Supplementary Material

We present the R code used in Section 3.

```
library(MASS)
library(mvtnorm)
library(copula)
set.seed(123)

##### FIGURE 1. Bivariate Normal ####
set.seed(123)

norms <- data.frame(mvrnorm(1000, c(10,5), matrix(c(10,5,5,10),2,2)))
colnames(norms) <- c("x", "z")

norms$xc <- norms$x - mean(norms$x)
norms$zc <- norms$z - mean(norms$z)

norms$xz <- norms$x * norms$z
norms$xz_c <- norms$xc * norms$zc

cor(norms$z, norms$xz)
cor(norms$zc, norms$xz_c)

##### FIGURE 2. Bivariate Normal Mixture ####
set.seed(123)
dat1 <-rmvnorm(1000,mean=c(4,4),sigma=matrix(data=c(1,0.6,0.6,1),nrow=2))
dat2 <- rmvnorm(100,mean=c(7,-1),sigma=matrix(data=c(1,0.1,0.1,1),nrow=2))
dat <- rbind(dat1,dat2)
d <- as.data.frame(dat)
colnames(d) <- c("x","z")
d$xc <- d$x - mean(d$x)
d$zc <- d$z - mean(d$z)
d$xz <- d$x*d$z
d$xzc <- d$xc*d$zc

cor(d$x,d$xz)
cor(d$xc,d$xzc)

##### FIGURE 3. Bivariate Joe copula with normal marginals ####
set.seed(125)

CopSEM <- function(copmvdc, Sigma, nw = 100000, np = 1000)
{
Xw <- rMvdc(nw,copmvdc)
Sw <- cov(Xw) ## warm-up VC matrix
Sigma.eigen <- eigen(Sigma)
Sigmaroot <- Sigma.eigen$vectors%*%sqrt(diag(Sigma.eigen$values))%*%t(Sigma.
   eigen$vectors)
Sx.eigen <- eigen(solve(Sw))
Sxroot <- Sx.eigen$vectors%*%sqrt(diag(Sx.eigen$values))%*%t(Sx.eigen$vectors
   )
X <- rMvdc(np,copmvdc)
Y <- (X%*%(Sxroot)%*% Sigmaroot)
list(Y = Y, covY = (cov(Y))) ## return Y and cov(Y)
}

## Sigma specification
Sigma <- matrix(c(10,5,5,10),2,2)
```

```
## Copula specification
coppar <- joeCopula(19, dim = 2)
copjoint <- mvdc(coppar, margins = c("norm","norm"), paramMargins = list(list
    (mean=15,sd=10), list(mean=5,sd=10)))

## Apply two-stage function
res.sim <- CopSEM(copjoint, Sigma, np = 1000, nw = 100000)
data <- res.sim$Y

d <- as.data.frame(data)
colnames(d) <- c("x","z")
d$xc <- d$x - mean(d$x)
d$zc <- d$z - mean(d$z)
d$xz <- d$x*d$z
d$xzc <- d$xc*d$zc

cor(d$x,d$xz)
cor(d$xc,d$xzc)
```