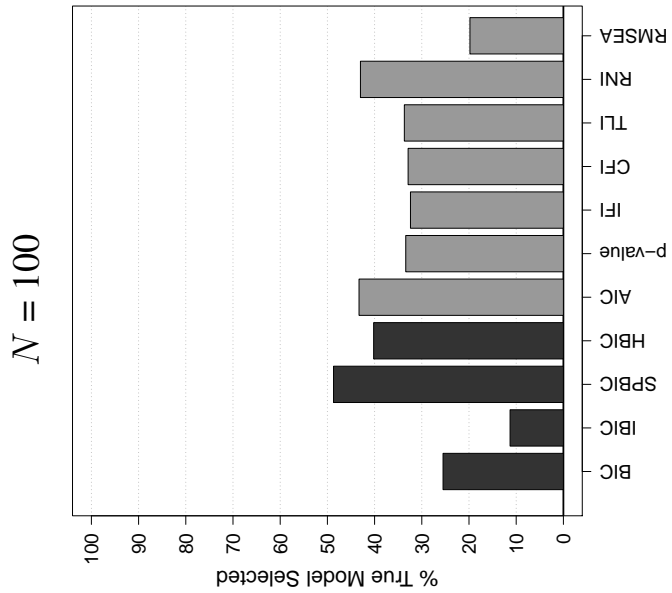
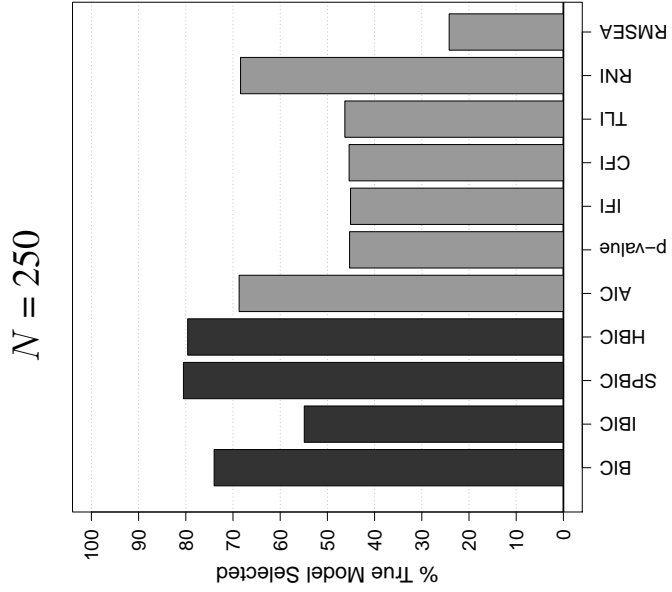
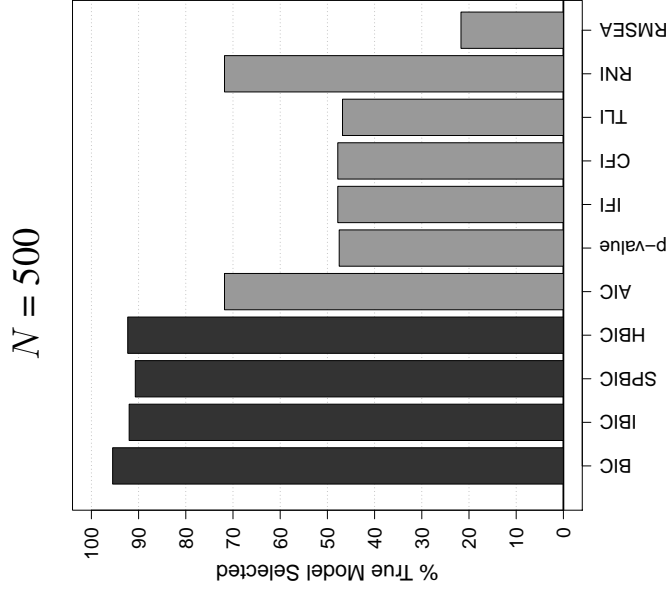


Figure A.2: SIM2 Percentage Selections of the True Model

Table A.1: SIM1 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 250$)

Models:	Extra Parameters			Dropped Loadings			Dropped and Extra			Dropped β Only M10	
	M6	M8	M9	M2	M3	M4	M5	M7	M11		M12
BIC											
Median	5.06	5.05	9.65	1.27	0.67	9.07	2.99	64.80	113.33	5.36	92.73
IQR	1.15	1.33	2.42	6.95	9.11	13.58	8.56	26.82	33.87	2.38	26.49
Median Rank	4	4	7	3	2	7	4	9	11	5	10
HBIC											
Median	3.22	3.21	5.97	3.10	4.35	14.58	4.82	64.80	120.68	3.52	96.41
IQR	1.15	1.33	2.42	6.95	9.11	13.58	8.56	26.82	33.87	2.38	26.49
Median Rank	3	3	6	4	5	8	5	9	11	4	10
IBIC											
Median	9.94	7.36	16.99	-1.10	-3.89	1.71	3.67	61.85	98.18	9.07	85.71
IQR	1.06	1.30	2.34	6.80	8.74	12.77	7.60	26.42	32.12	2.45	26.37
Median Rank	6	5	8	3	1	3	4	9	11	6	10
SPBIC											
Median	2.77	2.88	5.17	3.55	4.73	14.75	4.21	67.23	123.28	14.31	92.21
IQR	1.13	1.28	2.31	6.84	8.80	13.49	8.29	26.46	33.10	5.07	26.04
Median Rank	3	3	4	4	5	8	4	9	11	7	10

Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

Table A.2: SIM1 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 5000$)

Models:	Extra Parameters			Dropped Loadings			Dropped and Extra			Dropped β Only	
	M6	M8	M9	M2	M3	M4	M5	M7	M11	M12	M10
BIC											
Median	8.06	8.07	15.70	121.63	196.22	447.21	163.16	1312.60	2687.59	25.46	2068.00
IQR	1.11	1.17	2.04	31.16	39.14	60.01	38.34	109.78	155.93	11.49	121.84
Median Rank	1	2	3	5	7	8	6	9	11	4	10
HBIC											
Median	6.22	6.23	12.02	123.47	199.90	452.72	165.00	1312.60	2694.94	23.62	2071.68
IQR	1.11	1.17	2.04	31.16	39.14	60.01	38.34	109.78	155.93	11.49	121.84
Median Rank	1	2	3	5	7	8	6	9	11	4	10
IBIC											
Median	15.96	13.46	29.01	116.34	185.59	430.90	160.86	1309.84	2660.40	32.08	2054.88
IQR	1.12	1.18	2.02	31.11	39.06	59.65	38.18	109.94	155.54	11.39	121.79
Median Rank	2	1	3	5	7	8	6	9	11	4	10
SPBIC											
Median	5.80	5.89	11.23	123.91	200.26	452.99	164.50	1315.53	2697.25	34.31	2067.51
IQR	1.09	1.15	2.08	31.07	39.07	59.79	38.25	109.85	155.60	11.14	121.74
Median Rank	1	2	3	5	7	8	6	9	11	4	10

Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

Table A.3: SIM2 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 100$)

Models:	Extra Parameters			Dropped Loadings			Dropped and Extra			Dropped β Only	
	M6	M8	M9	M2	M3	M4	M5	M7	M11	M12	M10
BIC											
Median	4.15	4.09	7.74	-0.28	-0.05	2.33	2.96	55.87	89.50	16.92	47.94
IQR	1.43	1.30	2.59	5.70	8.25	10.95	8.35	22.77	31.00	7.22	19.28
Median Rank	4	4	7	3	2	3	4	10	11	8	9
HBIC											
Median	2.31	2.25	4.07	1.55	3.63	7.84	4.80	55.87	96.85	11.40	51.61
IQR	1.43	1.30	2.59	5.70	8.25	10.95	8.35	22.77	31.00	7.22	19.28
Median Rank	3	3	4	3	5	7	5	10	11	8	9
IBIC											
Median	8.65	5.91	14.21	-2.31	-4.31	-4.20	3.58	46.97	73.43	20.08	41.79
IQR	1.31	1.33	2.41	5.28	7.49	9.95	7.52	19.24	28.90	7.23	18.82
Median Rank	6	5	7	3	2	2	4	10	11	8	9
SPBIC											
Median	1.76	1.91	3.18	2.36	5.82	10.88	6.49	55.91	95.10	18.35	43.63
IQR	1.32	1.22	2.37	5.97	8.81	11.40	8.75	20.01	31.05	7.97	17.11
Median Rank	2	2	4	4	5	7	6	10	11	8	9

Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

Table A.4: SIM2 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 250$)

Models:	Extra Parameters			Dropped Loadings			Dropped and Extra			Dropped β Only	
	M6	M8	M9	M2	M3	M4	M5	M7	M11	M12	M10
BIC											
Median	5.05	5.06	9.60	5.34	10.94	21.07	13.62	140.89	242.75	30.00	130.44
IQR	1.18	1.26	2.16	9.48	13.54	17.12	13.28	34.59	49.85	12.51	30.45
Median Rank	2	2	4	3	5	7	6	10	11	8	9
HBIC											
Median	3.21	3.22	5.92	7.18	14.61	26.58	15.46	140.89	250.10	24.49	134.11
IQR	1.18	1.26	2.16	9.48	13.54	17.12	13.28	34.59	49.85	12.51	30.45
Median Rank	2	2	3	4	5	7	6	10	11	7	9
IBIC											
Median	10.49	7.87	17.98	2.40	4.72	11.88	13.18	132.71	223.22	35.82	122.17
IQR	1.11	1.16	2.13	9.19	12.84	16.63	12.42	33.26	48.60	11.74	30.30
Median Rank	4	3	7	2	2	5	5	10	11	8	9
SPBIC											
Median	2.69	2.80	4.99	8.22	16.68	29.52	16.99	139.93	249.17	31.08	125.98
IQR	1.19	1.19	2.06	9.59	13.29	17.68	13.19	33.19	49.76	11.84	29.17
Median Rank	2	2	3	4	5	7	6	10	11	8	9

Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

Table A.5: SIM2 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 500$)

Models:	Extra Parameters			Dropped Loadings			Dropped and Extra			Dropped β Only		
	M6	M8	M9	M2	M3	M4	M5	M7	M11	M12	M10	
BIC												
Median	5.79	5.76	11.11	15.95	29.33	52.74	32.18	285.49	503.97	48.88	264.15	
IQR	1.13	1.12	2.05	13.26	18.43	23.66	18.41	51.06	73.84	17.35	46.78	
Median Rank	2	2	3	4	5	8	6	10	11	7	9	
HBIC												
Median	3.95	3.92	7.43	17.78	33.01	58.26	34.02	285.49	511.32	43.37	267.82	
IQR	1.13	1.12	2.05	13.26	18.43	23.66	18.41	51.06	73.84	17.35	46.78	
Median Rank	2	2	3	4	6	8	6	10	11	7	9	
IBIC												
Median	11.92	9.30	20.85	12.25	21.85	41.59	31.13	277.34	481.61	56.93	254.34	
IQR	1.11	1.11	2.03	13.18	18.07	23.42	18.31	49.66	72.38	17.06	46.56	
Median Rank	2	1	4	3	5	7	6	10	11	8	9	
SPBIC												
Median	3.36	3.49	6.43	18.78	35.23	61.51	35.66	284.42	510.78	49.74	259.39	
IQR	1.12	1.11	2.07	13.40	18.61	23.77	18.72	49.56	73.51	17.13	45.54	
Median Rank	1	2	3	4	6	8	6	10	11	7	9	

Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

Table A.6: SIM2 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 1000$)

Models:	Extra Parameters			Dropped Loadings			Dropped and Extra			Dropped β Only	
	M6	M8	M9	M2	M3	M4	M5	M7	M11	M12	M10
BIC											
Median	6.43	6.36	12.26	38.30	71.26	122.24	70.98	574.86	1031.76	85.87	539.21
IQR	1.32	1.26	2.25	18.49	25.31	33.76	25.81	71.06	102.38	23.84	65.43
Median Rank	2	1	3	4	6	8	6	10	11	7	9
HBIC											
Median	4.59	4.52	8.59	40.14	74.94	127.76	72.82	574.86	1039.11	80.36	542.88
IQR	1.32	1.26	2.25	18.49	25.31	33.76	25.81	71.06	102.38	23.84	65.43
Median Rank	2	1	3	4	6	8	6	10	11	6	9
IBIC											
Median	13.24	10.62	23.38	33.90	62.38	108.75	69.11	566.65	1006.68	95.79	528.40
IQR	1.32	1.31	2.25	18.38	25.20	33.57	25.53	70.10	101.66	23.81	65.16
Median Rank	2	1	3	4	5	8	6	10	11	7	9
SPBIC											
Median	4.01	4.09	7.58	41.07	77.26	130.55	74.28	573.69	1038.86	86.47	535.21
IQR	1.30	1.27	2.24	18.53	25.67	33.83	26.06	70.11	102.20	23.64	64.71
Median Rank	1	2	3	4	6	8	6	10	11	7	9

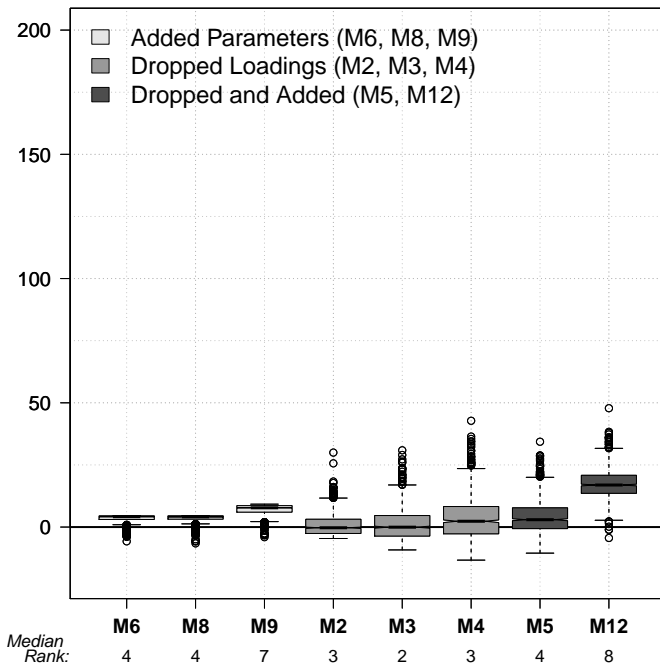
Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

Table A.7: SIM2 Fit Statistic Quantiles and Median Ranks with No True Model ($N = 5000$)

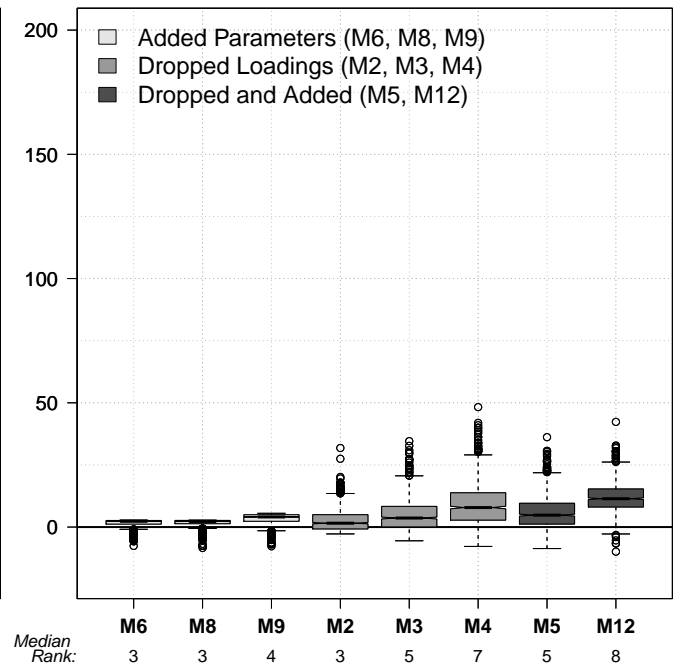
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	M6	M8	M9	M2	M3	M4	M5	M7	M11	M12	M10
BIC											
Median	8.07	8.11	15.74	217.41	407.71	680.70	377.48	2873.99	5265.19	366.31	2754.49
IQR	1.19	1.19	2.25	40.31	59.62	78.59	57.01	162.95	224.78	50.38	128.37
Median Rank	1	2	3	4	7	8	6	10	11	5	9
HBIC											
Median	6.24	6.28	12.07	219.25	411.39	686.21	379.31	2873.99	5272.54	360.80	2758.17
IQR	1.19	1.19	2.25	40.31	59.62	78.59	57.01	162.95	224.78	50.38	128.37
Median Rank	1	2	3	4	7	8	6	10	11	5	9
IBIC											
Median	16.48	14.00	30.01	211.36	395.43	662.42	373.82	2865.70	5233.53	381.35	2740.30
IQR	1.20	1.14	2.25	40.31	59.49	78.44	56.94	162.39	224.39	50.35	128.38
Median Rank	2	1	3	4	6	8	6	10	11	6	9
SPBIC											
Median	5.66	5.83	11.03	220.22	413.58	689.05	380.68	2872.47	5272.02	366.78	2750.09
IQR	1.21	1.18	2.22	40.33	59.66	78.43	56.91	162.52	225.05	50.30	128.34
Median Rank	1	2	3	4	7	8	6	10	11	5	9

Note: Cells report median and interquartile range (IQR) values of the fit statistics across the simulations, with median model ranks below. Lower numbers indicate better fit.

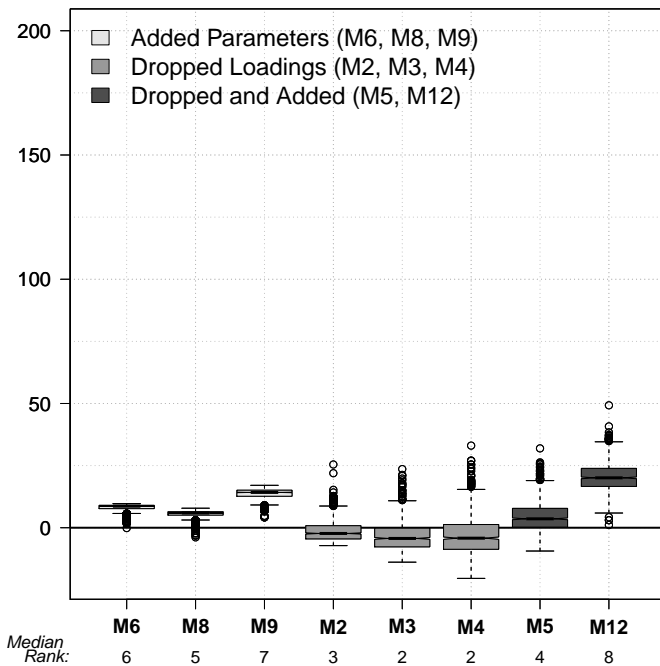
BIC



HBIC



IBIC



SPBIC

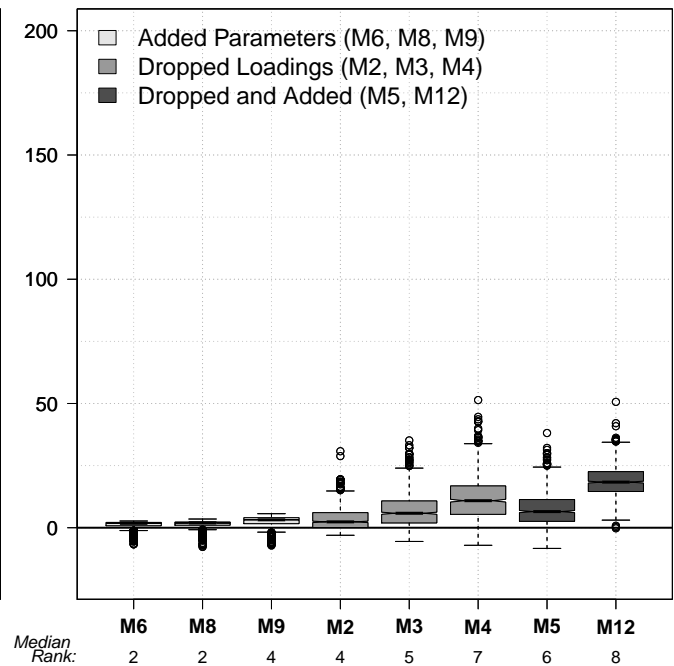
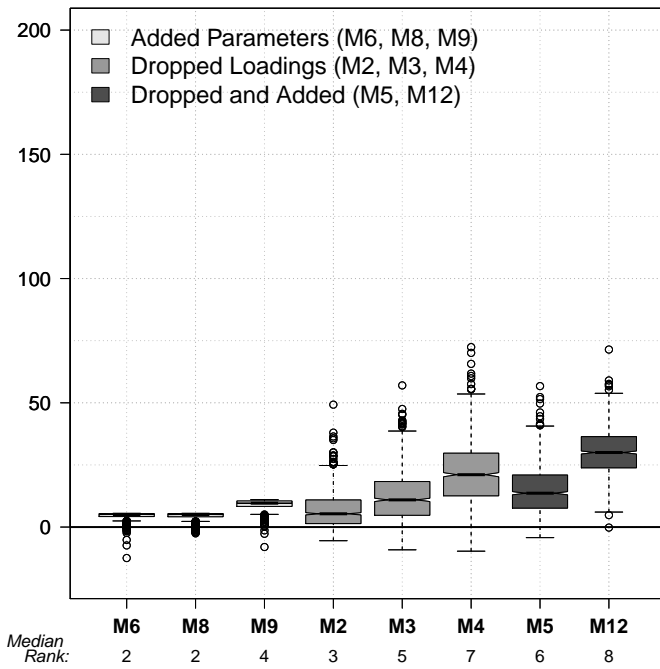
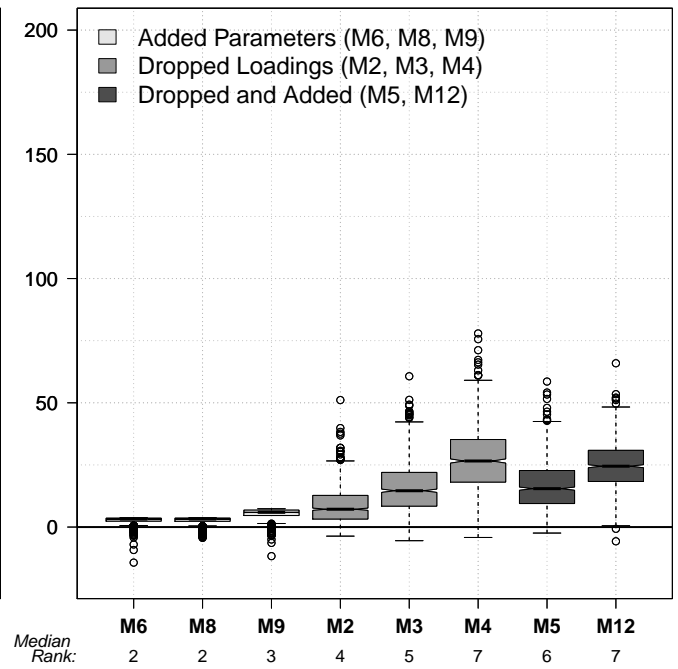


Figure A.3: SIM2 IC Box-and-Whisker Plots and Median Ranks with No True Model ($N = 100$). Box-and-Whisker plots represent each fit statistic for each model over the 1,000 simulations. Numbers below the model names represent the median rank of each fit statistic for each model over the 1000 simulations.

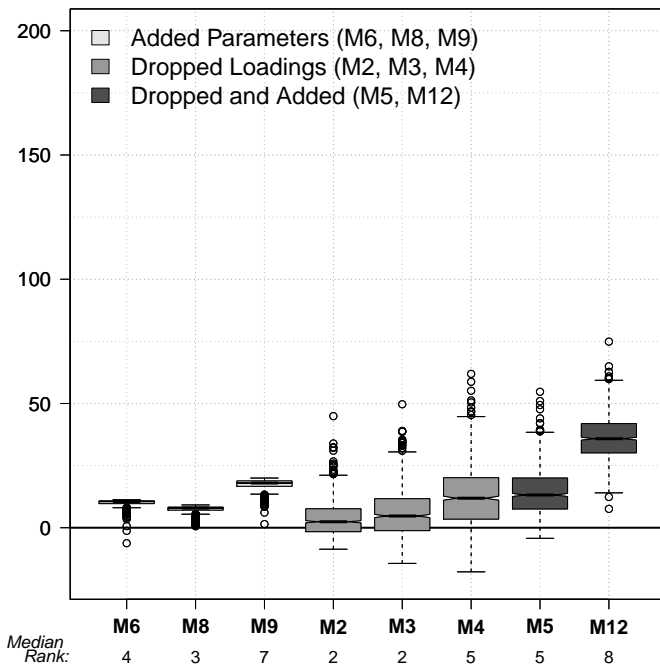
BIC



HBIC



IBIC



SPBIC

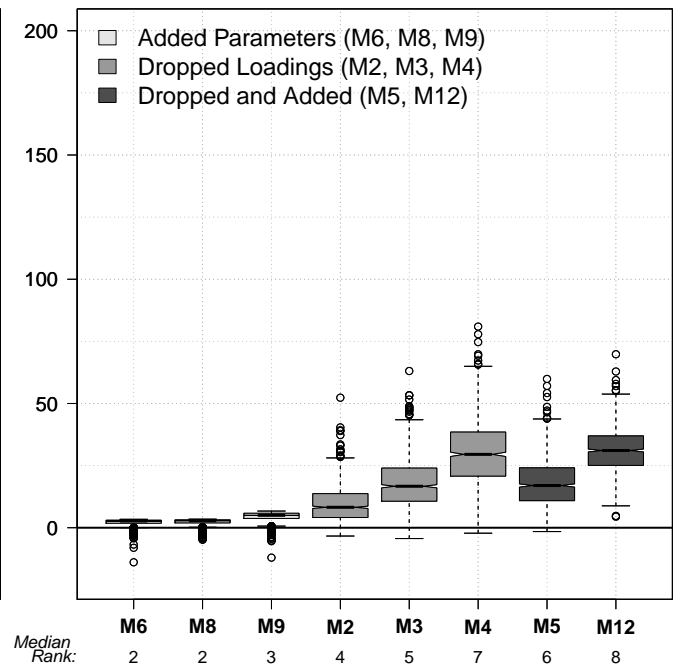
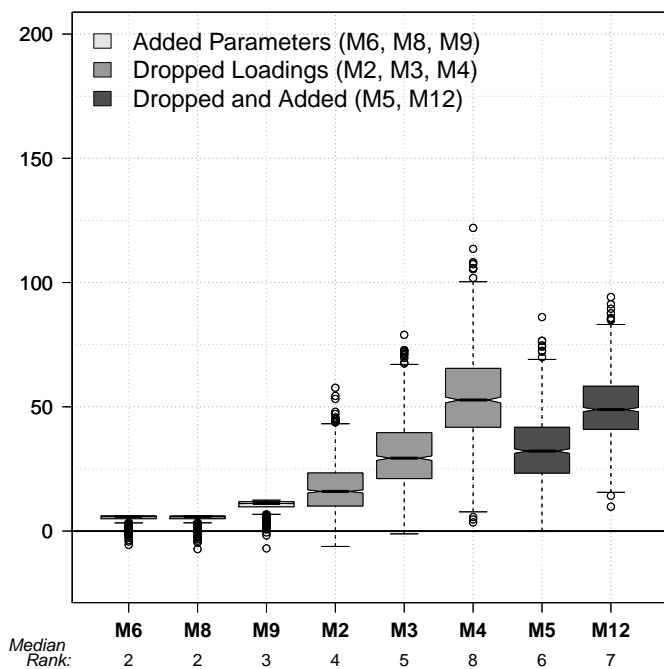
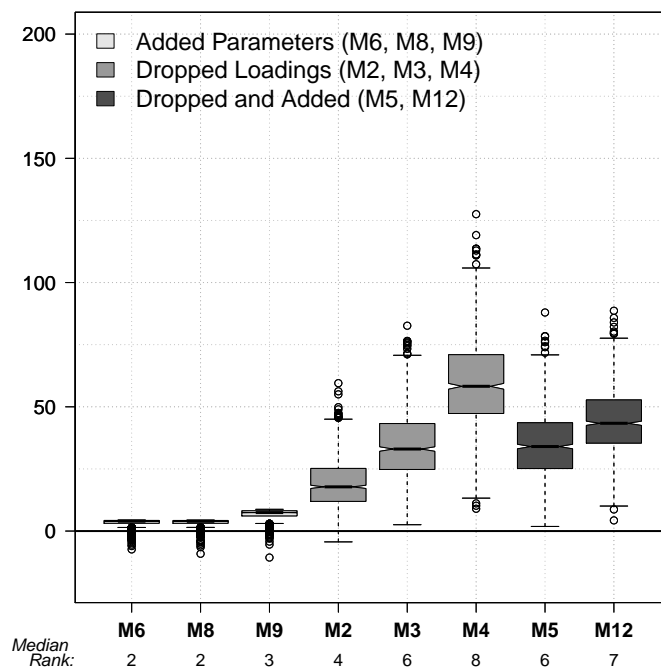


Figure A.4: SIM2 IC Box-and-Whisker Plots and Median Ranks with No True Model ($N = 250$). Box-and-Whisker plots represent each fit statistic for each model over the 1,000 simulations. Numbers below the model names represent the median rank of each fit statistic for each model over the 1000 simulations.

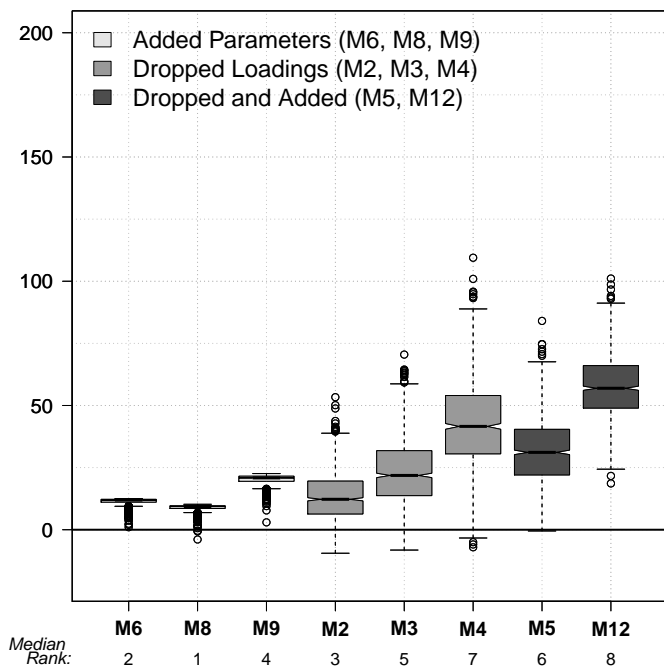
BIC



HBIC



IBIC



SPBIC

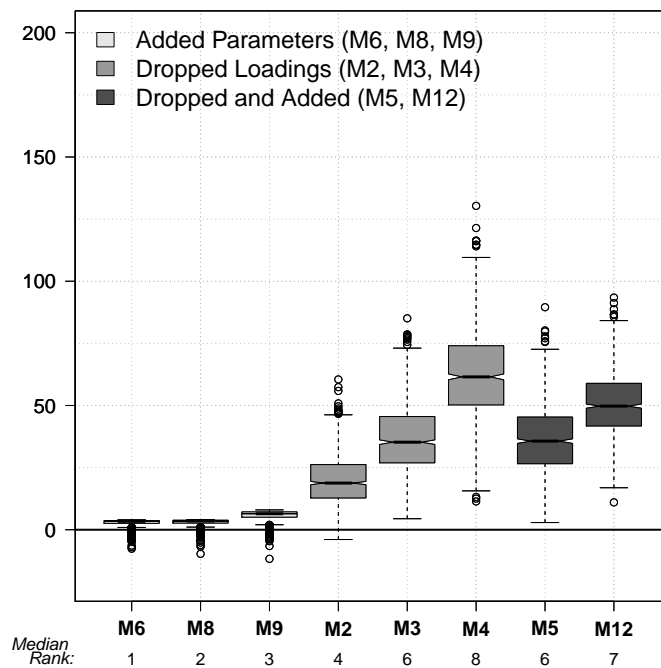


Figure A.5: SIM2 IC Box-and-Whisker Plots and Median Ranks with No True Model ($N = 500$). Box-and-Whisker plots represent each fit statistic for each model over the 1,000 simulations. Numbers below the model names represent the median rank of each fit statistic for each model over the 1000 simulations.

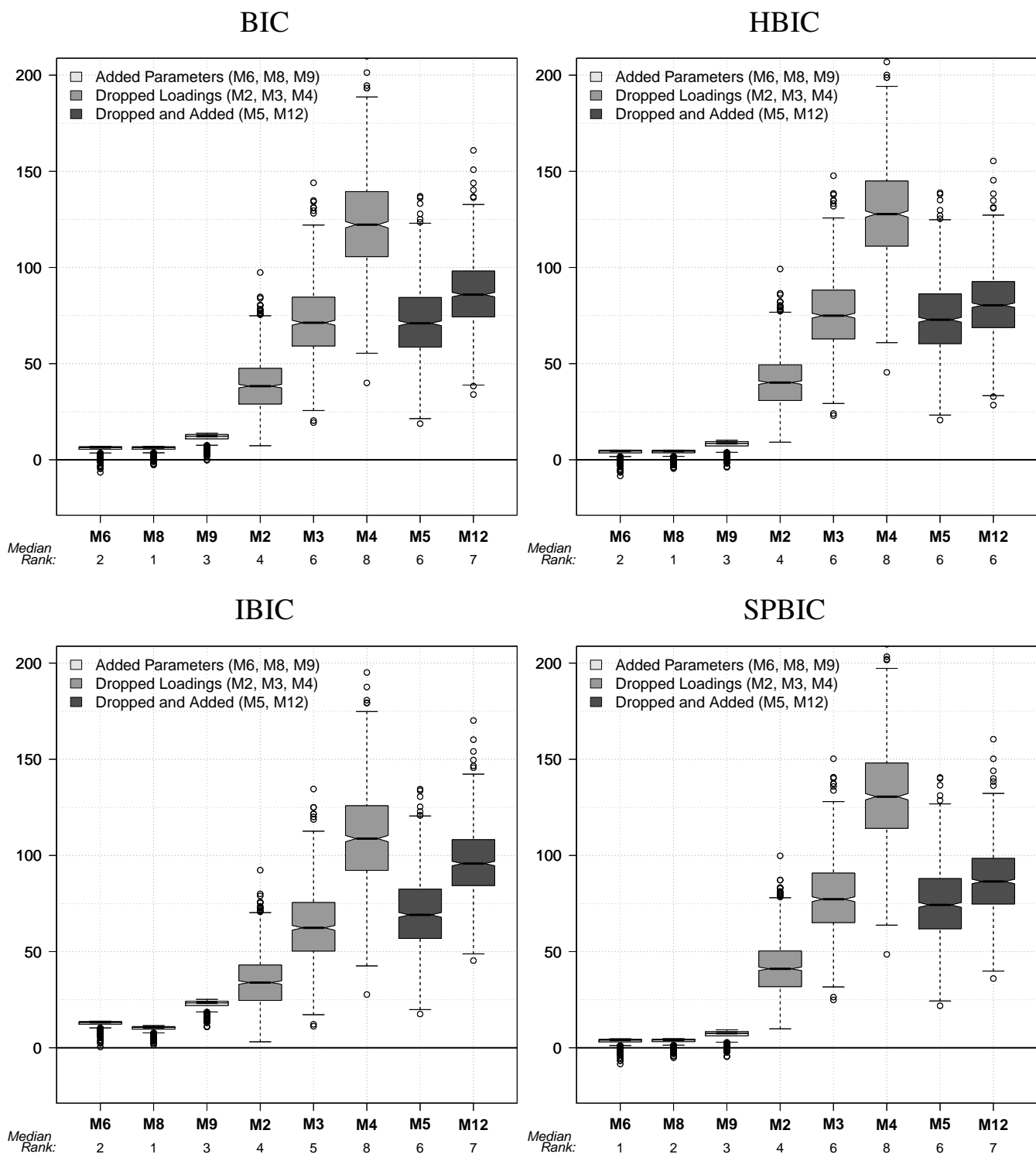


Figure A.6: SIM2 IC Box-and-Whisker Plots and Median Ranks with No True Model ($N = 1000$). Box-and-Whisker plots represent each fit statistic for each model over the 1,000 simulations. Numbers below the model names represent the median rank of each fit statistic for each model over the 1000 simulations.

References

Kashyap, R. L. (1982). Optimal choice of ar and ma parts in autoregressive moving average models. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 4(2):99–104.