

Appendix A1

Here we show how to extract the parameter estimates of the nonlinear mixed-effects model from the R output. For this, the statistical model (1), R output (2), and table of results presented in the manuscript (3), are linked in a new table (4).

(1) Nonlinear mixed-effects model (equation 9 in manuscript, page 12):

$$\text{LFMC}_{ji} \sim \mathcal{N}(\mu_i; \sigma^2)$$

$$\mu_i = \frac{A - w}{1 + e^{(m - t_i)/s}} + w$$

$$\sigma_i^2 = (\sigma_{base} \text{GW}_i + \sigma_{base} \delta_1 \text{GE}_i + \sigma_{base} \delta_2 \text{SM}_i + \sigma_{base} \delta_3 \text{SM}_i)^2$$

$$A = A_{0j} + A_1 \text{GE}_i + A_2 \text{SM}_i + A_3 \text{SS}_i$$

$$w = w_{0j} + w_1 \text{GE}_i + w_2 \text{SM}_i + w_3 \text{SS}_i$$

$$m = m_{0j} + m_1 \text{GE}_i + m_2 \text{SM}_i + m_3 \text{SS}_i$$

$$s = s_{0j} + s_1 \text{GE}_i + s_2 \text{SM}_i + s_3 \text{SS}_i$$

$$\text{Cor}(\text{LFMC}_{ji}; \text{LFMC}_{ji'}) = f(\text{ARMA}_{[s,q]})$$

$$A_{0j} \sim \mathcal{N}(\mu_{A_{0j}}; \sigma_{A_0}^2) ; w_{0j} \sim \mathcal{N}(\mu_{w_{0j}}; \sigma_{w_0}^2) ; m_{0j} \sim \mathcal{N}(\mu_{m_{0j}}; \sigma_{m_0}^2) ; s_{0j} \sim \mathcal{N}(\mu_{s_{0j}}; \sigma_{s_0}^2)$$

(2) R output (line 163 in script s2.R)

```
> M1 <- update(nl.me.ml.2, method = "REML")
> summary(M1)
```

```
Nonlinear mixed-effects model fit by REML
Model: lfmc ~ SSd1f(time, A, w, m, s)
Data: lfmc.data.gd
      AIC      BIC    logLik
1931.668 1983.689 -950.834

Random effects:
Formula: A ~ 1 | group
      A.(Intercept) Residual
StdDev:      9.408521 7.162899

Variance function:
Structure: Different standard deviations per stratum
Formula: ~1 | leaf.type
Parameter estimates:
      Grass W      Grass E      M. spinosum S. bracteolactus
      1.0000000      0.8851764      3.1796104      2.9647391

Fixed effects: list(A + w ~ leaf.type, m + s ~ 1)
      Value Std.Error DF t-value p-value
A.(Intercept)      54.25350  5.988181 226  9.060097 0e+00
A.leaf.typeGrass E      30.92293  8.835210 226  3.499966 6e-04
A.leaf.typeM. spinosum      223.24504 15.915557 226 14.026844 0e+00
A.leaf.typeS. bracteolactus      240.27654 16.857356 226 14.253513 0e+00
w.(Intercept)      29.05581  1.712960 226 16.962334 0e+00
w.leaf.typeGrass E     -20.68938  2.592469 226 -7.980569 0e+00
w.leaf.typeM. spinosum      31.67297  7.753176 226  4.085161 1e-04
w.leaf.typeS. bracteolactus      26.97508  8.071111 226  3.342177 1e-03
m      30.92881  2.065196 226 14.976213 0e+00
s     -16.15406  2.365392 226 -6.829339 0e+00

Correlation:
      A.(In) A.l.GE A..M.s A..S.b w.(In) w.l.GE w..M.s w..S.b m
A.leaf.typeGrass E      -0.527
A.leaf.typeM. spinosum     -0.146  0.535
A.leaf.typeS. bracteolactus     -0.115  0.537  0.746
w.(Intercept)      -0.217 -0.034 -0.200 -0.216
w.leaf.typeGrass E     -0.035 -0.283 -0.382 -0.399 -0.258
w.leaf.typeM. spinosum     -0.118 -0.227 -0.547 -0.454  0.157  0.573
w.leaf.typeS. bracteolactus     -0.129 -0.241 -0.462 -0.575  0.188  0.601  0.640
m      -0.217 -0.324 -0.633 -0.666  0.092  0.145  0.161  0.172
s     -0.246 -0.354 -0.710 -0.748  0.416  0.575  0.702  0.752  0.544

Standardized Within-Group Residuals:
      Min      Q1      Med      Q3      Max
-2.27999668 -0.59657156 -0.02042043  0.57273326  3.12017985

Number of Observations: 247
Number of Groups: 12
```

This output is a result of the modelling process, which suggests $m_1, m_2, m_3, s_1, s_2, s_3, \sigma_{w_0}^2, \sigma_{m_0}^2, \sigma_{s_0}^2$ to be zero.

(3) Table of results (Table 2 in manuscript)

Parameter	Estimate	Meaning
μ_{A_0}	54.3% [42.5 : 66.0]	Maximum LFMC (\hat{A}) of grasses in the W site. A varies with plot and therefore a hyperparameter is estimated.
A_1	30.9% [13.5 : 48.3]	Difference between \hat{A} of grasses in the E site and \hat{A} of grasses in W site.
A_2	223.4% [191.9 : 254.6]	Difference between \hat{A} of <i>M. spinosum</i> and \hat{A} of grasses in the W site.
A_3	240.3% [207.1 : 273.5]	Difference between \hat{A} of <i>S. filaginoides</i> and \hat{A} of grasses in the W site.
w_0	29.1% [25.7 : 32.4]	Minimum LFMC (\hat{w}) of grasses in the W site.
w_1	-20.7% [-25.8 : -15.6]	Difference between \hat{w} of grasses in the E site and \hat{w} of grasses in the W site.
w_2	31.7% [16.4 : 47.0]	Difference between \hat{w} of <i>M. spinosum</i> and \hat{w} of grasses in the W site.
w_3	26.7% [11.1 : 42.9]	Difference between \hat{w} of <i>S. filaginoides</i> and \hat{w} of grasses in the W site.
m	30.9 days [26.9 : 35.0]	Day when the LFMC half-maximum occurs (\hat{m}) in the study area (data suggest one general value for all the leaf types).
s	-16.1 [-20.8 : -11.5]	Parameter controlling speed of change (\hat{s}) of the LFMC (data suggest one general value for all the leaf types, the negative value indicates vegetation to be drying during the fire season).
σ_{base}	7.1% [6.0 : 8.6]	LFMC variability within-plots (residual standard error) of grass ($\hat{\sigma}_{gw}$) in Site W at the beginning the fire season.
δ_1	0.9 [0.7 : 1.2]	Ratio between the within-plots standard error of the LFMC of grasses in Site E ($\hat{\sigma}_{ge}$) and grasses in W site ($\hat{\sigma}_{gw}$).
δ_2	3.2 [2.4 : 4.0]	Ratio between the within-plots standard error of the initial LFMC <i>M. spinosum</i> ($\hat{\sigma}_{sm}$) and grasses in W site ($\hat{\sigma}_{gw}$).
δ_3	3.0 [2.4 : 3.9]	Ratio between the within-plots standard error of the initial LFMC <i>S. filaginoides</i> ($\hat{\sigma}_{ss}$) and grasses in W site ($\hat{\sigma}_{gw}$).
σ_{A_0}	9.4% [3.9 : 15.8]	Variability among plots (standard deviation) in the maximum LFMC (random effects on A).

(4) Connection between model/table in manuscript and R output

Parameter	R output												
μ_{A_0}	Fixed effects: list(A + w ~ leaf.type, m + s ~ 1) <table border="1"> <thead> <tr> <th></th> <th>value</th> <th>Std.Error</th> <th>DF</th> <th>t-value</th> <th>p-value</th> </tr> </thead> <tbody> <tr> <td>A.(Intercept)</td> <td>54.25350</td> <td>5.988181</td> <td>226</td> <td>9.060097</td> <td>0e+00</td> </tr> </tbody> </table>		value	Std.Error	DF	t-value	p-value	A.(Intercept)	54.25350	5.988181	226	9.060097	0e+00
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