Supplementary Info

1. Modeling for μwheel translation on flat and textured surfaces

We consider a μwheel that consists of *n* individual spheres with sphere *i* located at $\mathbf{r}_i$ (Fig. S2A). Its instantaneous velocity is

$$\dot{\mathbf{r}}_{CM} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{v}_i = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_{ij}\mathbf{F}_j \tag{S1}$$

where $\mathbf{r}_{CM}$ is the position vector of the center of mass. $\mathbf{v}_i$ is the velocity of sphere i, related to the force acting on particle *j*, $\mathbf{F}_j$, through the mobility matrix $\boldsymbol{\mu}_{ij}$, i.e. $\mathbf{v}_i = \sum_{j=1}^{n}\mu_{ij}\mathbf{F}_j$. To gain essential physical insight and for simplicity, we only consider the self-mobility terms, i.e. $\mu_{ii} = \left[\beta_i\left(\mathbf{I}-\hat{\mathbf{z}}\hat{\mathbf{z}}\right)+\alpha_i\hat{\mathbf{z}}\hat{\mathbf{z}}\right]/6\pi\eta a$, where η is the solvent viscosity. β_i and α_i are the mobilities of sphere *i* moving parallel and perpendicular to the substrate. When the sphere-wall separation h_i is large, the polynomial expansion for β_i and α_i are (10, 28, 29)

$$\begin{aligned}\beta_i &= 1-\frac{9}{16}\frac{a}{h_i}+\frac{1}{8}\left(\frac{a}{h_i}\right)^3-\frac{45}{256}\left(\frac{a}{h_i}\right)^4-\frac{1}{16}\left(\frac{a}{h_i}\right)^5 \\ \alpha_i &= 1-\frac{9}{8}\frac{a}{h_i}+\frac{1}{2}\left(\frac{a}{h_i}\right)^3-\frac{1}{8}\left(\frac{a}{h_i}\right)^5\end{aligned}. \tag{S2}$$

When h_i is small, numerically precise and convenient expressions based on asymptotic lubrication theory (31) are available:

$$\beta_i = \frac{-\frac{2}{5}\left[\ln\left(1-\frac{a}{h_i}\right)+0.37089\right]}{\left[-\frac{8}{15}\ln\left(1-\frac{a}{h_i}\right)+0.95429\right]\left[-\frac{2}{5}\ln\left(1-\frac{a}{h_i}\right)+0.37089\right]-\frac{4}{3}\left[\frac{1}{10}\ln\left(1-\frac{a}{h_i}\right)+0.19295\right]^2}$$

$$\alpha_i = \frac{1}{\frac{a}{h_i\left(1-\frac{a}{h_i}\right)}-\frac{1}{5}\ln\left(1-\frac{a}{h_i}\right)+0.97127} \tag{S3}$$

In our work, μwheels are pulled to the surface by gravity and are very close to the substrate; therefore, we use Eqn S3 which provides the correct limits that $\alpha_i, \beta_i \to 0$ when the contact distance $a / h_i \to 1$.

Assuming that the force F_i on each sphere has same magnitude, $F_i = F$, the torque generated by all forces must balance the hydrodynamic torque

$$FR_n\xi = 8\pi\eta\omega R_n^3 f(R_n / h) \tag{S4}$$

where h is the elevation of the μwheel center of mass, i.e. its vertical separation from the origin. In addition, $f\left(R_n / h\right) = \frac{2}{5}\ln\left[1/\left(h / R_n - 1\right)\right] + 0.3709$ is a correction factor (30) for the hydrodynamic torque due to the existence of wall. The force F can then be related to the angular rotation frequency of the μwheel ω and other system parameters

$$F = \frac{8\pi\eta\omega R_n^2 f(R_n / h)}{\xi}. \tag{S5}$$

After eliminating F in Eqn S1, we write the instantaneous μwheel velocities parallel (v) and perpendicular (v_z) to the substrate as

$$v / V_0 = \frac{dx}{dt} = \dot{\mathbf{r}}_{CM} \cdot \hat{\mathbf{x}} = -\frac{1}{n} \sum_{i=1}^{n} \beta_i \cos\phi_i$$
$$v_Z / V_0 = \frac{dh}{dt} = \dot{\mathbf{r}}_{CM} \cdot \hat{\mathbf{z}} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i \sin\phi_i \qquad \text{(S6)}$$

where $V_0 = \frac{4\omega R_n^{\ 2} f(R_n / h)}{3\xi a}$ and $\phi_i = \omega t + \frac{2\pi}{n}(i-1)$ is the angle between $\mathbf{r}_{CM} - \mathbf{r}_i$ and $\hat{\mathbf{z}}$.

The sphere-wall separation h_i is related to the elevation of the µwheel center of mass h by

$$h_i = R_n \cos\phi_i + h \qquad \text{(S7)}$$

For a dimer rolling on a flat substrate, *n*=2 and Eqn. S6-7 simplifies to Eqn. 2 in the main text. It can be numerically solved with Eqn. S3.

As shown in Fig. S3B, the geometric features on a textured substrate can be described by the topographic surface function $h_s(x)$. Here, we model the surface features by a series of small trapezoidal bumps that are evenly spaced. Clearly, a flat substrate corresponds to $h_s(x) = 0$. To simulate the rolling of a dimer on the textured substrate, Eqn. S6 can still be used. As shown in Fig. S3A, the sphere-wall separation h_i in Eqn. S3 is related to the elevation of the µwheel center of mass h by

$$h_1 = h - h_s(x + a\sin\omega t) + a\cos\omega t$$
$$h_2 = h - h_s(x - a\sin\omega t) - a\cos\omega t \qquad \text{(S8)}$$

Solving Eqn. S6, S8, and S3 numerically yields the motion of a dimer on a textured substrate.

2. Calculation of the ideal substrate for µwheels of different shape and symmetry

Using a square μwheel (4,4) as an example (Fig. S4), we mark four circles, O_1-O_4. When the μwheel rolls smoothly, its center of mass does not vary along the *z* direction. The distance traveled at time t_i, will be $x_i = \omega t_i R_n = \theta_i R_n$ assuming *OP* as the starting point. From geometry, the height of wheel circumference to the wall will be $h_i = \overline{P_1 P_2}$, where P_1 and P_2 are the intersections between *OP* and circle *O* and circle O_1 and are calculated using the Matlab Mapping toolbox (*linecirc* function). Because of symmetry, we need only calculate (x_i, h_i) from $\theta_i = 0\ to\ \angle POP'$, i.e. $\theta_i = \pi/4$. Similar calculation procedures can be followed for obtaining the commensurate microroads for μwheels of differing geometry (2,2) and (7,6).

4. Force analysis for μwheels climbing inclined surfaces

To understand the "rectifier" effect that we have observed for μwheels climbing an inclined surface, we use a force balance (33, 34) shown in the inset of Fig. 4a:

$$\begin{cases} F_f + F_h - F_g \sin\gamma = 0 \\ N_c - F_g \cos\gamma = 0 \end{cases} \quad \text{(S9)}$$

where the hydrodynamic lift force has been neglected. The friction force is thus $F_f = \mu_k N_c = \mu_k F_g \cos\gamma$. The hydrodynamic force has two components: translation-translation $F_{h,tt}$ and $F_{h,tr}$ which, for a sphere, can be expressed as $F_{h,tt} = 6\pi\eta a V f_{tt}$ and $F_{h,tr} = 6\pi\eta\omega a^2 f_{tr}$ following Goldman et al. (5), where correction functions due to the wall are $f_{tt} = -\frac{8}{15} ln\left(\frac{\delta}{a}\right) - 0.9588$ and $f_{tr} = \frac{2}{15} ln\left(\frac{\delta}{a}\right) - 0.2526$. The lowest frequency where μ wheels roll and where $F_f + F_{h,tr} = F_g \sin\gamma$, is $\omega_c = \frac{F_g(\sin\gamma - \mu_k \cos\gamma)}{6\pi\eta a^2 f_{tr}}$. Under experimental conditions where $\delta \sim 200nm$, $\mu_k \sim 0.5$ and $\gamma \sim 30^\circ$, $f_c \sim 0.58$ Hz, consistent with experimental data.

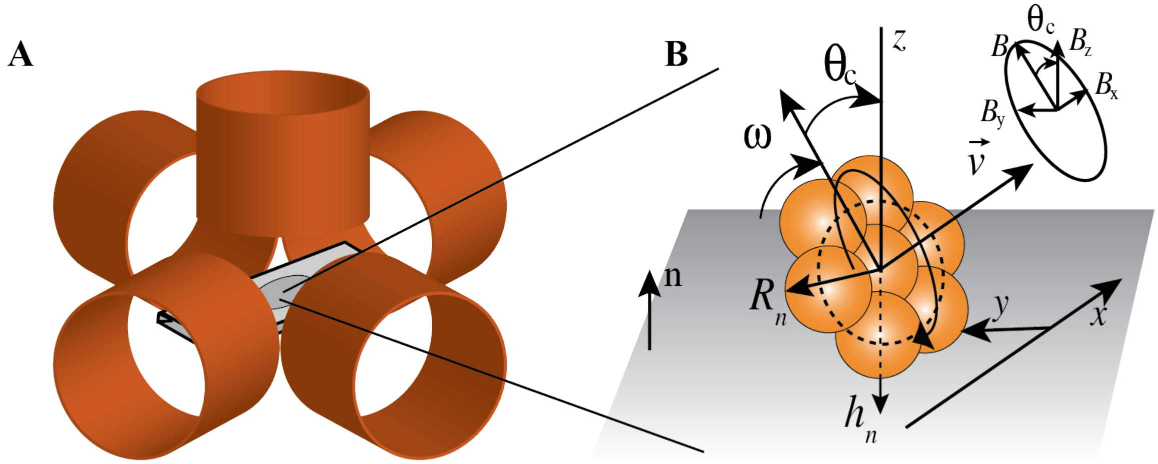

Fig. S1 (A) Experimental setup. (B) The rolling of a μwheel under a 3D magnetic field.

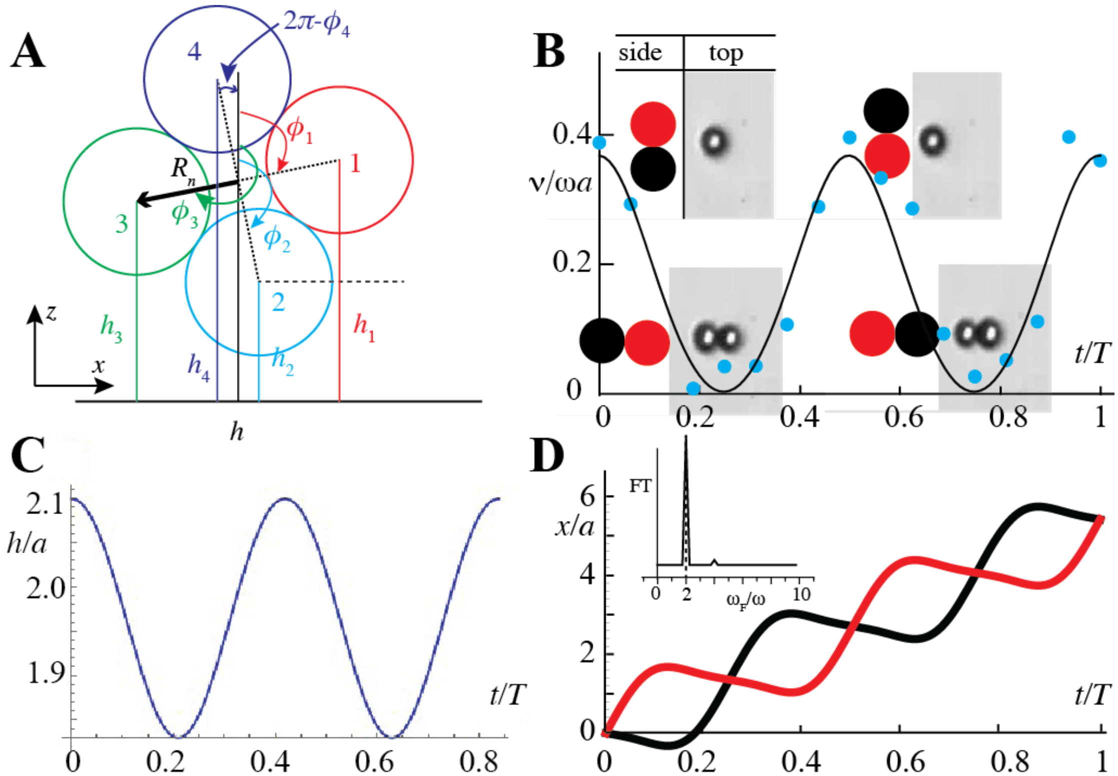

Fig. S2. μWheels translating on flat surfaces. (A) Model for a μwheel rolling on a flat surface with a square (4,4) as an example. (B) A comparison between experimental data (dots) and modeling (line) for the instantaneous velocity *v(t)* parallel to the substrate. Snapshots correspond to instantaneous dimer configurations obtained from experiments (top view) and simulation (side view). (C) The calculated dimer center of mass with time. (D) Displacement of the centers of mass of each lobe. Inset: Fourier transform of instantaneous velocity.

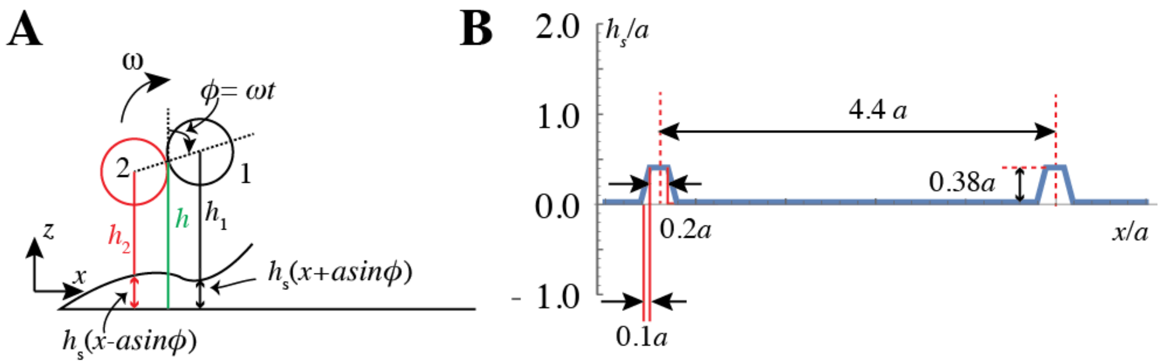

Fig. S3 (A) Schematics for modelling the rolling of a dimer on a textured surface. (B) Textured surface with trapezoidal bumps used in the simulations.

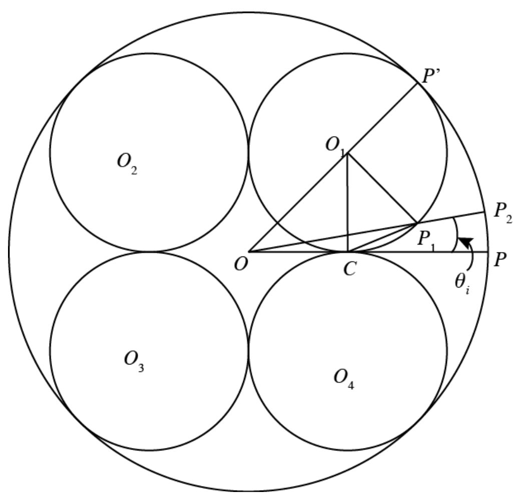

Fig. S4 Calculation of an ideal road for a square (4,4) μwheel translating without slip.

List of supplementary movies

Movie S1 Translation of a 7-mer and a 5-mer under a 3D magnetic field on a flat surface (B_{xy}=1.95 mT, B_z=5.31 mT, and $\omega_M = 3.14\,\mathrm{rad/s}$). The movie is displayed in real time.

Movie S2 Top: Simulation of a dimer translating on flat surface (by solving Eqn. 2) with same rotation frequency in experiments. The initial separation between the dimer's center of mass and substrate is 2.2*a*. Bottom: Translation of a dimer under a 3D magnetic field on a flat surface (B_{xy}=1.95 mT, B_z=5.31 mT, and $\omega_M = 0.628\,\mathrm{rad/s}$). The movies are accelerated 5x.

Movie S3 Translation of a dimer under a 3D magnetic field on a topographic surface along the blaze direction (B_{xy}=1.95 mT, B_z=5.31 mT, and $\omega_M = 0.628\,\mathrm{rad/s}$). The movie is accelerated 5x.

Movie S4 The simulated translation (slip and flip) of a dimer on a topographic surface with evenly spaced trapezoidal bumps. The geometric dimensions of the bumps are shown in Fig. S3B. The initial separation between the dimer edge and the substrate is $0.2a$. Both horizontal and vertical dimensions are scaled by the sphere radius *a*.

Movie S5 The simulated translation (sequential flip of two lobes) of a dimer on a flat substrate. The initial separation between the dimer edge and the substrate is $0.05a$. Both horizontal and vertical dimensions are scaled by the sphere *a.*

Movie S6 The simulated translation (continuous flip) of dimer on a topographic surface with trapezoidal bumps spaced by d=1.39a. The height and length of the bumps are the same as in Movie S4. The initial separation between the dimer edge and the substrate is $0.2a$. Both horizontal and vertical dimensions are scaled by the sphere radius *a*.

Movie S7 The rectifier effect for the translation of a dimer under a 3D magnetic field on a topographic surface (B_{xy}=1.95 mT, B_z=5.31 mT, and $\omega_M = 0.628\,\mathrm{rad/s}$). Top: dimer translating in the blaze direction; bottom: dimer translating against the blaze direction. The movie is accelerated 5x.

Movie S8 Comparison of a diamond and square µwheel translation on the flat (top) vs. topographic surface against the blaze direction (bottom) (B_{xy}=1.95 mT, B_z=5.31 mT, and $\omega_M = 6.28\,\mathrm{rad/s}$). The movie is in real time.

Movie S9 Translation of a diamond and square against the blaze direction (B_{xy}=1.95 mT, B_z= 5.31 mT, and $\omega_M = 1.256\,\mathrm{rad/s}$). The movie is accelerated 5x.