Science Advances
AAAS

advances.sciencemag.org/cgi/content/full/5/10/eaaw8586/DC1

Supplementary Materials for

High-efficiency single-photon generation via large-scale active time multiplexing

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Published 4 October 2019, *Sci. Adv.* **5**, eaaw8586 (2019)
DOI: 10.1126/sciadv.aaw8586

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Table S1. Experimental parameters.
Reference (*37*)

Supplementary Materials

Theory of a time-multiplexed HSPS

Here we outline the theory for modeling our time-multiplexed HSPS. First we define the following three probabilities

$$G(k) = \frac{\mu^k}{(1+\mu)^{k+1}} \tag{S1}$$

$$D(1|k) = \sum_{l=1}^{k} \eta_t^l (1-\eta_t)^{k-l} \binom{k}{l} \left(\frac{1}{d}\right)^{l-1} \tag{S2}$$

$$T(M|j,k) = (t_i {t_{DL}}^{N-j+1})^M (1 - t_i {t_{DL}}^{N-j+1})^{k-M} \binom{k}{M} \tag{S3}$$

$G(k)$ is the probability that an SPDC source generates k-photon pairs in a time bin with a mean photon number μ. Our SPDC source with 90% indistinguishability can be well approximated by a thermal distribution (*38*), because ~ $0.9^{0.5}$ = 95% of our SPDC photon pairs are generated into a single Schmidt mode. $D(1|k)$ is the trigger-photon detection probability conditioned by k photon pairs created. η_t is the total transmission of the signal photons from the SPDC crystal to SPDs, and d is the number of SPDs used for a detector cascade. The SPDs are assumed to work as “bucket” detectors discriminating between zero and one-or-more photons. With Eq. S2, a photon-number-resolving (PNR) detector can be simulated with $d \to \infty$, in which case $D(1|k) \to k\eta_t(1-\eta_t)^{k-1}$. $T(M|j, k)$ is the M-photon emission probability conditioned by k photon-pair generation in the j-th time bin, where t_{DL} denotes the transmission of an adjustable delay line for a delay time τ, and t_i is the net transmission of the other optics (including an initial delay line, fiber coupling efficiency, etc.). With Eqs. S1-S3, the multiplexed heralding probability is given by

$$P_H = 1 - \left\{1 - \sum_{k_1=1}^{\infty} G(k_1) D(1|k_1)\right\}^N \tag{S4}$$

where N is the number of multiplexed time bins. The probability of producing an M-photon state after time multiplexing is given by

$$P_M = \sum_{j=1}^{N} \left[1 - \sum_{k_1=1}^{\infty} \{1 - G(k_1) D(1|k_1)\}\right]^{N-j} \sum_{k_2=1}^{\infty} G(k_2) D(1|k_2) T(M|j,k_2) \tag{S5}$$

The first part in Eq. S5 is the probability that no trigger photon detection has occurred from the (j + 1)-th to N-th time bins. The second part is the probability that one of the trigger SPDs clicks in the j-th time slot, and M heralded photons are released at the output time window. Thus, the product of the first and second parts for each j describes the probability that the latest heralding signal is generated in the j-th time bin and an M-photon state is emitted from an adjustable delay line after $(N - j)\tau$ delay. While previous time-multiplexing schemes (*3*, *5*) heralded photons in the earliest time bins, multiplexing the latest-born heralded photons reduces the effective loss in the adjustable delay line. Table S1 shows a list of our experimental parameters to simulate the theoretical curves shown in Fig. 2A-2C and the possible improved source characteristics in the main text.

Table S1. Experimental parameters.

Parameter	Symbol	Value			
		This work			Possible improvement
Number of multiplexed time bins	N		40		100
Trigger-photon system detection efficiency	η_t		0.53		0.85
Number of trigger SPDs	d		4		16
Adjustale delay line cycle transmission	t_{DL}		0.988		0.992
Other optics transmission	t_i		0.83		0.85
Multiplexing system repetition rate (kHz)	R		500		5000
Mean photon number per pulse	μ	0.004	0.05	0.18	0.1
Single-photon probability	P_1	0.051(2)	0.412(13)	0.667(24)	0.75
Second-order auto-correlation function	$g^{(2)}(t = 0)$	0.007(7)	0.088(7)	0.269(7)	0.05
Spectral temporal indistinguishability	I	0.91(4)	0.90(3)	0.92(3)	0.99
Single-photon generation rate (kcps)	C_1 (= P_1R)	25.5(10)	206(7)	334(12)	3800