

Online supplemental materials for “On standardizing within-person effects: Potential problems of global standardization” by Wang, Zhang, Maxwell, and Bergeman (2018; *Multivariate Behavioral Research*)

Part A. Review of standardization approaches reported in recent empirical papers with time-varying covariates (TVCs)

Part B. Technical materials for the derivations (Appendices A, B, and C included)

Part C. The distribution of the number of time points under different Poisson distributions (meanT = sdT = 5, 10, 20, 30, 56, or 100) for the simulation study

Part D. Applying the derived formulas to calculate the sample correlations for the empirical example

Part E. Simulation results when N=50 or N=300 or when the standardized coefficients are nonnormally distributed

Part A. Review of standardization approaches reported in recent empirical papers with time-varying covariates (TVCs)

The review was conducted in January of 2018. We focused on the empirical papers that (1) cited Curran and Bauer (2011) and reported standardization approaches for predictors, outcomes, and/or standardized fixed-effects estimates; or (2) cited Schuurman et al. (2016) and reported standardization approaches for predictors, outcomes, and/or standardized fixed-effects estimates (in the tables and reference list, they are italicized).

1. Global standardization clearly described and used.

Paper	Descriptions on the standardization approaches	Note
Aafjes-van Doorn et al. (2017)	"To obtain standardized estimates of the within-person effects of our predictors in Models 2 and 3, we calculated β coefficients using the standard formula: $\beta = B (SD_x / SD_y)$."	Global standardization.
Armeli et al. (2014)	"To aid in the evaluation of the strength of the effects, we calculated standardized coefficients as per Hox (2010)."	Global standardization.
Foshee et al. (2013)	"Standardized regression coefficients were calculated by multiplying the estimate by the ratio of the standard deviations of the independent and dependent variables."	Global standardization.

2. Global standardization vaguely described and we suspect that global standardization was used.

Paper	Descriptions on the standardization approaches	Note
Hill et al. (2015)	"To allow for an easier way to interpret the coefficients, we standardized the outcome and predictor variables." "Within-client days-in-clinic and	Global standardization because if it's within-person standardization, no group mean centering is needed.

	within-therapist days-in-clinic were centered around the group mean”	
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3. Global standardization done on predictors and raw scores for outcomes

Paper	Descriptions on the standardization approaches	Note
Braun et al. (2015)	“the beta obtained for each predictor when the predictor was first standardized (M= 0, SD =1) prior to being entered in the model.”	Global standardization on predictors and raw scores for outcomes.
Wurpts (2016)	“Unlike in OLS linear regression, the formulae for calculating standardized regression coefficients are not as straightforward. However, one can obtain pseudo-standardized coefficients by multiplying the unstandardized coefficient by its sample standard deviation and dividing it by the residual variance of Y at its level.”	Global SD of X was used for the numerator and level-1 residual standard deviation was used for the denominator for standardizing within-person effects.

4. Within-person standardization clearly described and used

Paper	Descriptions on the standardization approaches	Note
Ramseyer et al. (2014)	Idiographic modeling “averaging the standardized regression weights across individuals”	Idiographic modeling (not multilevel modeling) with within-person standardization
<i>Dejonckheere et al. (2017)</i>	<i>“Variables were within-person standardized (Schuurman et al., 2016)”</i>	<i>Within-person standardization was implemented on the variables and then multilevel modeling was conducted on the WP standardized variables.</i>
<i>Dejonckheere et al. (2018)</i>	<i>“For comparison, we estimated all reported relationships also using multilevel models with within-person standardized outcome and predictor (both PA on NA and NA on PA) and report the results in the Supplementary</i>	<i>Within-person standardization was implemented on the variables and then modeling was conducted on the WP standardized variables.</i>

	<i>Materials (Tables 1–3). All models replicate our correlational findings, showing robustness across approaches.</i>	
<i>Lydon-Staley (2018)</i>	<i>“Both outcome and predictor variables were withinperson standardized before the analysis to minimize the extent to which associations between symptoms of depression and network density were driven by individual differences in emotion variance (Pe et al., 2015). A second motivation for using withinperson standardized variables was to render the coefficients representing different edges in the network comparable to one another, as raw regression coefficients are sensitive to scale and variance differences across variables (see Bringmann et al., 2016; Bulteel et al., 2016; Pe et al., 2015; Schuurman et al., 2016 for further discussions of this approach).”</i>	<i>Within-person standardization was implemented on the variables and then modeling was conducted on the WP standardized variables</i>

5. Within-person standardization done on predictors and raw scores for outcomes

Paper	Descriptions on the standardization approaches	Note
Berenson et al. (2011)	“Because momentary perceived rejection showed significant diagnostic group differences in both mean and variance, we standardized it within each individual to enable equating within-person momentary fluctuations in this variable across the entire sample (Std rejection).”	Within-person standardization on predictors and raw scores for outcomes.
Miller et al. (2017)	“Within-person deviations in (Level 1) depression and strain were calculated as a given assessment’s value minus a girl’s unique person mean across all visits divided by the girl’s unique	Within-person standardization on predictors and raw scores for outcomes. Unstandardized coefficients were reported.

	standard deviation (i.e., person-standardized).”	
Wilson (2017)	“we mean-standardized (i.e., z-scored) PTSD severity to statistically partial out effects of within-person daily PTSD symptoms opposed to overall, between-person PTSD symptoms over the entire monitoring period. Within-person PTSD severity was person-mean standardized (PMS) to capture the extent to which PTSD symptoms deviated from each participant’s personal mean on each day of monitoring. In other words, PMS PTSD reflects how mild/severe the participants’ PTSD symptoms were each day compared with their own personal average.”	Within-person standardization on predictors and raw scores for outcomes. Unstandardized coefficients were reported.

6. Procedure and purpose of standardization were not clearly described

Paper	Descriptions on the standardization approaches	Note
Ambwani et al. (2016)	“All self-efficacy variables were standardized to facilitate interpretation.”	Procedure and purpose of standardization were not clearly described. Only unstandardized coefficients were reported
Berry et al. (2017)		Procedure and purpose of standardization were not described. Standardized coefficients of TVCs were reported.
Buyukcan-Tetik et al. (2018)		Procedure and purpose of standardization were not described. Standardized coefficients of TVCs were reported.
Conklin et al. (2015)	“ β is the estimate obtained in the same model when predictors were standardized to a mean of 0 and an SD of 1. These standardized estimates	Procedure and purpose of standardization were not clearly described. Not sure how the predictors were standardized and

	show the change in BDI points associated with a one SD increase in the predictor (at each session)."	whether the outcomes were standardized. Standardized coefficients of TVCs were reported.
Freeman et al. (2017)	"All predictors were standardized before analysis for interpretation of effect sizes. Standardization permits us to interpret effect sizes without changing the nature or the pattern of significance of the estimated effects"	Not sure how the predictors were standardized and whether the outcomes were standardized.
Gills Jr. et al. (2016)		Procedure and purpose of standardization were not described. Standardized coefficients of TVCs were reported.
Sasso et al. (2016)	"We standardized raw, within-, and between-patient process scores to a M = 0 and SD = 1."	Procedure and purpose of standardization were not clearly described. Not sure how the predictors were standardized and whether the outcomes were standardized. Standardized coefficients of TVCs were reported.
Solmeyer et al. (2014)		Procedure and purpose of standardization were not described. Standardized coefficients of TVCs were reported.
Zilcha-Mano et al. (2017)		Procedure and purpose of standardization were not described. A standardized interaction effect between two time-varying variables was reported.
Zuroff et al. (2012)	"Self-Criticism was standardized prior to the analysis."	Procedure and purpose of standardization were not clearly described.

7. Others

Paper	Descriptions on the standardization approaches	Note
Falkenström et al. (2013)	"Because standardized estimates are not available for random coefficient models, only unstandardized estimates are reported."	No standardization because of its unavailability for random coefficient models.

References

- Aafjes-van Doorn, K., Lilliengren, P., Cooper, A., Macdonald, J., & Falkenström, F. (2017). Patients' affective processes within initial experiential dynamic therapy sessions. *Psychotherapy, 54*(2), 175-183.
- Ambwani, S., Berenson, K. R., Simms, L., Li, A., Corfield, F., & Treasure, J. (2016). Seeing things differently: An experimental investigation of social cognition and interpersonal behavior in anorexia nervosa. *International Journal of Eating Disorders, 49*(5), 499-506.
- Armeli, S., O'Hara, R. E., Ehrenberg, E., Sullivan, T. P., & Tennen, H. (2014). Episode-specific drinking-to-cope motivation, daily mood, and fatigue-related symptoms among college students. *Journal of Studies on Alcohol and Drugs, 75*(5), 766-774.
- Berenson, K. R., Downey, G., Rafaeli, E., Coifman, K. G., & Paquin, N. L. (2011). The rejection-rage contingency in borderline personality disorder. *Journal of Abnormal Psychology, 120*(3), 681-690.
- Berry, D., & Willoughby, M. T. (2017). On the practical interpretability of cross-lagged panel models: Rethinking a developmental workhorse. *Child Development, 88*(4), 1186-1206.
- Braun, J. D., Strunk, D. R., Sasso, K. E., & Cooper, A. A. (2015). Therapist use of Socratic questioning predicts session-to-session symptom change in cognitive therapy for depression. *Behaviour Research and Therapy, 70*, 32-37.
- Buyukcan-Tetik, A., Finkenauer, C., & Bleidorn, W. (2018). Within-person variations and between-person differences in self-control and wellbeing. *Personality and Individual Differences, 122*, 72-78.
- Conklin, L. R., & Strunk, D. R. (2015). A session-to-session examination of homework engagement in cognitive therapy for depression: Do patients experience immediate benefits? *Behaviour Research and Therapy, 72*, 56-62.
- Dejonckheere, E., Bastian, B., Fried, I. E., Murphy, S., & Kuppens, P. (2017). Perceiving social pressure not to feel negative predicts depressive symptoms in daily life. *Depression and Anxiety, 34*(9), 836-844.

Dejonckheere, E., Mestdagh, M., Houben, M., Erbas, Y., Pe, M., Bastian, B., Koval, P., Brose, A., & Kuppens, P. (2018). The bipolarity of affect and depressive symptoms. *Journal of Personality and Social Psychology, 114* (2), 323-341.

Falkenström, F., Granström, F., & Holmqvist, R. (2014). Working alliance predicts psychotherapy outcome even while controlling for prior symptom improvement. *Psychotherapy Research, 24*(2), 146-159.

Foshee, V. A., Benefield, T. S., Reyes, H. L. M., Ennett, S. T., Faris, R., Chang, L. Y., ... & Suchindran, C. M. (2013). The peer context and the development of the perpetration of adolescent dating violence. *Journal of Youth and Adolescence, 42*(4), 471-486.

Freeman, L. K., & Gottfredson, N. C. (2018). Using ecological momentary assessment to assess the temporal relationship between sleep quality and cravings in individuals recovering from substance use disorders. *Addictive Behaviors, 83*, 95-101.

Gillis, H. L. (L.), Jr., Kivlighan, D. M., Jr., & Russell, K. C. (2016). Between-client and within-client engagement and outcome in a residential wilderness treatment group: An actor partner interdependence analysis. *Psychotherapy, 53*(4), 413-423.

Hill, C. E., Baumann, E., Shafran, N., Gupta, S., Morrison, A., Rojas, A. E. P., ... & Gelso, C. J. (2015). Is training effective? A study of counseling psychology doctoral trainees in a psychodynamic/interpersonal training clinic. *Journal of Counseling Psychology, 62*(2), 184-201.

Lydon-Staley, D. M., Xia, M., Mak, H. W., & Fosco, G. (2018). *Adolescent Emotion Network Dynamics in Daily Life and Implications for Depression*. Obtained from psyarxiv.com.

Miller, A. B., Eisenlohr-Moul, T., Giletta, M., Hastings, P. D., Rudolph, K. D., Nock, M. K., & Prinstein, M. J. (2017). A within-person approach to risk for suicidal ideation and suicidal behavior: Examining the roles of depression, stress, and abuse exposure. *Journal of Consulting and Clinical Psychology, 85*(7), 712-722.

Ramseyer, F., Kupper, Z., Caspar, F., Znoj, H., & Tschacher, W. (2014). Time-series panel analysis (TSPA): Multivariate modeling of temporal associations in psychotherapy process. *Journal of Consulting and Clinical Psychology, 82*(5), 828-838.

Sasso, K. E., Strunk, D. R., Braun, J. D., DeRubeis, R. J., & Brotman, M. A. (2016). A re-examination of process–outcome relations in cognitive therapy for depression: Disaggregating within-patient and between-patient effects. *Psychotherapy Research, 26*(4), 387-398.

Solmeyer, A. R., McHale, S. M., & Crouter, A. C. (2014). Longitudinal associations between sibling relationship qualities and risky behavior across adolescence. *Developmental Psychology, 50*(2), 600-610.

Wilson, S. M., Krenek, M., Dennis, P. A., Yard, S. S., Browne, K. C., & Simpson, T. L. (2017). Daily associations between PTSD, drinking, and self-appraised alcohol-related problems. *Psychology of Addictive Behaviors, 31*(1), 27-35.

Wurpts, I. C. (2016). *Performance of Contextual Multilevel Models for Comparing Between-Person and Within-Person Effects* (Doctoral dissertation, Arizona State University).

Zilcha-Mano, S., Lipsitz, I., & Errázuriz, P. (2018). When is it Effective to Focus on the Alliance? Analysis of a Within-Client Moderator. *Cognitive Therapy and Research, 42*(2), 159-171.

Zuroff, D. C., Koestner, R., Moskowitz, D. S., McBride, C., & Bagby, R. M. (2012). Therapist's autonomy support and patient's self-criticism predict motivation during brief treatments for depression. *Journal of Social and Clinical Psychology, 31*(9), 903-932.

Part B: Technical materials for the derivations

Deriving the population correlations

Using the within-person means and standard deviations defined in the main text, we can express raw data, x_{it} and y_{it} , as

$$\begin{aligned} x_{it} &= s_{xi} \times x_{it}^{PS} + x_i, \\ y_{it} &= s_{yi} \times y_{it}^{PS} + y_i. \end{aligned} \tag{1}$$

where x_{it}^{PS} and y_{it}^{PS} are WP standardized scores of X and Y respectively. Due to within-person standardization, we have $E(X_{it}^{PS}|i) = E(Y_{it}^{PS}|i) = 0$ and $\sigma(X_{it}^{PS}|i) = \sigma(Y_{it}^{PS}|i) = 1$, indicating that for each individual, the population within-person means of X^{PS} and Y^{PS} are 0 and the population within-person standard deviations of X^{PS} and Y^{PS} are 1. Moreover, across all individuals and all time points, we always have $E(X^{PS}) = E(Y^{PS}) = 0$ and $\sigma(X^{PS}) = \sigma(Y^{PS}) = 1$, indicating the overall means are 0 and the overall standard deviations are 1 for both X^{PS} and Y^{PS} .

In Eq (1), $x_{it}^{PC} = s_{xi} \times x_{it}^{PS}$ and $y_{it}^{PC} = s_{yi} \times y_{it}^{PS}$ are the person-mean centered scores of X and Y respectively. Due to person-mean centering, we have $E(X_{it}^{PC}|i) = E(Y_{it}^{PC}|i) = 0$, indicating that for each individual, the population within-person means of X^{PC} and Y^{PC} are 0.

The population correlation between person-mean-SD standardized variables X^{PS} and Y^{PS}

As defined in the main text, $\rho_{w,i}$ is the population WP correlation between X and Y for individual i and $\mu_{\rho w}$ is the population average WP correlation. Mathematically, $\rho_{w,i}$ is also the the population correlation between X^{PS} and Y^{PS} of individual i . When WP correlations are homogeneous across individuals, we have $\rho_{w,i} = \mu_{\rho w}$ and thus $\mu_{\rho w}$ is also the homogeneous correlation between X^{PS} and Y^{PS} . When WP correlations are heterogeneous across individuals, at the population level (T is infinity and N is infinity), the correlation of stacked long data in X^{PS} and Y^{PS} is the population average WP correlation $\mu_{\rho w}$. Therefore, the population correlation of X^{PS} and Y^{PS} is $\mu_{\rho w}$, regardless of whether the WP correlations are homogeneous or heterogeneous.

The population correlation between raw variables X and Y

When σ_X , σ_Y , X^{PS} , and Y^{PS} follow a joint multivariate normal distribution, the correlation between stacked long raw variables (X and Y) at the population level is

$$\rho_{X,Y|normal} = \frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X,\sigma_Y})\mu_{\rho w} + \sigma_{\mu_X,\mu_Y}}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2 + \sigma_{\mu_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2 + \sigma_{\mu_Y}^2)}} = \frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X,\sigma_Y})\mu_{\rho w} + \sigma_{\mu_X}\sigma_{\mu_Y}\rho_b}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2 + \sigma_{\mu_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2 + \sigma_{\mu_Y}^2)}}. \quad (2)$$

When the normality assumption is relaxed, we have

$$\rho_{X,Y} = \frac{\mu_{\sigma_X}\mu_{\sigma_Y}\mu_{\rho w} + PROD_4 + \mu_{\sigma_X}PROD_{31} + \mu_{\sigma_Y}PROD_{32} + \sigma_{\mu_X}\sigma_{\mu_Y}\rho_b}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2 + \sigma_{\mu_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2 + \sigma_{\mu_Y}^2)}}. \quad (3)$$

$PROD_4 = E[(\sigma_X - E(\sigma_X))(\sigma_Y - E(\sigma_Y))(X^{PS})(Y^{PS})]$, $PROD_{31} = E[(\sigma_Y - E(\sigma_Y))(X^{PS})(Y^{PS})]$,

and $PROD_{32} = E[(\sigma_X - E(\sigma_X))(X^{PS})(Y^{PS})]$. Under the joint normality assumption for σ_X , σ_Y ,

X^{PS} , and Y^{PS} , $PROD_{31} = PROD_{32} = 0$ and $PROD_4 = \sigma_{\sigma_X,\sigma_Y}\mu_{\rho w}$. When WX and WY are not correlated, we have $\mu_{\rho w} = 0$ (average within-person correlation is 0) and

$PROD_4 = PROD_{31} = PROD_{32} = 0$. The derivations are shown in Appendix A of this document.

From Eqs (2; under the normality assumption) and (3; relaxing the normality assumption), we can see that $\rho_{X,Y}$, the population correlation between raw variables X and Y , is a combination of both the average within-person correlation $\mu_{\rho w}$ and the between-person correlation ρ_b . Even when the average within-person correlation $\mu_{\rho w}$ is 0, we have $\rho_{X,Y} = \frac{\sigma_{\mu_X}\sigma_{\mu_Y}\rho_b}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2 + \sigma_{\mu_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2 + \sigma_{\mu_Y}^2)}}$, which may not equal the between-person correlation ρ_b . Thus, $\rho_{X,Y}$ reflects neither the average within-person correlation $\mu_{\rho w}$ nor the between-person correlation ρ_b . Instead, it measures a conflated and often meaningless relation.

The population correlation between person-mean centered variables X^{PC} and Y^{PC}

The population correlation between person-mean centered variables (X^{PC} and Y^{PC}) with stacked long data under the joint normality assumption is

$$\rho_{CX,CY|normal} = \frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X,\sigma_Y})\mu_{\rho w}}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2)}}. \quad (4)$$

It can be shown that $\frac{(\mu_{\sigma X}\mu_{\sigma Y} + \sigma_{\sigma X,\sigma Y})}{\sqrt{(\mu_{\sigma X}^2 + \sigma_{\sigma X}^2)(\mu_{\sigma Y}^2 + \sigma_{\sigma Y}^2)}}$ ranges between -1 and 1, inclusively (See Appendix B of this document). Therefore, $|\rho_{CX,CY}|_{normal} \leq |\mu_{\rho w}|$.

Relaxing the joint normality assumption, we have

$$\rho_{CX,CY} = \frac{\mu_{\sigma X}\mu_{\sigma Y}\mu_{\rho w} + PROD4 + \mu_{\sigma X}PROD31 + \mu_{\sigma Y}PROD32 + \sigma_{\mu X,\mu Y}}{\sqrt{(\mu_{\sigma X}^2 + \sigma_{\sigma X}^2)(\mu_{\sigma Y}^2 + \sigma_{\sigma Y}^2)}}. \quad (5)$$

The derivations are shown in Appendix A of this document.

The population correlation between person-mean centered variables X^{PC} and Y^{PC} , $\rho_{CX,CY}$, is shown in Eq 4 (under normality) and Eq 5 (relaxing normality) The expressions for $\rho_{CX,CY}$ do not involve any terms related to the within-person mean variables. Thus, person-mean centering successfully removes the between-person correlation ρ_b from $\rho_{CX,CY}$. This also indicates that if one wants to disaggregate between- and within-person relations, person-mean centering is necessary unless there are no individual differences in WP means (both $\sigma_{\mu X}$ and $\sigma_{\mu Y}$ are 0). However, even after person-mean centering, $\rho_{CX,CY}$ is not always equal to the average within-person correlation $\mu_{\rho w}$. For example, under normality, only when $\frac{(\mu_{\sigma X}\mu_{\sigma Y} + \sigma_{\sigma X,\sigma Y})}{\sqrt{(\mu_{\sigma X}^2 + \sigma_{\sigma X}^2)(\mu_{\sigma Y}^2 + \sigma_{\sigma Y}^2)}} = 1$ and/or $\mu_{\rho w} = 0$, $\rho_{CX,CY}$ is equal to $\mu_{\rho w}$. When the normality assumption is relaxed, with $\mu_{\rho w} = 0$, $\rho_{CX,CY}$ is equal to $\mu_{\rho w}$; whereas with $\mu_{\rho w} \neq 0$, $\rho_{CX,CY}$ is generally not equal to $\mu_{\rho w}$.

Summary of the population correlation derivation results

Our derivation results revealed that (1) $\rho_{X,Y}$, the population correlation between raw variables X and Y , reflects neither the average within-person correlation $\mu_{\rho w}$ nor the between-person correlation ρ_b ; instead, it measures a conflated and often meaningless relation; (2) person-mean centering successfully removes the between-person correlation ρ_b from the population correlation between person-mean centered variables (X^{PC} and Y^{PC}), $\rho_{CX,CY}$; however, $\rho_{CX,CY}$ is still generally not equal to the the average within-person correlation $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$; and (3) the population correlation between within-person standardized variables (X^{PS} and Y^{PS}), $\rho_{WX,WY}$, is equal to $\mu_{\rho w}$, regardless of whether the data are

normally distributed or not and whether the within-person correlations are homogeneous or heterogeneous. In addition, we did not assume the covariances between a person mean variable and a person SD variable to be zero in the derivations and those covariances do not appear explicitly in the population correlation formulas.

Asymptotic performance of global standardization and P-S for estimating within-person relations

In this section, we analytically evaluate the asymptotic performance of global standardization and P-S for estimating the average within-person relations ($\mu_{\rho w}$), under homogeneous or heterogeneous within-person relation conditions.

Asymptotic performance of P-S under the homogeneous WP relation condition

Appendix C shows that with within-person standardization, the GLS estimator of γ_{10}^{PS} in Eq (9) of the main text under the homogeneous within-person relation condition is the sample correlation from stacked long data in X^{PS} and Y^{PS} , $r_{wx,wy}$. Therefore, when using the within-person standardized variables X^{PS} and Y^{PS} in multilevel modeling (Eq 9 of the main text), the GLS estimate is a consistent estimate of the homogeneous within-person correlation between the two variables, $\mu_{\rho w}$.

Asymptotic performance of global standardization under the homogeneous WP relation condition

$r_{cx,cy}$ is the GLS estimate of γ_{10}^{G3*} from global standardization under the homogeneous within-person relation condition (see Appendix C of this document). In addition, $\hat{\gamma}_{10,homo}^{G3*} = r_{cx,cy}$ and $\hat{\gamma}_{10,homo}^{G1*}$ asymptotically approach the same parameter, $\rho_{CX,CY}$. Earlier, we have shown that $\rho_{CX,CY}$ equals $\mu_{\rho w}$ only under strict conditions (e.g., $\mu_{\rho w} = 0$). Therefore, under the homogeneous within-person relation condition, $\hat{\gamma}_{10,homo}^{G3*}$ or $\hat{\gamma}_{10,homo}^{G1*}$ are generally inconsistent estimators of $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$. Now we discuss three specific scenarios for understanding how between-person differences in within-person

standard deviations play a role in the difference between $\rho_{CX,CY}$ and $\mu_{\rho w}$. Note that the three scenarios are ideal cases, under which for at least one of the time-varying variables, there are no individual differences in the within-person standard deviations. We determine under these ideal-case scenarios and under the homogeneous within-person relation condition, whether $\hat{\gamma}_{10,homo}^{G3*}$ and $\hat{\gamma}_{10,homo}^{G1*}$ are consistent estimators of $\mu_{\rho w}$.

Scenario 1: There are no between-person differences in the within-person standard deviations for either X or Y . In this case, we have $\sigma_{\sigma X}^2 = \sigma_{\sigma Y}^2 = 0$, $Var(X^{PC}) = \mu_{\sigma X}^2$, $Var(Y^{PC}) = \mu_{\sigma Y}^2$, $Cov(X^{PC}, Y^{PC}) = \mu_{\sigma X}\mu_{\sigma Y}\mu_{\rho w}$, and thus $\rho_{CX,CY} = \mu_{\rho w}$. Therefore, under Scenario 1, $\hat{\gamma}_{10,homo}^{G3*}$ and $\hat{\gamma}_{10,homo}^{G1*}$ are consistent estimators of $\mu_{\rho w}$.

Scenario 2: There are between-person differences in the within-person standard deviations for the outcome, but no such differences for the predictor. In this case, we have $\sigma_{\sigma X}^2 = 0$ but $\sigma_{\sigma Y}^2 \neq 0$. Then, $\rho_{CX,CY} = \frac{\mu_{\sigma Y}\mu_{\rho w} + PROD31}{\sqrt{(\mu_{\sigma Y}^2 + \sigma_{\sigma Y}^2)}}$. Therefore, under Scenario 2, $\rho_{CX,CY}$ can be different from $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$. For example, under the normality assumption, we have $\rho_{CX,CY} = \frac{\mu_{\sigma Y}\mu_{\rho w}}{\sqrt{(\mu_{\sigma Y}^2 + \sigma_{\sigma Y}^2)}} < \mu_{\rho w}$ when $\sigma_{\sigma X}^2 = 0$, $\sigma_{\sigma Y}^2 \neq 0$, and $\mu_{\rho w} \neq 0$.

Scenario 3: There are between-person differences in the within-person standard deviations for the predictor, but no such differences for the outcome. In this case, we have $\sigma_{\sigma Y}^2 = 0$ but $\sigma_{\sigma X}^2 \neq 0$. Then $\rho_{CX,CY} = \frac{\mu_{\sigma X}\mu_{\rho w} + PROD32}{\sqrt{(\mu_{\sigma X}^2 + \sigma_{\sigma X}^2)}}$. Therefore, under Scenario 3, $\rho_{CX,CY}$ can also be different from $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$. For example, under the normality assumption, we have $\rho_{CX,CY} = \frac{\mu_{\sigma X}\mu_{\rho w}}{\sqrt{(\mu_{\sigma X}^2 + \sigma_{\sigma X}^2)}} < \mu_{\rho w}$ when $\sigma_{\sigma Y}^2 = 0$, $\sigma_{\sigma X}^2 \neq 0$, and $\mu_{\rho w} \neq 0$.

The derivation results under Scenarios 2 and 3 clearly show that even when every person has the same WP standard deviation in one (not both) of the time varying variables, the *asymptotic* difference in $(\rho_{CX,CY} - \mu_{\rho w})$ may not be 0 when $\mu_{\rho w} \neq 0$. Thus, even under the ideal-case scenarios (Scenarios 2 and 3), $\hat{\gamma}_{10,homo}^{G3*}$ or $\hat{\gamma}_{10,homo}^{G1*}$ generally are inconsistent estimators of $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$.

Furthermore, $\hat{\gamma}_{10,homo}^{G2*}$ in Eq (6) of the main text asymptotically approaches $\rho_{CX,CY} \frac{\sigma_{CY}}{\sigma_Y}$, where

σ_{CY} and σ_Y are the population standard deviation of person-mean centered outcome Y^{PC} and original Y using stacked long data, respectively. Because $\sigma_{CY} \leq \sigma_Y$, $\rho_{CX,CY} \frac{\sigma_{CY}}{\sigma_Y}$ is equal to $\mu_{\rho w}$, also only under strict conditions. Actually, even under Scenario 1, $\rho_{CX,CY} \frac{\sigma_{CY}}{\sigma_Y}$ generally does not equal $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$. Thus, even under the ideal-case scenarios (Scenarios 1, 2, and 3), $\hat{\gamma}_{10,homo}^{G2*}$ generally is an inconsistent estimator of $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$.

Asymptotic performance of global standardization and P-S under the heterogeneous WP relation condition

When the within-person relations are heterogeneous across individuals, $u_{1i} \neq 0$ is true in reality and thus u_{1i} should be included in the multilevel models. Here, we consider a condition in which all individuals have the same number of time points, $T_i = T$. We focused on this scenario because the derivation results are simple and clear for one to easily evaluate the estimation performance of the standardization approaches. When $T_i = T$, the GLS estimate of γ_{10}^{PS} has a very simple form (see Appendix C for the derivation). That is,

$$\hat{\gamma}_{10,GLS|T_i=T}^{PS} = \hat{\gamma}_{10,GLS|T_i=T}^{PS*} = r_{wx,wy}, \quad (6)$$

Again, $r_{wx,wy}$ is the sample correlation between the stacked long person-mean-SD standardized variables and is a consistent estimate of the average within-person correlation between X^{PS} and Y^{PS} , $\mu_{\rho w}$. Now we use two of the previously discussed special scenarios to show that global standardization may not accurately estimate the average within-person relation under the heterogeneous within-person relation condition either. The third scenario was not included here because its results do not have a simple form. When $T_i = T$, the GLS estimate of $\hat{\gamma}_{10|T_i=T}^{G3}$ is given in Eq (10) of Appendix C.

Scenario 1: There are no between-person differences in the within-person standard deviations

for either X or Y . With $T_i = T$, $\hat{\gamma}_{10,GLS}^{G3} = r_{wx,wy} \frac{s_{cy}}{s_{cx}}$ and the standardized coefficient estimate

$\hat{\gamma}_{10,GLS}^{G3*} = r_{wx,wy} \frac{s_{cy}}{s_{cx}} \frac{s_{cx}}{s_{cy}} = r_{wx,wy}$ (see Appendix C for the derivation). Therefore, asymptotically, we

have $E(\hat{\gamma}_{10,GLS}^{G3*}) = \mu_{\rho w}$. This indicates that the standardized coefficient estimate, $\hat{\gamma}_{10,GLS}^{G3*}$, from global

standardization, is a consistent estimate of the average within-person relation $\mu_{\rho w}$ when there are no individual differences in the WP standard deviations for either of the variables.

Scenario 2: There are between-person differences in the within-person standard deviations for the outcome, but no such differences for the predictor. With $T_i = T$, we have $\hat{\gamma}_{10,GLS}^{G3} = r_{cx,cy} \frac{s_{cy}}{s_{cx}}$ and the standardized coefficient estimate $\hat{\gamma}_{10,GLS}^{G3*} = r_{cx,cy}$ (see Appendix C for the derivation). From the previous section, we have known that $r_{cx,cy}$ generally is an inconsistent estimate of $\mu_{\rho w}$ under Scenario 2 when $\mu_{\rho w} \neq 0$. Thus, even under this ideal-case scenario (no BP difference in WP SD for the predictor), we have shown that $\hat{\gamma}_{10,GLS}^{G3*}$ generally is an inconsistent estimate of $\mu_{\rho w}$ when u_{1i} is included/modeled in the multilevel models for the heterogeneous within-person relation condition and $\mu_{\rho w} \neq 0$.

$\hat{\gamma}_{10,GLS}^{G3*}$ and $\hat{\gamma}_{10,GLS}^{G1*}$ asymptotically approach the same parameter. So the above consistency results apply to $\hat{\gamma}_{10,GLS}^{G1*}$ as well. With regard to $\hat{\gamma}_{10,GLS}^{G2*}$ in the heterogeneous within-person relation condition, under Scenarios 1 and 2, $\hat{\gamma}_{10,GLS}^{G2*}$ asymptotically approaches to $\mu_{\rho w} \frac{\sigma_{CY}}{\sigma_Y}$ and $\rho_{CX,CY} \frac{\sigma_{CY}}{\sigma_Y}$ respectively. Therefore, under both Scenarios 1 and 2, $\hat{\gamma}_{10,GLS}^{G2*}$ generally is an inconsistent estimator of $\mu_{\rho w}$ when $\mu_{\rho w} \neq 0$ because of $\sigma_{CY} \leq \sigma_Y$ and the strict conditions for $\rho_{CX,CY}$ to equal $\mu_{\rho w}$.

Summary of the consistency derivation results

Our derivation results revealed that regardless of whether within-person relations are homogeneous or heterogeneous, global standardization (M_{G1} and M_{G3}) generally yields inconsistent estimates of the average within-person correlation ($\mu_{\rho w}$) when (1) $\mu_{\rho w} \neq 0$ and (2) there are between-person differences in the WP standard deviations of one or both of the time-varying variables. For M_{G2} , even under the ideal case that there are no BP differences in the WP standard deviations of either variables, the standardized estimates are generally inconsistent for $\mu_{\rho w}$ when the population grand SD of Y is different from that of person-mean centered Y ($\sigma_{CY} \neq \sigma_Y$) and $\mu_{\rho w} \neq 0$. In contrast, P-S yields consistent estimates of $\mu_{\rho w}$.

Note that the normality assumption was not used in the derivations. Under the heterogeneous within-person relations, the derivations were based on the condition in which all individuals have the same number of time points so that simple-form results can be obtained. In the next section, we evaluated the performance of the standardization methods for estimating and inferring within-person relations under both equal and unequal number of assessments conditions.

The aforementioned derivation results apply to the multilevel models with only one time-varying predictor or bivariate within-person relations. With two or more predictors, the standardized coefficients are functions of the relevant bivariate correlations. When the bivariate within-person relations are inconsistently recovered by a global standardization approach, one can infer that the standardized coefficient estimates from global standardization for multilevel models with two or more predictors are generally inconsistent estimates of the multivariate within-person relations.

References

- Bohrnstedt, G. W., & Goldberger, A. S. (1969). On the exact covariance of products of random variables. *Journal of the American Statistical Association*, 64, 1439-1442. doi: 10.2307/2286081
- Raudenbush, S., & Bryk, A. (2002). *Hierarchical linear models (second edition)*. Thousand Oaks, CA, US: Sage Publications.

Appendix A: Deriving the population correlations between raw variables or between person-mean centered variables

For raw variables, we have

$$X_{it} = \sigma_{X_i} \times X_{it}^{PS} + \mu_{X_i}$$

$$Y_{it} = \sigma_{Y_i} \times Y_{it}^{PS} + \mu_{Y_i}$$

Using the exact covariance of products of random variables derived by Bohrnstedt and Goldberger (1969) and the fact that neither person mean variable μ_X nor person SD variable σ_X is correlated with WP standardized variable X^{PS} (and neither μ_Y nor σ_Y is correlated with Y^{PS} , used later; note that μ_X and σ_X are allowed to be correlated and μ_Y and σ_Y are allowed to be correlated), the grand variance of person-mean centered X ($X^{PC} = \sigma_X \times X^{PS}$) is

$$\begin{aligned} Var(X^{PC}) &= [E(\sigma_X)]^2 Var(X^{PS}) + [E(X^{PS})]^2 Var(\sigma_X) + Var(X^{PS}) Var(\sigma_X) \\ &= \mu_{\sigma_X}^2 + 0 + \sigma_{\sigma_X}^2. \end{aligned}$$

Similarly, $Var(Y^{PC}) = \mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2$. Thus, there are two sources for the grand variance of a person-mean centered variable: the average of the person standard deviations and the variance in the person standard

deviations. The covariance of stacked long $X^{PC} = \sigma_X \times X^{PS}$ and μ_X is

$$\begin{aligned} Cov(X^{PC}, \mu_X) &= E(\sigma_X)Cov(X^{PS}, \mu_X) + E(X^{PS})Cov(\sigma_X, \mu_X) \\ &\quad + E[(\sigma_X - E(\sigma_X))(X^{PS} - E(X^{PS}))](\mu_X - E(\mu_X)) = 0 \end{aligned}$$

Thus, the grand variance of raw X is

$$\begin{aligned} Var(X) &= Var(\sigma_X \times X^{PS}) + 2Cov(\sigma_X \times X^{PS}, \mu_X) + Var(\mu_X) \\ &= \mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2 + \sigma_{\mu_X}^2. \end{aligned}$$

Similarly, $Var(Y) = \mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2 + \sigma_{\mu_Y}^2$. This expression indicates that there are three sources for the grand variance of a raw variable: the average of the person standard deviations, the variance in the person standard deviations, and the variance in the person means. Applying Eq (11) in Bohrnstedt and Goldberger (1969), with many of the terms being 0, we have

$$\begin{aligned} Cov(X^{PC}, Y^{PC}) &= E(\sigma_X)E(\sigma_Y)Cov(X^{PS}, Y^{PS}) \\ &\quad + E[(\sigma_X - E(\sigma_X))(\sigma_Y - E(\sigma_Y))(X^{PS} - E(X^{PS}))(Y^{PS} - E(Y^{PS}))] \\ &\quad + E(\sigma_X)E[(\sigma_Y - E(\sigma_Y))(X^{PS} - E(X^{PS}))(Y^{PS} - E(Y^{PS}))] \\ &\quad + E(\sigma_Y)E[(\sigma_X - E(\sigma_X))(X^{PS} - E(X^{PS}))(Y^{PS} - E(Y^{PS}))] \\ &= \mu_{\sigma_X}\mu_{\sigma_Y}\mu_{\rho_w} + PROD_4 + \mu_{\sigma_X}PROD_{31} + \mu_{\sigma_Y}PROD_{32}. \end{aligned}$$

$PROD_4 = E[(\sigma_X - E(\sigma_X))(\sigma_Y - E(\sigma_Y))(X^{PS})(Y^{PS})]$, $PROD_{31} = E[(\sigma_Y - E(\sigma_Y))(X^{PS})(Y^{PS})]$, and $PROD_{32} = E[(\sigma_X - E(\sigma_X))(X^{PS})(Y^{PS})]$. Because μ_Y and σ_X are not correlated with X^{PS} and X^{PS} is not correlated with μ_Y , the covariance of stacked long X and Y is

$$\begin{aligned} Cov(X, Y) &= Cov(X^{PC}, Y^{PC}) + Cov(\mu_X, \mu_Y) \\ &= \mu_{\sigma_X}\mu_{\sigma_Y}\mu_{\rho_w} + PROD_4 + \mu_{\sigma_X}PROD_{31} + \mu_{\sigma_Y}PROD_{32} + \sigma_{\mu_X, \mu_Y}. \end{aligned}$$

When σ_X , σ_Y , X^{PS} , and Y^{PS} follow a joint multivariate normal distribution, applying Eq (13) in

Bohrnstedt and Goldberger (1969), we have

$$\begin{aligned} Cov(X^{PC}, Y^{PC}) &= E(\sigma_X)E(\sigma_Y)Cov(X^{PS}, Y^{PS}) + Cov(\sigma_X, \sigma_Y)Cov(X^{PS}, Y^{PS}) \\ &= \mu_{\sigma_X}\mu_{\sigma_Y}\mu_{\rho_w} + \sigma_{\sigma_X, \sigma_Y}\mu_{\rho_w}, \end{aligned}$$

and

$$Cov(X, Y) = \mu_{\sigma_X}\mu_{\sigma_Y}\mu_{\rho_w} + \sigma_{\sigma_X, \sigma_Y}\mu_{\rho_w} + \sigma_{\mu_X, \mu_Y}.$$

Appendix B: Inequality in the numerator and denominator of

$$\frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X, \sigma_Y})}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2)}}$$

Here, we study inequality in the numerator and denominator of $\frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X, \sigma_Y})}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2)}}$. For the square of the numerator, we have

$$(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X, \sigma_Y})^2 = \mu_{\sigma_X}^2\mu_{\sigma_Y}^2 + \sigma_{\sigma_X, \sigma_Y}^2 + 2\mu_{\sigma_X}\mu_{\sigma_Y}\sigma_{\sigma_X, \sigma_Y}.$$

For the square of the denominator, we have

$$(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2) = \mu_{\sigma_X}^2\mu_{\sigma_Y}^2 + \mu_{\sigma_X}^2\sigma_{\sigma_Y}^2 + \sigma_{\sigma_X}^2\mu_{\sigma_Y}^2 + \sigma_{\sigma_X}^2\sigma_{\sigma_Y}^2.$$

Let $\Delta = (\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2) - (\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X, \sigma_Y})^2$. Then

$$\Delta = (\mu_{\sigma_X}\sigma_{\sigma_Y} - \sigma_{\sigma_X}\mu_{\sigma_Y})^2 + 2\mu_{\sigma_X}\mu_{\sigma_Y}\sigma_{\sigma_X}\sigma_{\sigma_Y}(1 - \rho_{\sigma_X, \sigma_Y}) + \sigma_{\sigma_X}^2\sigma_{\sigma_Y}^2(1 - \rho_{\sigma_X, \sigma_Y}^2). \quad (7)$$

Clearly, $\Delta \geq 0$ and thus the square of the denominator of $\frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X, \sigma_Y})}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2)}}$ is always greater than or equal to the squared numerator. Therefore, $\left| \frac{(\mu_{\sigma_X}\mu_{\sigma_Y} + \sigma_{\sigma_X, \sigma_Y})}{\sqrt{(\mu_{\sigma_X}^2 + \sigma_{\sigma_X}^2)(\mu_{\sigma_Y}^2 + \sigma_{\sigma_Y}^2)}} \right| \leq 1$.

Appendix C: GLS estimators of within-person effects and relations (γ_{10} s and

γ_{10}^* s) under various conditions

The GLS estimator of γ_{10}^{C2} (e.g., Raudenbush & Bryk, 2002) in Eq (4) of the main text is

$$\hat{\gamma}_{10, GLS}^{C2} = \left\{ \sum_i [\sigma_{C,u}^2 + \sigma_{C,e}^2 (CX_i' CX_i)^{-1}]^{-1} \right\}^{-1} \sum_i \{ [\sigma_{C,u}^2 + \sigma_{C,e}^2 (CX_i' CX_i)^{-1}]^{-1} (CX_i' CX_i)^{-1} CX_i' CY_i \}, \quad (8)$$

where $\sigma_{C,e}^2 = Var(e_{it}^{C2}|i)$, $\sigma_{C,u}^2 = Var(u_{1i}^{C2})$, $CX_i = X_i^{PC}$, and $CY_i = Y_i^{PC}$. When the multivariate normality assumption is met, the GLS estimator is also the ML estimator. When $\sigma_{C,u}^2 = 0$, the GLS estimator is the same as the OLS estimator, which is a function of the sample correlation between the two stacked long person-mean centered variables:

$$\hat{\gamma}_{10,GLS|\sigma_{C,u}^2=0}^{C2} = \left\{ \sum_i (CX_i'CX_i)^{-1} \sum (CX_i'CY_i) = r_{cy,cx} \frac{s_{cy}}{s_{cx}} \right\}. \quad (9)$$

When $T_i = T$, we have $CX_i'CX_i = (T-1) \times s_{xi}^2$ and $CX_i'CY_i = (T-1) \times r_i \times s_{xi}s_{yi}$. Define $A_i = \Pi_{i' \neq i} [(T-1)\sigma_{C,u}^2 s_{xi'}^2 + \sigma_{C,e}^2]$. Then Eq (8) can be reduced to

$$\hat{\gamma}_{10,GLS|T_i=T}^{C2} = \frac{\sum_i r_i s_{xi} s_{yi} A_i}{\sum_i s_{xi}^2 A_i}. \quad (10)$$

Scenario 1: When $T_i = T$, $s_{xi} = s_{cx}$, and $s_{yi} = s_{cy}$, we have $A_i = [(T-1)\sigma_u^2 s_{cx} + \sigma_e^2]^{N-1}$ and thus the coefficient estimate $\hat{\gamma}_{10}^{C2} = r_{wx,wy} \frac{s_{cy}}{s_{cx}}$ and the standardized coefficient estimate

$$\hat{\gamma}_{10}^{G3*} = r_{wx,wy} \frac{s_{cy}}{s_{cx}} \frac{s_{cx}}{s_{cy}} = r_{wx,wy}.$$

Scenario 2: When $T_i = T$, $s_{xi} = s_{cx}$, and $s_{yi} \neq s_{cy}$, we have

$$\hat{\gamma}_{10}^{C2} = \sum_i (r_i s_{yi}) / (N s_{cx}) = r_{cy,cx} \frac{s_{cy}}{s_{cx}}. \text{ The standardized coefficient estimate } \hat{\gamma}_{10}^{G3*} = r_{cy,cx}.$$

When $Var(CY_{it}|i) = 1$ and $Var(CX_{it}|i) = 1$ and thus the data have been person-mean-SD standardized, we have $CX_i'CX_i = T_i - 1$ and $CX_i'CY_i = (T_i - 1)r_i$. Then we have

$$\begin{aligned} \hat{\gamma}_{10,GLS}^{PS} &= \left\{ \sum_i [\sigma_{PS,u}^2 + \sigma_{PS,e}^2 / (T_i - 1)]^{-1} \right\}^{-1} \sum_i \{ [\sigma_{PS,u}^2 + \sigma_{PS,e}^2 / (T_i - 1)]^{-1} r_i \} \\ &= \sum_i w_i r_i, \end{aligned} \quad (11)$$

where $w_i = (\sigma_{PS,u}^2 + \sigma_{PS,e}^2 / (T_i - 1))^{-1} / [\sum_i (\sigma_{PS,u}^2 + \sigma_{PS,e}^2 / (T_i - 1))^{-1}]$. Thus $\hat{\gamma}_{10,GLS}^{PS}$ is a weighted average of the intra-individual correlations and the weight is a function of the number of time points of an individual.

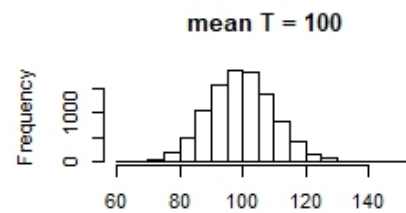
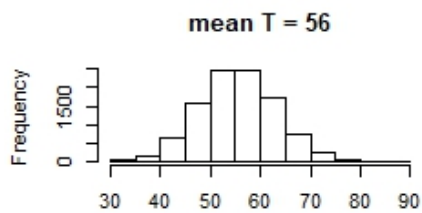
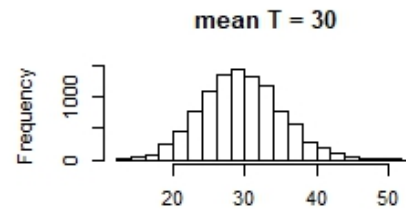
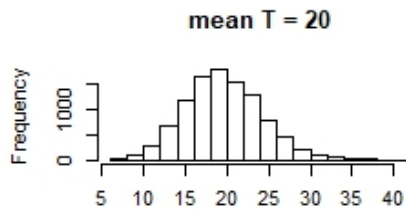
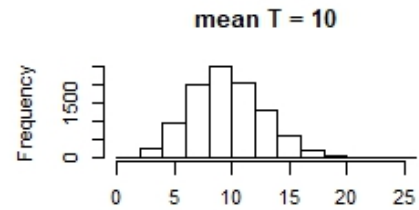
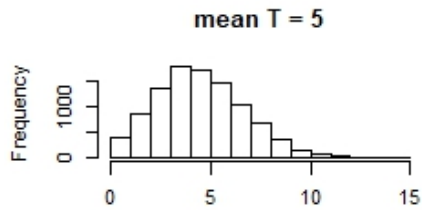
When $T_i = T$, regardless of the value for $\sigma_{PS,u}^2$ and $\sigma_{PS,e}^2$, we have $w_i = 1/N$ and thus

$$\hat{\gamma}_{10,GLS|T_i=T}^{PS} = \sum_i r_i / N = r_{wx,wy}, \text{ which is the sample correlation between the two stacked long}$$

person-mean-SD standardized variables.

When $\sigma_{PS,u}^2 = 0$, $\hat{\gamma}_{10,GLS|\sigma_{PS,u}^2=0}^{PS} = \sum_i [(T_i - 1) / \sum (T_i - 1)] r_i = r_{wx,wy}$, which is also the sample correlation between the two stacked long person-mean-SD standardized variables.

Part C. The distribution of the number of time points under different Poisson distributions ($meanT = sdT = 5, 10, 20, 30, 56, \text{ or } 100$) for the simulation study



Part D. Applying the derived formulas to calculate the sample correlations for the empirical example

The sample correlation between the within-person standardized NA and the within-person standardized Stress was .55: $\hat{\mu}_{\rho w} = .55$. The sample correlation of the person-mean centered NA and the person-mean centered stress was .65 and can be calculated as below.

$$\begin{aligned} r_{cx,cy} &= \frac{\hat{\mu}_{\sigma X} \hat{\mu}_{\sigma Y} \hat{\mu}_{\rho w} + PR\hat{O}D4 + \hat{\mu}_{\sigma X} PR\hat{O}D31 + \hat{\mu}_{\sigma Y} PR\hat{O}D32}{\sqrt{(\hat{\mu}_{\sigma X}^2 + \hat{\sigma}_{\sigma X}^2)(\hat{\mu}_{\sigma Y}^2 + \hat{\sigma}_{\sigma Y}^2)}} \\ &= \frac{2.68 \times 3.44 \times .55 + 1.05 + 2.68 \times .18 + 3.44 \times .26}{\sqrt{(2.68^2 + 1.72^2)(3.44^2 + 1.29^2)}} \\ &= .65. \end{aligned}$$

The sample correlation between raw NA and raw Stress was .72 and can be calculated as below.

$$\begin{aligned} r_{x,y} &= \frac{\hat{\mu}_{\sigma X} \hat{\mu}_{\sigma Y} \hat{\mu}_{\rho w} + PR\hat{O}D4 + \hat{\mu}_{\sigma X} PR\hat{O}D31 + \hat{\mu}_{\sigma Y} PR\hat{O}D32 + \hat{\sigma}_{\mu X, \mu Y}}{\sqrt{(\hat{\mu}_{\sigma X}^2 + \hat{\sigma}_{\sigma X}^2 + \hat{\sigma}_{\mu X}^2)(\hat{\mu}_{\sigma Y}^2 + \hat{\sigma}_{\sigma Y}^2 + \hat{\sigma}_{\mu Y}^2)}} \\ &= \frac{2.68 \times 3.44 \times .55 + 1.05 + 2.68 \times .18 + 3.44 \times .26 + 19.62}{\sqrt{(2.68^2 + 1.72^2 + 4.36^2)(3.44^2 + 1.29^2 + 5.92^2)}} \\ &= .72, \end{aligned}$$

where $PR\hat{O}D4 = Mean[(SNA - 2.68) \times (SStress - 3.44) \times (StdNA) \times (StdStress)]$, $PR\hat{O}D31 = Mean[(SStress - 3.44) \times (StdNA) \times (StdStress)]$, and $PR\hat{O}D32 = Mean[(SNA - 2.68) \times (StdNA) \times (StdStress)]$.

Part E. Simulation results when $N = 50$ or $N = 300$ or when the standardized coefficients are nonnormally distributed

Table 1: Simulation results: Biases and coverage rates of $\mu_{\rho w}$ estimates when $N = 50$. One predictor is included. M_{G1} : the person-mean centering (P-C) model

in Eq (2) followed by global standardization in Eq (5); M_{G2} : the P-C model in Eq (2) followed by global standardization in Eq (6); M_{G3} : the P-C model in Eq (4)

followed by global standardization in Eq (7); M_{PS} : the P-S approach in Eq (9); M_{EB} : the P-C model in Eq (2) with EB standardization.

Nper	Ntime	$\mu_{\rho w}$	Individual differences in within-person relations exist										No individual differences in within-person relations									
			Bias					Coverage rates					Bias				Coverage rates					
			M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{G1}	M_{G2}	M_{G3}	M_{PS}		
Between-person differences in within-person standard deviations of X and Y exist																						
50	5	0	0.00	0.00	0.00	0.00	0.00	92.0%	92.0%	91.8%	93.7%	34.6%	0.00	0.00	0.00	0.00	95.5%	95.5%	92.3%	93.6%		
50	10	0	0.00	0.00	0.00	0.00	0.00	93.3%	93.3%	93.9%	94.4%	58.2%	0.00	0.00	0.00	0.00	93.4%	93.4%	92.1%	94.2%		
50	20	0	0.00	0.00	0.00	0.00	0.00	93.7%	93.7%	94.2%	94.6%	73.8%	0.00	0.00	0.00	0.00	94.0%	94.0%	93.4%	95.5%		
50	30	0	0.00	0.00	0.00	0.00	0.00	93.0%	93.0%	93.4%	94.0%	75.7%	0.00	0.00	0.00	0.00	93.8%	93.8%	93.5%	96.2%		
50	56	0	0.00	0.00	0.00	0.00	0.00	94.9%	94.9%	94.6%	94.3%	88.8%	0.00	0.00	0.00	0.00	93.4%	93.4%	93.3%	95.3%		
50	100	0	0.00	0.00	0.00	0.00	0.00	94.6%	94.6%	94.6%	94.4%	93.4%	0.00	0.00	0.00	0.00	93.4%	93.4%	93.3%	95.4%		
50	5	-0.5	1.6%	-49.2%	8.0%	-8.6%	-9.9%	92.8%	2.7%	93.8%	90.9%	66.7%	-15.3%	-57.5%	-15.3%	-9.2%	81.5%	0.0%	72.9%	87.2%		
50	10	-0.5	12.5%	-40.5%	15.9%	-3.4%	-11.3%	88.4%	4.5%	85.5%	93.6%	58.9%	-15.4%	-55.3%	-15.4%	-4.3%	56.8%	0.0%	52.3%	93.0%		
50	20	-0.5	29.4%	-29.7%	30.9%	-1.5%	-8.4%	61.0%	19.6%	58.0%	94.1%	73.2%	-15.3%	-53.9%	-15.3%	-1.7%	25.4%	0.0%	23.9%	95.2%		
50	30	-0.5	38.2%	-24.3%	39.1%	-0.9%	-6.5%	41.1%	32.5%	39.7%	94.5%	79.1%	-15.2%	-53.3%	-15.2%	-0.8%	11.9%	0.0%	11.3%	96.3%		
50	56	-0.5	51.6%	-15.5%	52.0%	-0.7%	-4.3%	21.4%	56.2%	21.5%	94.7%	88.1%	-14.9%	-52.9%	-14.9%	-0.3%	2.7%	0.0%	2.5%	96.6%		
50	100	-0.5	62.6%	-9.9%	62.8%	0.1%	-2.2%	12.7%	67.1%	12.6%	94.2%	91.6%	-15.0%	-52.8%	-15.0%	0.1%	0.2%	0.0%	0.2%	98.1%		
Between-person differences in within-person standard deviations of Y exist but of X do not exist																						
50	5	0	0.00	0.00	0.00	0.00	0.00	94.2%	94.2%	94.0%	94.4%	40.2%	0.00	0.00	0.00	0.00	95.7%	95.7%	93.1%	92.7%		
50	10	0	0.00	0.00	0.00	0.00	0.00	93.9%	93.9%	94.1%	94.5%	50.8%	0.00	0.00	0.00	0.00	96.1%	96.1%	94.4%	94.8%		
50	20	0	0.00	0.00	0.00	0.00	0.00	94.4%	94.4%	94.5%	94.2%	67.8%	0.00	0.00	0.00	0.00	95.7%	95.7%	95.2%	94.8%		
50	30	0	0.00	0.00	0.00	0.00	0.00	94.5%	94.5%	94.4%	93.8%	76.5%	0.00	0.00	0.00	0.00	96.2%	96.2%	95.7%	94.5%		
50	56	0	0.00	0.00	0.00	0.00	0.00	94.8%	94.8%	94.9%	95.1%	86.0%	0.00	0.00	0.00	0.00	95.1%	95.1%	95.0%	94.7%		
50	100	0	0.00	0.00	0.00	0.00	0.00	94.6%	94.6%	94.5%	94.2%	89.2%	0.00	0.00	0.00	0.00	96.5%	96.5%	96.5%	95.6%		
50	5	-0.5	-3.5%	-51.9%	-3.4%	-7.2%	13.9%	97.7%	0.0%	97.4%	92.4%	59.0%	-4.2%	-52.1%	-4.2%	-9.9%	96.1%	0.0%	93.0%	85.4%		
50	10	-0.5	-4.1%	-49.3%	-4.1%	-3.4%	5.5%	95.1%	0.0%	94.9%	93.1%	68.6%	-3.8%	-49.1%	-3.8%	-3.9%	95.6%	0.0%	94.5%	92.9%		
50	20	-0.5	-4.4%	-48.1%	-4.4%	-1.8%	2.2%	93.1%	0.0%	93.0%	93.2%	78.6%	-3.7%	-47.5%	-3.7%	-1.6%	91.8%	0.0%	91.1%	95.9%		
50	30	-0.5	-3.8%	-47.2%	-3.8%	-0.7%	2.0%	95.3%	0.0%	95.2%	94.7%	86.1%	-3.5%	-46.9%	-3.5%	-0.6%	91.0%	0.0%	89.9%	97.6%		
50	56	-0.5	-4.4%	-47.0%	-4.4%	-0.8%	0.6%	93.2%	0.0%	93.2%	94.1%	90.6%	-3.7%	-46.6%	-3.7%	-0.3%	81.6%	0.0%	81.2%	96.9%		
50	100	-0.5	-3.9%	-46.4%	-3.9%	0.0%	0.8%	93.2%	0.0%	93.2%	92.9%	90.9%	-3.7%	-46.5%	-3.7%	0.0%	67.0%	0.0%	66.8%	97.6%		
Between-person differences in within-person standard deviations of X and Y do not exist																						
50	5	0	0.00	0.00	0.00	0.00	0.00	93.2%	93.2%	93.9%	95.1%	41.0%	0.00	0.00	0.00	0.00	96.0%	96.0%	93.3%	92.5%		
50	10	0	0.00	0.00	0.00	0.00	0.00	92.8%	92.8%	93.0%	94.1%	52.2%	0.00	0.00	0.00	0.00	96.6%	96.6%	95.3%	94.6%		
50	20	0	0.00	0.00	0.00	0.00	0.00	94.5%	94.5%	94.4%	94.6%	70.7%	0.00	0.00	0.00	0.00	96.3%	96.3%	95.9%	95.8%		
50	30	0	0.00	0.00	0.00	0.00	0.00	94.5%	94.5%	94.5%	94.4%	79.9%	0.00	0.00	0.00	0.00	94.1%	94.1%	93.9%	94.4%		
50	56	0	0.00	0.00	0.00	0.00	0.00	94.7%	94.7%	94.7%	94.9%	88.8%	0.00	0.00	0.00	0.00	95.3%	95.3%	95.2%	95.7%		
50	100	0	0.00	0.00	0.00	0.00	0.00	95.0%	95.0%	95.0%	95.1%	90.6%	0.00	0.00	0.00	0.00	96.4%	96.4%	96.3%	96.2%		
50	5	-0.5	0.1%	-51.3%	0.0%	-7.6%	7.4%	96.5%	0.0%	96.3%	92.4%	64.1%	0.2%	-51.3%	0.2%	-9.8%	97.0%	0.0%	95.6%	84.3%		
50	10	-0.5	0.0%	-48.4%	0.0%	-3.3%	1.0%	96.9%	0.0%	97.1%	94.7%	69.7%	0.4%	-48.1%	0.4%	-3.9%	97.9%	0.0%	97.6%	94.2%		
50	20	-0.5	-0.1%	-47.2%	-0.1%	-1.6%	-0.4%	94.9%	0.0%	95.0%	93.8%	80.2%	0.5%	-46.7%	0.5%	-1.5%	97.0%	0.0%	96.7%	96.0%		
50	30	-0.5	-0.3%	-46.7%	-0.3%	-1.2%	-0.7%	94.7%	0.0%	94.7%	93.8%	83.1%	0.4%	-46.5%	0.4%	-0.9%	97.4%	0.0%	97.4%	97.3%		
50	56	-0.5	-0.3%	-46.1%	-0.3%	-0.8%	-0.6%	94.2%	0.0%	94.2%	93.6%	88.2%	0.3%	-46.3%	0.3%	-0.4%	97.9%	0.0%	97.8%	96.5%		
50	100	-0.5	-0.2%	-46.3%	-0.2%	-0.5%	-0.4%	95.5%	0.0%	95.5%	95.0%	93.4%	0.6%	-45.8%	0.6%	0.2%	98.3%	0.0%	98.2%	98.0%		

Table 2: Simulation results: Biases and coverage rates of μ_{pw} estimates when $N = 300$. One predictor is included. M_{G1} : the person-mean centering (P-C) model in Eq (2) followed by global standardization in Eq (5); M_{G2} : the P-C model in Eq (2) followed by global standardization in Eq (6); M_{G3} : the P-C model in Eq (4) followed by global standardization in Eq (7); M_{PS} : the P-S approach in Eq (9); M_{EB} : the P-C model in Eq (2) with EB standardization.

Nper	Ntime	μ_{pw}	Individual differences in within-person relations exist										No individual differences in within-person relations									
			Bias					Coverage rates					Bias				Coverage rates					
			M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{G1}	M_{G2}	M_{G3}	M_{PS}		
Between-person differences in within-person standard deviations of X and Y exist																						
300	5	0	0.00	0.00	0.00	0.00	0.00	93.5	93.5	93.5	94.7	27.2	0.00	0.00	0.00	0.00	92.3	92.3	87.8	91.0		
300	10	0	0.00	0.00	0.00	0.00	0.00	95.1	95.1	94.3	93.1	45.4	0.00	0.00	0.00	0.00	93.8	93.8	91.5	94.5		
300	20	0	0.00	0.00	0.00	0.00	0.00	94.8	94.8	94.6	95.6	65.7	0.00	0.00	0.00	0.00	92.1	92.1	91.4	94.1		
300	30	0	0.00	0.00	0.00	0.00	0.00	93.7	93.7	94.0	94.7	75.0	0.00	0.00	0.00	0.00	93.3	93.3	92.9	95.3		
300	56	0	0.00	0.00	0.00	0.00	0.00	94.4	94.4	94.4	94.6	87.3	0.00	0.00	0.00	0.00	93.3	93.3	93.2	95.7		
300	100	0	0.00	0.00	0.00	0.00	0.00	95.8	95.8	95.9	94.8	91.1	0.00	0.00	0.00	0.00	91.7	91.7	91.3	94.6		
300	5	-0.5	2.3	-49.4	9.5	-7.6	-9.1	93.7	0.0	78.6	69.2	33.6	-15.0	-58.0	-15.0	-8.9	15.9	0.0	10.3	48.9		
300	10	-0.5	13.1	-40.7	16.4	-3.1	-11.2	50.6	0.0	34.0	86.7	7.8	-15.3	-55.6	-15.3	-3.6	0.2	0.0	0.1	77.0		
300	20	-0.5	28.4	-30.7	29.9	-1.4	-8.6	0.8	0.0	0.3	92.1	15.6	-15.6	-54.5	-15.6	-1.6	0.0	0.0	0.0	88.1		
300	30	-0.5	37.5	-25.1	38.4	-0.9	-6.7	0.0	0.5	0.0	92.7	31.2	-15.6	-54.1	-15.6	-0.8	0.0	0.0	0.0	91.1		
300	56	-0.5	51.5	-17.0	51.9	-0.5	-4.2	0.0	10.2	0.0	95.3	62.7	-15.6	-53.7	-15.6	-0.2	0.0	0.0	0.0	95.0		
300	100	-0.5	60.9	-11.6	61.1	-0.3	-2.6	0.0	35.2	0.0	94.8	79.6	-15.5	-53.6	-15.5	0.1	0.0	0.0	0.0	96.0		
Between-person differences in within-person standard deviations of Y exist but of X do not exist																						
300	5	0	0.00	0.00	0.00	0.00	0.00	93.5	93.5	93.9	94.6	29.8	0.00	0.00	0.00	0.00	95.9	95.9	93.7	92.9		
300	10	0	0.00	0.00	0.00	0.00	0.00	94.2	94.2	93.9	94.2	47.5	0.00	0.00	0.00	0.00	95.3	95.3	93.2	93.1		
300	20	0	0.00	0.00	0.00	0.00	0.00	96.0	96.0	96.1	96.0	72.4	0.00	0.00	0.00	0.00	94.7	94.7	94.1	94.2		
300	30	0	0.00	0.00	0.00	0.00	0.00	94.4	94.4	94.3	95.1	77.7	0.00	0.00	0.00	0.00	95.7	95.7	95.1	95.4		
300	56	0	0.00	0.00	0.00	0.00	0.00	95.1	95.1	95.1	95.5	85.1	0.00	0.00	0.00	0.00	95.4	95.4	95.2	95.7		
300	100	0	0.00	0.00	0.00	0.00	0.00	95.2	95.2	95.2	95.6	91.3	0.00	0.00	0.00	0.00	95.9	95.9	95.7	95.8		
300	5	-0.5	-3.9	-52.6	-3.9	-7.4	13.9	91.8	0.0	92.0	71.0	14.8	-4.1	-52.6	-4.1	-9.6	89.9	0.0	83.9	41.6		
300	10	-0.5	-4.4	-49.9	-4.4	-3.6	5.2	87.1	0.0	86.8	84.8	50.1	-3.7	-49.5	-3.7	-3.8	82.1	0.0	78.6	75.4		
300	20	-0.5	-4.0	-48.3	-4.0	-1.4	2.8	83.8	0.0	83.9	92.2	75.1	-3.8	-48.1	-3.8	-1.7	59.0	0.0	57.1	87.0		
300	30	-0.5	-4.0	-47.8	-4.0	-0.9	1.9	81.0	0.0	81.0	93.6	84.3	-3.7	-47.6	-3.7	-0.9	38.9	0.0	37.8	90.1		
300	56	-0.5	-4.2	-47.5	-4.2	-0.5	1.0	76.4	0.0	76.4	93.9	90.5	-3.7	-47.3	-3.7	-0.2	12.5	0.0	12.5	95.7		
300	100	-0.5	-4.1	-47.3	-4.1	-0.3	0.5	74.5	0.0	74.5	93.9	92.3	-3.7	-47.0	-3.7	0.2	0.9	0.0	0.9	96.3		
Between-person differences in within-person standard deviations of X and Y do not exist																						
300	5	0	0.00	0.00	0.00	0.00	0.00	95.2	95.2	95.2	94.6	29.8	0.00	0.00	0.00	0.00	94.3	94.3	91.4	91.7		
300	10	0	0.00	0.00	0.00	0.00	0.00	93.8	93.8	93.9	93.5	51.9	0.00	0.00	0.00	0.00	95.1	95.1	94.0	95.0		
300	20	0	0.00	0.00	0.00	0.00	0.00	94.6	94.6	94.7	95.3	70.0	0.00	0.00	0.00	0.00	94.6	94.6	94.1	94.0		
300	30	0	0.00	0.00	0.00	0.00	0.00	94.2	94.2	94.2	94.0	79.0	0.00	0.00	0.00	0.00	94.4	94.4	94.0	94.6		
300	56	0	0.00	0.00	0.00	0.00	0.00	94.6	94.6	94.6	94.6	87.3	0.00	0.00	0.00	0.00	94.9	94.9	94.9	95.2		
300	100	0	0.00	0.00	0.00	0.00	0.00	95.2	95.2	95.2	95.0	91.3	0.00	0.00	0.00	0.00	94.7	94.7	94.6	94.6		
300	5	-0.5	0.1	-52.0	0.0	-7.7	7.4	96.9	0.0	97.0	67.3	40.8	0.4	-51.7	0.4	-9.3	98.2	0.0	95.9	43.8		
300	10	-0.5	0.0	-49.2	0.0	-3.4	0.9	95.7	0.0	95.6	84.3	62.7	0.6	-48.8	0.6	-3.6	97.8	0.0	97.2	76.9		
300	20	-0.5	0.0	-47.7	0.0	-1.5	-0.3	95.5	0.0	95.5	91.4	77.5	0.5	-47.4	0.5	-1.5	98.0	0.0	97.5	88.3		
300	30	-0.5	-0.1	-47.2	-0.1	-1.1	-0.5	93.0	0.0	93.0	90.3	80.0	0.4	-47.0	0.4	-0.9	97.8	0.0	97.6	90.9		
300	56	-0.5	0.1	-46.7	0.1	-0.5	-0.3	96.4	0.0	96.4	94.9	88.8	0.4	-46.6	0.4	-0.3	97.2	0.0	96.9	94.1		
300	100	-0.5	0.0	-46.6	0.0	-0.3	-0.2	95.0	0.0	95.0	95.0	92.0	0.5	-46.3	0.5	0.1	97.7	0.0	97.7	95.9		

Table 3: Simulation results from the models with two predictors included when $N = 50$. M_{G1} : the person-mean centering (P-C) model in Eq (2) followed by global standardization in Eq (5); M_{G2} : the P-C model in Eq (2) followed by global standardization in Eq (6); M_{G3} : the P-C model in Eq (4) followed by global standardization in Eq (7); M_{PS} : the P-S approach in Eq (9); M_{EB} : the P-C model in Eq (2) with EB standardization.

Nper	Ntime	X_1 on Y (True value =0)										X_2 on Y (True value = -0.4)									
		Bias					Coverage rates (%)					Relative bias (%)					Coverage rates (%)				
		M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}
Between-person differences in within-person standard deviations of X and Y exist																					
Equal number of assessments across individuals																					
50	5	-0.02	0.00	-0.02	-0.02	-0.02	92.8	93.4	92.9	94.1	50.9	-5.3	-52.8	-2.0	-13.0	8.3	92.2	3.9	94.9	90.9	84.6
50	10	-0.01	0.00	-0.01	-0.01	-0.01	94.8	94.0	94.3	94.3	66.4	-1.6	-48.0	0.2	-6.9	-2.0	93.5	2.3	94.2	92.7	80.8
50	20	-0.01	0.00	-0.01	-0.01	-0.01	93.9	94.2	94.2	94.4	80.7	4.3	-43.2	4.7	-3.6	-3.5	94.6	5.0	95.2	94.4	88.0
50	30	-0.01	0.00	-0.01	0.00	0.00	92.5	92.6	92.7	92.8	86.4	8.5	-40.4	8.9	-1.9	-2.7	93.1	9.1	93.2	93.1	90.6
50	56	-0.01	0.00	-0.01	0.00	-0.01	93.7	94.1	93.7	94.2	91.9	10.9	-38.6	11.0	-1.8	-2.7	93.8	11.4	93.6	94.7	93.9
50	100	-0.01	0.00	-0.01	0.00	0.00	94.8	94.3	94.7	94.4	92.8	14.2	-36.6	14.3	-1.3	-1.8	92.8	17.9	92.7	96.2	96.6
Unequal number of assessments across individuals																					
50	5	-0.02	0.00	-0.01	-0.02	0.00	93.3	93.3	92.7	94.9	50.1	-0.7	-50.2	-0.8	-12.6	51.8	93.2	7.4	92.4	92.0	92.3
50	10	-0.01	0.00	-0.01	-0.01	-0.02	93.2	93.2	94.0	94.9	65.7	-2.5	-48.2	-1.1	-8.3	0.1	93.0	3.3	94.4	92.3	81.6
50	20	-0.01	0.00	-0.01	-0.01	-0.01	93.6	93.6	93.6	94.5	79.1	4.0	-43.4	4.7	-3.6	-3.7	94.0	6.1	93.7	93.5	86.4
50	30	-0.01	0.00	-0.01	0.00	0.00	93.6	93.6	93.7	93.4	84.7	8.4	-40.2	8.8	-1.7	-2.3	93.4	8.1	93.6	93.5	90.6
50	56	-0.01	0.00	-0.01	0.00	0.00	93.2	93.2	93.5	94.1	90.7	13.0	-37.6	13.1	-0.7	-1.2	92.4	13.0	92.4	95.0	94.8
50	100	-0.01	0.00	-0.01	0.00	0.00	94.1	94.1	94.3	94.5	93.1	14.2	-36.6	14.2	-0.4	-0.7	92.4	17.9	92.3	93.8	96.6
Between-person differences in within-person standard deviations of X and Y do not exist																					
Equal number of assessments across individuals																					
50	5	-0.01	0.00	-0.01	-0.02	-0.01	93.2	93.9	94.5	94.7	51.2	-4.1	-53.2	-3.4	-12.5	4.5	93.8	1.7	94.6	92.3	82.5
50	10	-0.01	0.00	-0.01	-0.01	-0.01	93.7	94.6	94.3	94.1	68.3	-3.3	-50.4	-3.2	-6.8	-1.8	93.3	0.3	93.5	93.3	81.3
50	20	-0.01	0.00	-0.01	-0.01	-0.01	93.3	93.9	93.4	93.6	81.0	-2.6	-48.2	-2.5	-4.1	-2.7	94.4	0.2	94.4	93.9	89.1
50	30	0.00	0.00	0.00	0.00	0.00	94.6	94.9	94.7	94.9	88.4	-1.5	-47.4	-1.4	-2.4	-1.7	94.2	0.1	94.5	94.7	92.0
50	56	0.00	0.01	0.00	0.00	0.00	94.0	93.8	93.9	93.8	91.0	0.1	-46.2	0.1	-0.4	-0.2	94.9	0.5	94.9	94.5	96.0
50	100	0.00	0.01	0.00	0.00	0.00	94.5	94.4	94.5	94.7	93.3	-0.5	-46.3	-0.5	-0.7	-0.7	94.1	0.6	94.1	93.9	94.9
Unequal number of assessments across individuals																					
50	5	-0.01	0.00	-0.01	-0.02	-0.05	94.2	94.4	94.6	94.2	53.5	-3.76	-52.88	-2.70	-12.42	40.45	94.8	2.2	95.5	93.0	88.0
50	10	-0.01	0.00	-0.01	-0.01	-0.01	93.5	93.5	93.7	93.5	66.2	-2.63	-49.80	-2.34	-6.71	0.25	93.2	0.7	93.5	92.2	83.7
50	20	0.00	0.00	0.00	0.00	0.00	92.9	93.1	92.8	92.4	79.4	-1.72	-47.81	-1.61	-3.48	-1.81	93.5	0.5	93.6	92.8	87.6
50	30	0.00	0.00	0.00	0.00	0.00	95.3	95.6	95.1	94.8	87.6	-1.66	-47.08	-1.60	-2.50	-1.82	95.2	0.1	95.3	94.3	91.8
50	56	0.00	0.01	0.00	0.00	0.00	94.9	94.6	94.9	95.1	92.1	-0.07	-46.12	-0.06	-0.49	-0.29	94.3	0.4	94.4	94.4	94.8
50	100	0.00	0.01	0.00	0.00	0.00	94.9	95.3	94.9	94.8	93.7	0.38	-45.70	0.38	0.17	0.24	94.8	0.2	94.8	94.3	96.5

Table 4: Simulation results from the models with two predictors included when $N = 300$. M_{G1} : the person-mean centering (P-C) model in Eq (2) followed by global standardization in Eq (5); M_{G2} : the P-C model in Eq (2) followed by global standardization in Eq (6); M_{G3} : the P-C model in Eq (4) followed by global standardization in Eq (7); M_{PS} : the P-S approach in Eq (9); M_{EB} : the P-C model in Eq (2) with EB standardization.

Nper	Ntime	X_1 on Y (True value =0)										X_2 on Y (True value = -0.4)									
		Bias					Coverage rates (%)					Relative bias (%)					Coverage rates (%)				
		M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}
Between-person differences in within-person standard deviations of X and Y exist																					
Equal number of assessments across individuals																					
300	5	-0.01	0.00	-0.01	-0.02	-0.01	92.4	91.8	93.6	93.3	35.0	-7.1	-54.2	-2.5	-12.7	2.5	88.6	0.0	94.8	73.2	83.7
300	10	-0.01	0.00	-0.01	-0.01	-0.01	93.6	93.8	93.6	93.2	63.4	-1.9	-48.6	-0.2	-6.7	-4.2	94.3	0.0	94.4	83.9	67.4
300	20	0.00	0.00	0.00	0.00	0.00	95.8	94.7	95.9	95.7	83.1	4.9	-43.5	5.6	-3.0	-3.7	91.4	0.0	90.7	92.6	79.7
300	30	0.00	0.00	0.00	0.00	0.00	94.6	93.9	94.9	94.1	86.7	8.1	-41.3	8.5	-1.9	-3.0	84.2	0.0	83.2	93.8	84.9
300	56	-0.01	0.00	-0.01	0.00	0.00	94.2	93.8	94.3	94.2	90.5	12.4	-38.6	12.6	-1.1	-1.9	69.3	0.0	68.9	94.7	91.3
300	100	-0.01	0.00	-0.01	0.00	0.00	94.8	94.6	94.7	94.6	92.6	15.2	-36.7	15.2	-0.4	-1.0	56.8	0.0	56.6	94.5	94.9
Unequal number of assessments across individuals																					
300	5	-0.01	0.00	-0.01	-0.02	-0.02	94.0	94.0	94.0	92.4	40.6	-5.6	-53.4	-1.4	-12.7	39.3	92.8	0.0	95.1	74.9	98.3
300	10	-0.01	0.00	-0.01	-0.01	-0.01	94.2	94.2	94.2	94.3	58.9	-1.7	-48.5	-0.1	-7.1	-2.5	95.2	0.0	95.1	85.1	74.5
300	20	-0.01	0.00	-0.01	0.00	-0.01	93.3	93.3	93.7	93.8	80.8	3.9	-44.0	4.5	-3.7	-4.1	93.2	0.0	92.6	92.3	76.7
300	30	-0.01	0.00	-0.01	0.00	0.00	93.4	93.4	93.4	93.9	85.8	8.2	-41.3	8.5	-2.2	-3.1	83.2	0.0	82.1	93.3	85.3
300	56	-0.01	0.00	-0.01	0.00	0.00	94.2	94.2	94.4	95.7	91.2	12.6	-38.2	12.7	-1.0	-1.9	66.7	0.0	66.4	95.6	91.4
300	100	-0.01	0.00	-0.01	0.00	0.00	93.9	93.9	94.0	94.6	92.7	15.3	-36.7	15.4	-0.4	-0.9	56.5	0.0	56.5	95.1	94.2
Between-person differences in within-person standard deviations of X and Y do not exist																					
Equal number of assessments across individuals																					
300	5	-0.01	0.00	-0.01	-0.02	-0.01	94.6	95.3	94.9	93.3	41.1	-4.1	-54.0	-3.3	-12.9	4.1	94.9	0.0	96.0	69.9	87.2
300	10	-0.01	0.00	-0.01	-0.01	-0.01	94.7	95.5	95.3	93.4	66.3	-2.8	-50.6	-2.5	-6.6	-1.5	95.5	0.0	95.5	86.6	79.9
300	20	-0.01	0.00	-0.01	-0.01	-0.01	95.4	96.2	95.7	95.7	83.4	-1.8	-48.7	-1.7	-3.4	-1.9	94.3	0.0	94.7	92.3	87.4
300	30	0.00	0.00	0.00	0.00	0.00	94.6	93.6	94.6	94.2	85.8	-1.5	-48.0	-1.4	-2.4	-1.7	93.8	0.0	94.1	93.0	88.7
300	56	0.00	0.00	0.00	0.00	0.00	95.2	95.4	95.2	95.2	92.7	-1.0	-47.4	-1.0	-1.5	-1.3	97.9	0.0	97.8	97.0	95.3
300	100	0.00	0.01	0.00	0.00	0.00	95.2	94.6	95.2	95.2	93.5	-0.3	-46.8	-0.2	-0.5	-0.4	95.9	0.0	95.9	96.2	96.4
Unequal number of assessments across individuals																					
300	5	-0.01	0.00	-0.01	-0.02	-0.02	94.3	94.5	94.1	92.4	41.8	-4.06	-53.87	-3.37	-12.89	46.36	95.3	0.0	95.0	72.9	98.6
300	10	-0.01	0.00	-0.01	-0.01	-0.01	94.8	94.0	94.9	93.7	64.8	-3.09	-50.71	-2.81	-7.13	-0.24	94.4	0.0	94.7	84.4	82.1
300	20	0.00	0.00	0.00	-0.01	-0.01	94.2	93.4	94.4	94.8	80.4	-1.70	-48.55	-1.61	-3.35	-1.71	94.4	0.0	94.6	90.9	85.3
300	30	0.00	0.00	0.00	0.00	0.00	92.6	92.6	92.6	93.0	84.6	-1.83	-48.23	-1.77	-2.86	-2.13	94.4	0.0	94.7	93.2	88.5
300	56	0.00	0.00	0.00	0.00	0.00	94.6	93.9	94.5	94.7	90.7	-0.88	-47.30	-0.86	-1.39	-1.17	94.4	0.0	94.4	93.6	92.9
300	100	0.00	0.01	0.00	0.00	0.00	94.8	93.6	94.9	94.7	92.9	-0.33	-46.80	-0.32	-0.58	-0.51	95.2	0.0	95.2	95.0	96.1

Table 5: Simulation results from the models with one predictor included and the unstandardized coefficients are normally distributed but the standardized coefficients are nonnormally distributed. M_{G1} : the person-mean centering (P-C) model in Eq (2) followed by global standardization in Eq (5); M_{G2} : the P-C model in Eq (2) followed by global standardization in Eq (6); M_{G3} : the P-C model in Eq (4) followed by global standardization in Eq (7); M_{PS} : the P-S approach in Eq (9); M_{EB} : the P-C model in Eq (2) with EB standardization.

Nper	Ntime	$\mu_{pw} = -.40$									
		Bias					Coverage rates (%)				
		M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}	M_{G1}	M_{G2}	M_{G3}	M_{PS}	M_{EB}
Between-person differences in within-person standard deviations of X and Y exist											
50	5	39.3%	-61.2%	37.9%	-3.9%	9.7%	71.3%	0.1%	76.1%	94.4%	81.8%
50	10	37.1%	-59.2%	35.4%	-2.1%	3.8%	69.7%	0.0%	74.6%	94.6%	89.5%
50	20	38.5%	-58.2%	36.9%	-1.0%	2.2%	68.9%	0.2%	73.1%	94.8%	92.1%
50	30	37.4%	-57.8%	36.5%	-0.2%	2.4%	69.5%	0.0%	73.0%	94.7%	92.4%
50	56	37.0%	-57.6%	36.1%	0.2%	2.0%	69.1%	0.0%	70.4%	94.8%	93.3%
50	100	36.8%	-57.9%	35.5%	-1.0%	0.7%	67.5%	0.1%	70.1%	94.5%	93.9%
100	5	37.4%	-62.4%	36.9%	-5.4%	7.7%	51.5%	0.0%	58.7%	94.4%	83.4%
100	10	36.9%	-60.2%	35.4%	-2.9%	2.6%	48.8%	0.0%	54.9%	94.2%	89.8%
100	20	36.7%	-58.8%	36.0%	-0.8%	1.6%	46.3%	0.0%	48.9%	94.8%	91.5%
100	30	36.5%	-58.5%	35.7%	-0.8%	0.9%	44.2%	0.0%	47.2%	96.2%	94.8%
100	56	35.3%	-58.7%	35.3%	-0.9%	0.0%	46.8%	0.0%	48.1%	95.7%	94.8%
100	100	36.8%	-58.0%	36.2%	-0.1%	0.8%	43.0%	0.0%	44.5%	95.8%	94.8%
300	5	36.2%	-62.5%	35.8%	-5.4%	7.0%	11.3%	0.0%	15.3%	90.0%	76.5%
300	10	35.6%	-60.5%	35.4%	-2.6%	2.0%	6.7%	0.0%	8.2%	95.2%	92.1%
300	20	36.2%	-59.3%	36.2%	-1.1%	1.0%	4.7%	0.0%	5.1%	94.0%	91.6%
300	30	35.8%	-59.1%	36.0%	-0.8%	0.5%	5.8%	0.0%	5.2%	94.8%	93.4%
300	56	36.1%	-58.5%	35.9%	-0.3%	0.5%	3.2%	0.0%	3.6%	95.4%	94.3%
300	100	35.9%	-58.4%	35.9%	0.0%	0.4%	3.8%	0.0%	3.7%	94.5%	93.9%