Supporting Material, File S1 Text

Homeostatic Controllers Compensating for Growth and Perturbations

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Steady state of cell-internal-generated compound A without negative feedback

Integrating $\dot{V}{=}k_1$ from V_0 to $V(t)$ and from time zero to time t gives

$$V(t) = k_1 t + V_0 \tag{S1}$$

Inserting Eq. S1 into the equation $\dot{E} = -E\dot{V}/V$ (Eq. 10), we get

$$\dot{E} = -E\left(\frac{k_1}{k_1 t{+}V_0}\right) = -E\left(\frac{1}{t{+}\alpha}\right) \tag{S2}$$

where $\alpha = V_0/k_1$. Separation of variables gives

$$\frac{\mathrm{d}E}{E} = \frac{\mathrm{d}t}{t{+}\alpha} \tag{S3}$$

Integrating from time zero to t and from E_0 to $E(t)$ we get

$$\ln\left(\frac{E}{E_0}\right) = \ln\left(\frac{\alpha}{\alpha{+}t}\right) \quad \Rightarrow \quad E = E_0\left(\frac{\alpha}{\alpha{+}t}\right) \tag{S4}$$

Inserting E from Eq. S4 into the rate equation for A (Eq. 11)

$$\dot{A} = k_2{\cdot}E - A{\cdot}\frac{\dot{V}}{V} = k_2{\cdot}E - A{\cdot}\frac{k_1}{V_0{+}k_1{\cdot}t} = k_2{\cdot}E - A{\cdot}\frac{1}{\alpha{+}t} \tag{S5}$$

we get the following rate equation for A

$$\dot{A} + \frac{A}{\alpha{+}t} = k_2 E_0\left(\frac{\alpha}{\alpha{+}t}\right) \tag{S6}$$

The solution of Eq. S6 is given by Eq. 11, i.e.,

$$A(t) = k_2 E_0 \alpha + (A_0 - k_2 E_0 \alpha)\left(\frac{\alpha}{\alpha{+}t}\right) \tag{S7}$$