Supplementary Information for Adaptive Zone Model Predictive Control of Artificial Pancreas Based on Glucose- and Velocity-Dependent Control Penalties

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This document provides supplementary information for the manuscript "Adaptive Zone Model Predictive Control of Artificial Pancreas Based on Glucose- and Velocity-Dependent Control Penalties," submitted for publication in IEEE Transactions on Biomedical Engineering. In Section I, we review the zone MPC algorithm with velocity-weighting and velocity-penalty developed in [1], based on which the adaptive zone MPC approach described in the manuscript is developed. The implementation details of the developed parameter adaptation approach are presented in Section II.

I. Zone Model Predictive Control

In [1], a periodic zone MPC with velocity-weighting and velocity-penalty is developed for the artificial pancreas to achieve safe and satisfactory closed-loop blood glucose regulation for patients with T1DM. Driven by the arrival of glucose measurements, the controller runs in discrete time and calculates a numeric control law every 5 minutes. Specifically, at discrete time instant i, the control law of the zone MPC with velocity-weighting and velocity-penalty is obtained by solving a constrained optimization problem of the following form:

$$\mathsf{u}^*_{0:N_\mathsf{u}-1} := \arg\min_{\mathsf{u}_{0:N_\mathsf{u}-1}} J\left(x_i, \mathsf{u}_{0:N_\mathsf{u}-1}\right) \tag{1}$$

subject to

$$\mathsf{x}_0 := x_i \tag{2.1}$$
$$\mathsf{x}_{k+1} := A\mathsf{x}_k + B\mathsf{u}_k \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}-1} \tag{2.2}$$
$$\mathsf{y}_k := C_\mathsf{y}\mathsf{x}_k \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}} \tag{2.3}$$
$$\mathsf{v}_k := C_\mathsf{v}\mathsf{x}_k \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}} \tag{2.4}$$
$$\mathsf{u}_k \le \hat{\xi}_{i+k} \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{u}-1} \tag{2.5}$$
$$\mathsf{u}_k \ge \check{\xi}_{i+k} \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{u}-1} \tag{2.6}$$
$$\mathsf{u}_k = 0 \quad \forall k \in \mathbb{Z}_{N_\mathsf{u}:N_\mathsf{p}-1} \tag{2.7}$$
$$\mathsf{z}_k := Z\left(\mathsf{y}_k,\ i+k\right) \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}} \tag{2.8}$$
$$\hat{\mathsf{z}}_k := \max\left(\mathsf{z}_k,\ 0\right) \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}} \tag{2.9}$$
$$\check{\mathsf{z}}_k := \min\left(\mathsf{z}_k,\ 0\right) \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}} \tag{2.10}$$
$$\hat{\mathsf{u}}_k := \max\left(\mathsf{u}_k,\ 0\right) \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{u}-1} \tag{2.11}$$
$$\check{\mathsf{u}}_k := \min\left(\mathsf{u}_k,\ 0\right) \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{u}-1} \tag{2.12}$$
$$\hat{\mathsf{v}}_k := \max\left(\mathsf{v}_k,\ 0\right) \quad \forall k \in \mathbb{Z}_{0:N_\mathsf{p}} \tag{2.13}$$
$$\hat{D} := \begin{cases} D > 0 & \text{if } y_0 + y_s \in \mathbb{D} \\ 0 & \text{otherwise.} \end{cases} \tag{2.14}$$

In eq. (1), the cost function $J\left(\cdot,\cdot\right)$ is defined as

$$J\left(\cdot,\cdot\right) := \sum_{k=1}^{N_\mathsf{p}} \left(\check{\mathsf{z}}_k^2 + Q\left(\mathsf{v}_k\right)\hat{\mathsf{z}}_k^2 + \hat{D}\hat{\mathsf{v}}_k^2\right) + \sum_{k=0}^{N_u-1}\left(\hat{R}\hat{\mathsf{u}}_k^2 + \check{R}\check{\mathsf{u}}_k^2\right), \tag{3}$$

where v_k, $\hat{\mathsf{z}}_k$, $\check{\mathsf{z}}_k$, $\hat{\mathsf{u}}_k$, $\check{\mathsf{u}}_k$ and $\hat{\mathsf{v}}_k$ are respectively defined in (2.4), (2.9), (2.10), (2.11), (2.12) and (2.13); in particular, $\hat{\mathsf{z}}_k$, $\check{\mathsf{z}}_k$ are determined on the basis of a diurnal zone-excursion function $Z\left(y,i\right)$ (see (2.8) for an illustration) defined by

$$Z\left(y,i\right) \quad := \quad \arg\min_{\alpha\in\mathbb{R}}\left\{\alpha^2 \ \middle|\ y-\alpha \in [\check{\zeta}_i, \hat{\zeta}_i]\right\},$$

with $[\check{\zeta}_i, \hat{\zeta}_i]$ representing the diurnal glucose target zone (see Fig. 1). $Q(\mathsf{v}_k)$ denotes a velocity-dependent weighting matrix [2] satisfying

$$Q(\mathsf{v}) := \begin{cases} 1 & \text{if } \mathsf{v} \ge 0 \\ \epsilon & \text{if } v \le 1 \\ \frac{1}{2}[\cos(\mathsf{v}\pi)(1-\epsilon)+(1+\epsilon)] & \text{otherwise.} \end{cases} \tag{4}$$

An illustration of $Q(\mathsf{v})$ is provided in Fig. 2. Parameter $\hat{D}$ in (3) determines a glucose-dependent cost on glucose velocity, which is defined in (2.14) with $\mathbb{D} := [140, 180]$ and $D := 1000$. The prediction horizon and control horizon in the MPC are set to $N_p := 9$ and $N_\mathsf{u} := 5$, which correspond to 45 and 25 minutes, respectively. The control input weighting parameters in [1] are set to $\hat{R} := 6500$ and $\check{R} := 100$, respectively. The state space model in (2.2)-(2.4) is parameterized by

$$A := \begin{bmatrix} p_1+2p_2 & -2p_1p_2-p_2^2 & p_1p_2^2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$
$$B := \frac{1800K}{u_{\text{TDI}}}\begin{bmatrix}1 & 0 & 0\end{bmatrix}^\top,$$
$$C_\mathsf{y} := \begin{bmatrix}0 & 0 & 1\end{bmatrix},$$
$$C_\mathsf{v} := \begin{bmatrix}0.1 & 0 & -0.1\end{bmatrix},$$
$$K := 90(p_1-1)(p_2-1)^2,\ p_1 := 0.98,\ p_2 := 0.965,$$

where u_{TDI} denotes the subject-specific total daily insulin; $\hat{\xi}_{i+k}$ and $\check{\xi}_{i+k}$ in (2.5)-(2.6) denote the upper and lower bounds on the control input u_k; we refer the interested reader to [3], [4] for more details about the model. The detailed description of this MPC algorithm can be found in [1].

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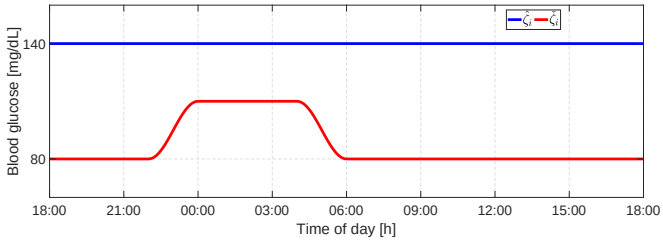

Fig. 1: An illustration of the upper and lower bounds $\hat{\zeta}_i$ and $\check{\zeta}_i$ of diurnal BG target zone (reproduced from Fig. A.1 in [1]).

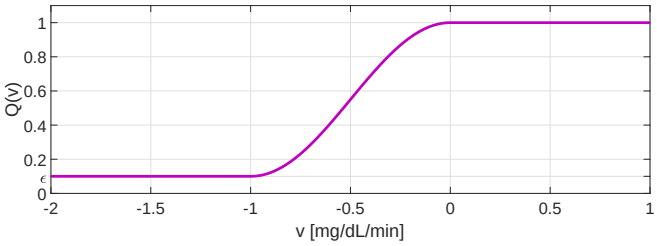

Fig. 2: An illustration of velocity-weighting function $Q(\mathsf{v})$ (reproduced from Fig. 1 in [1]).

We note that u_k in the MPC optimization problem denotes the relative correction of insulin infusion $\mathsf{u}_{\text{abs},k}$ from the basal rate $\mathsf{u}_{\text{basal}}$, namely, $\mathsf{u}_k := \mathsf{u}_{\text{abs},k} - \mathsf{u}_{\text{basal}}$. As the amount of insulin infusion is non-negative, it holds that $\check{\xi}_{i+k} \geq -\mathsf{u}_{\text{basal}}$ and $\mathsf{u}_k \geq -\mathsf{u}_{\text{basal}}$. By definition, $\hat{\mathsf{u}}_k$ and $\check{\mathsf{u}}_k$ indicate delivery rates above and below the basal rate $\mathsf{u}_{\text{basal}}$, respectively; thus the separate weighting parameters $\hat{R}$ and $\check{R}$ on $\hat{\mathsf{u}}_k$ and $\check{\mathsf{u}}_k$ allows independent penalization of the costs of insulin delivery above and below basal rate.

II. Implementation Details

In terms of implementation, the proposed adaptive MPC method basically replaces the original cost function in (3) with

$$J(\cdot,\cdot) := \sum_{k=1}^{N_{\mathsf{p}}} \left(\check{\mathsf{z}}_k^2 + Q(\mathsf{v}_k)\,\hat{\mathsf{z}}_k^2 + \hat{D}\hat{\mathsf{v}}_k^2 \right) + \sum_{k=0}^{N_{\mathsf{u}}-1} \left(\hat{R}(\mu_k,\mathsf{y}_k)\hat{\mathsf{u}}_k^2 + \check{R}(\mu_k,\mathsf{y}_k)\check{\mathsf{u}}_k^2 \right), \tag{5}$$

which adds to the non-convexity of the MPC optimization problem. To ensure the convergence of the optimization algorithm and speed up the computation, a heuristic technique is introduced to implement the proposed adaptive MPC based on the physiological properties of the insulin-glucose metabolic process. To aid the description, we use the notation $\{\cdot\}_i$ to denote a data sequence obtained by the zone MPC at controller update time instant i (e.g., $\{y_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$). The motivating observation is that a lag of $10-30$ minutes exists between the plasma insulin concentrations and the effect of insulin [5]. As the control horizon N_{u} is equal to 5 and the sampling time is 5 minutes, the predictions $\{\mathsf{y}_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ and $\{\mu_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ are dominated by the historical glucose measurements at time instant i rather than the optimal inputs $\{\mathsf{u}_k^*\}_i$, which was also observed in [6]. In this regard, we estimate $\{\mathsf{y}_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ and $\{\mu_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ with $\{\mathsf{y}_k : k \in \mathbb{Z}_{1:N_{\mathsf{u}}}\}_{i-1}$ and $\{\mathsf{y}_k - \mathsf{y}_{k-1} : k \in \mathbb{Z}_{1:N_{\mathsf{u}}}\}_{i-1}$, respectively, and calculate $\{\hat{R}(\mu_k,\mathsf{y}_k) : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ and $\{\check{R}(\mathsf{y}_k) : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ based on the obtained estimates for $\{\mathsf{y}_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$ and $\{\mu_k : k \in \mathbb{Z}_{0:N_{\mathsf{u}}-1}\}_i$. An important property of these estimates is that they can be calculated before solving the MPC optimization problem formed by (1), (5) and (2) at time instant i and are constant during the solution procedure of the optimization problem. In this way, the resultant structure of constrained optimization problem remains unchanged compared with that in [2], [6] and thus the optimization problem can be solved following the same procedure proposed in Section 3 of [6].

Note that the glucose velocity sequence $\{\mu_k\}$ we adopt in this work is different from $\{\mathsf{v}_k\}$ defined according to (2.4), which was proposed to quantify the velocity weighting and velocity penalties in [1]. By definition, $\{\mu_k\}$ provides a closer approximation of the velocity sequence of the noiseless glucose prediction $\{\mathsf{y}_k\}$. Another major consideration here, however, is to avoid introducing $\{\mu_k\}$-induced disturbances to the convergence of the $\{v_k\}$-driven sequential optimization procedure utilized to solve non-convex MPC optimization problem (see Section 3 in [6] for details). In particular, during the sequential optimization procedure, the sequence $\{v_k\}$ is updated in each iteration until the convergence conditions are satisfied. As the estimates for $\{\mu_k\}$ remain constant and do not change with $\{v_k\}$ throughout this procedure, the adopted $\{\mu_k\}$ sequence does not affect the convergence of the sequential optimization algorithm.

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