



**FIGURE S4: Simulations of the Fast Gauss Transform and deviations from the Gaussian-diffusion model, related to Figures 3-6.** Fractional error histogram (A) and correlogram (B) using an adapted Fast Gauss Transform on simulated data. The Fast Gauss Transform allows sums of  $O(N^2)$  Gaussian interactions between point-sources (here, UMIs) to be calculated in  $O(N)$  time. The Fast Gauss Transform is applied above to the weight sums  $w_i \equiv \sum_j w_{ij}$  of 2000 simulated UMIs (1000 beacons and 1000 targets) uniformly distributed in a two-dimensional box of side length  $10L_{\text{diff}}$  and with amplitudes  $A_i, A_j$  (from equation 8) normally distributed with  $\sigma = 1$ . Maximum fractional error bound is set to 30% for weights  $w_i$ . Physical simulations (C) with diffusion following the long-tailed Laplace distribution  $\sim e^{-\|\vec{x}\|/L_{\text{diff}}}$  in place of the Gaussian distribution ( $\sim e^{-\|\vec{x}\|^2/L_{\text{diff}}^2}$ ) used in Figure 3G. Grid lines indicate the respective lengths  $L_{\text{diff}}$  for both simulations, as in the simulation in Figure 3.