Supplementary Material for
Instrumental Variable Approach to Estimating the Scalar-on-Function Regression Model with Measurement Error with Application to Energy Expenditure Assessment in Childhood Obesity

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The Supplementary Materials are organized as follows. Section S.1 contains additional tables summarizing the effect of the sample size on the performance of the estimators of $\beta_2(t)$. Section S.2 presents additional table describing the performance of the estimators $\widehat{\beta}_X$, $\widehat{\beta}_W$, $\widehat{\beta}_{WS}$, $\widehat{\beta}_{WRS}$ with respect to varying measurement error(ME) variance (σ_u) and instrument variable (IV) variance σ_ω, assuming the data was generated based on $\beta_1(t)$. Finally, Section S.3 shows additional tables illustrating the effect of varying correlation in X (ρ_u), the measurement error (ρ_u), and the instrumental variable ρ_ω.

S.1 Sample Size Effect for $\beta_2(t)$

Table S.1: This table assesses the impact of sample sizes on the estimators. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_X$ for different sample sizes n. The true function is $\beta_2(t)$

	$\widehat{\beta}_X$		
n	ABias^2	Avar	AIMSE
100	0.0340	0.2500	0.2840
200	0.0245	0.1360	0.1604
500	0.0115	0.0593	0.0707
1000	0.0070	0.0354	0.0423
	$\widehat{\beta}_W$		
n	ABias^2	Avar	AIMSE
100	0.0637	0.1396	0.2032
200	0.0557	0.0745	0.1302
500	0.0416	0.0329	0.0746
1000	0.0391	0.0191	0.0582
	$\widehat{\beta}_{WRS}$		
n	ABias^2	Avar	AIMSE
100	0.0641	0.1392	0.2033
200	0.0561	0.0744	0.1305
500	0.0419	0.0330	0.0749
1000	0.0393	0.0191	0.0584
	$\widehat{\beta}_{WS}$		
n	ABias^2	Avar	AIMSE
100	0.0344	0.5301	0.5645
200	0.0334	0.2503	0.2837
500	0.0190	0.1213	0.1403
1000	0.0160	0.0746	0.0905
	$\widehat{\beta}_{IV}$		
n	ABias^2	Avar	AIMSE
100	0.0343	0.2709	0.3052
200	0.0251	0.1470	0.1721
500	0.0115	0.0656	0.0771
1000	0.0071	0.0386	0.0456

Table S.2: This table assesses the impact of sample sizes on the estimators. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_X$ for different sample sizes n. The response error is assumed to have a $Gamma(1, 1.5)$ distribution, where $Gamma(\alpha, \beta)$ denotes a distribution with mean $\alpha\beta$. The true function is $\beta_2(t)$.

β_2	$\widehat{\beta}_X$		
n	ABias^2	Avar	AIMSE
100	0.0494	0.4825	0.5319
200	0.0365	0.2575	0.2939
500	0.0171	0.1088	0.1260
1000	0.0139	0.0630	0.0769
β_2	$\widehat{\beta}_W$		
n	ABias^2	Avar	AIMSE
100	0.0733	0.2669	0.3402
200	0.0650	0.1414	0.2063
500	0.0444	0.0611	0.1055
1000	0.0442	0.0349	0.0791
β_2	$\widehat{\beta}_{WRS}$		
n	ABias^2	Avar	AIMSE
100	0.0735	0.2672	0.3407
200	0.0658	0.1404	0.2062
500	0.0453	0.0611	0.1064
1000	0.0447	0.0348	0.0796
β_2	$\widehat{\beta}_{WS}$		
n	ABias^2	Avar	AIMSE
100	0.0429	1.0618	1.1048
200	0.0388	0.4594	0.4981
500	0.0207	0.2309	0.2517
1000	0.0202	0.1269	0.1471
β_2	$\widehat{\beta}_{IV}$		
n	ABias^2	Avar	AIMSE
100	0.0495	0.5044	0.5539
200	0.0370	0.2742	0.3112
500	0.0171	0.1173	0.1344
1000	0.0140	0.0676	0.0816

S.2 Error variance effect

Table S.3: Impacts of varying magnitudes of measurement error and instrumental variable variance on our proposed estimator. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_X$ for sample size $n = 500$. The true function is $\beta_1(t)$.

$\sigma_\omega = 1$				$\sigma_u = 1$			
σ_u	ABias^2	Avar	AIMSE	σ_ω	ABias^2	Avar	AIMSE
0.5	0.0001	0.0479	0.0480	0.50	0.0001	0.0477	0.0478
1	0.0001	0.0474	0.0475	1.00	0.0001	0.0474	0.0475
4	0.0001	0.0408	0.0408	4.00	0.0001	0.0432	0.0432
16	0.0001	0.0411	0.0412	16.00	0.0001	0.0396	0.0397

Table S.4: Impacts of varying magnitudes of measurement error and instrumental variable variance on our proposed estimator. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_W$ for sample size $n = 500$. The true function is $\beta_1(t)$.

$\sigma_\omega = 1$				$\sigma_u = 1$			
σ_u	ABias^2	Avar	AIMSE	σ_ω	ABias^2	Avar	AIMSE
0.5	0.0001	0.0475	0.0476	0.50	0.0004	0.0458	0.0462
1	0.0004	0.0455	0.0458	1.00	0.0004	0.0455	0.0458
4	0.0392	0.0246	0.0638	4.00	0.0003	0.0412	0.0415
16	0.3664	0.0036	0.3701	16.00	0.0003	0.0376	0.0380

Table S.5: Impacts of varying magnitudes of measurement error and instrumental variable variance on our proposed estimator. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_{WSR}$ for sample size $n = 500$ (WSR). The true function is $\beta_1(t)$.

$\sigma_\omega = 1$				$\sigma_u = 1$			
σ_u	ABias^2	Avar	AIMSE	σ_ω	ABias^2	Avar	AIMSE
0.5	0.0001	0.0477	0.0478	0.50	0.0004	0.0460	0.0463
1	0.0003	0.0455	0.0459	1.00	0.0003	0.0455	0.0459
4	0.0392	0.0247	0.0638	4.00	0.0003	0.0413	0.0417
16	0.3664	0.0037	0.3701	16.00	0.0003	0.0381	0.0384

Table S.6: Impacts of varying magnitudes of measurement error and instrumental variable variance on our proposed estimator. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_{WS}$ for sample size $n = 500$. The true function is $\beta_1(t)$.

$\sigma_\omega = 1$				$\sigma_u = 1$			
σ_u	ABias^2	Avar	AIMSE	σ_ω	ABias^2	Avar	AIMSE
0.50	0.0148	0.1877	0.2025	0.50	0.0113	0.1855	0.1968
1	0.0114	0.1816	0.1929	1.00	0.0114	0.1816	0.1929
4	0.0144	0.0867	0.1011	4.00	0.0112	0.1484	0.1596
16	0.3512	0.0160	0.3672	16.00	0.0112	0.1363	0.1475

Table S.7: Impacts of varying magnitudes of measurement error and instrumental variable variance on our proposed estimator. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_{IV}$ for sample size $n = 500$. The true function is $\beta_1(t)$.

$\sigma_\omega = 1$				$\sigma_u = 1$			
σ_u	ABias^2	Avar	AIMSE	σ_ω	ABias^2	Avar	AIMSE
0.5	0.0001	0.0506	0.0507	0.50	0.0001	0.0488	0.0489
1	0.0001	0.0504	0.0505	1.00	0.0001	0.0504	0.0505
4	0.0001	0.0497	0.0498	4.00	0.0001	0.0794	0.0795
16	0.0028	0.1070	0.1097	16.00	0.0003	0.4784	0.4787

S.3 Case of Varying Correlation

Table S.8: The impact of correlation structures on the parameter estimates. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_X$ for sample size $n = 500$. The true function is $\beta_1(t)$

		ρ_X			ρ_u			ρ_ω		
	0	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
ABias^2	0.0001	0.0001	0.0002	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Avar	0.0408	0.0437	0.0614	0.1125	0.0430	0.0446	0.0466	0.0408	0.0409	0.0411
AIMSE	0.0408	0.0438	0.0615	0.1128	0.0431	0.0446	0.0466	0.0408	0.0410	0.0412

Table S.9: The impact of correlation structures on the parameter estimates. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_W$ for sample size $n = 500$. The true function is $\beta_1(t)$.

		ρ_X			ρ_u			ρ_ω		
	0	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
ABias^2	0.0392	0.0454	0.0532	0.0633	0.0255	0.0133	0.0039	0.0392	0.0392	0.0392
Avar	0.0246	0.0237	0.0259	0.0285	0.0246	0.0284	0.0337	0.0247	0.0247	0.0248
AIMSE	0.0638	0.0691	0.0791	0.0918	0.0501	0.0417	0.0376	0.0639	0.0640	0.0641

Table S.10: The impact of correlation structures on the parameter estimates. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_{WRS}$ for sample size $n = 500$. The true function is $\beta_1(t)$.

		ρ_X			ρ_u			ρ_ω		
	0	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
ABias^2	0.0392	0.0454	0.0532	0.0632	0.0255	0.0132	0.0039	0.0392	0.0392	0.0392
Avar	0.0247	0.0238	0.0261	0.0289	0.0248	0.0287	0.0339	0.0248	0.0249	0.0249
AIMSE	0.0638	0.0692	0.0793	0.0921	0.0502	0.0419	0.0378	0.0639	0.0640	0.0641

Table S.11: The impact of correlation structures on the parameter estimates. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_{WS}$ for sample size $n = 500$. The true function is $\beta_1(t)$

		ρ_X			ρ_u			ρ_ω		
	0	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
ABias^2	0.0144	0.0188	0.0252	0.0355	0.0059	0.0008	0.0024	0.0144	0.0144	0.0144
Avar	0.0867	0.0907	0.0973	0.1064	0.1106	0.1339	0.1687	0.0872	0.0880	0.0897
AIMSE	0.1011	0.1095	0.1225	0.1419	0.1164	0.1347	0.1711	0.1015	0.1024	0.1041

Table S.12: The impact of correlation structures on the parameter estimates. The averaged squared bias (ABias^2), averaged sample variance (Avar) and averaged integrated mean squared error (AIMSE) of $\widehat{\beta}_{IV}$ for sample size $n = 500$. The true function is $\beta_1(t)$.

		ρ_X			ρ_u			ρ_ω		
	0	0.25	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.75
ABias^2	0.0001	0.0001	0.0002	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Avar	0.0497	0.0540	0.0775	0.1521	0.0485	0.0483	0.0482	0.0552	0.0603	0.0652
AIMSE	0.0498	0.0541	0.0776	0.1524	0.0486	0.0484	0.0483	0.0553	0.0604	0.0653