

APPENDIX

Strain is a measure of deformation, defined by the one-dimensional Lagrangian equation as relative change in dimension: $\varepsilon = \frac{L-L_0}{L_0} = \frac{\Delta L}{L}$ where ε is strain, and L_0 and L are length before and after deformation, respectively. In systolic deformation this means L_0 is end-diastolic length (L_d) and L is end-systolic length (L_s). As seen from the equation, decrease will result in negative strain, increase in positive strain. $\varepsilon_L = \frac{L_s-L_d}{L_d} < 1$ if the wall shortens, i.e. negative strain. $\varepsilon_T = \frac{WT_s-WT_d}{WT_d} > 1$ if the wall thickens, i.e. positive strain, and $\varepsilon_C = \frac{C_s-C_d}{C_d} < 1$ if the circumference shortens, i.e. negative strain. The circumference (C) of a circle is given by $C = \pi \times D$, where D is the diameter. Thus, circumferential strain is $\varepsilon_C = \frac{C_s-C_d}{C_d} = \frac{\pi \times D_s - \pi \times D_d}{\pi \times D_d} = \frac{D_s-D_d}{D_d}$. As fractional shortening is percent diameter shortening, defined as $FS = \frac{D_d-D_s}{D_d}$, it follows that $\varepsilon_c = -FS$. This can be measured at the epicardial, mid wall or endocardial level, respectively.

The strain tensor in the cardiac coordinate system looks like this:

$$\begin{pmatrix} \varepsilon_L & \varepsilon & \varepsilon_{LC} \\ \varepsilon_{TL} & \varepsilon_T & \varepsilon_{TC} \\ \varepsilon_{CL} & \varepsilon_{CT} & \varepsilon_C \end{pmatrix}$$

where ε_L , ε_T and ε_C are the three normal strain components, while ε_{LT} , ε_{TL} , ε_{LC} , ε_{CL} , ε_{TC} and ε_{CT} are shear strains, usually considered pairwise symmetric.

In an incompressible object, the volume has to remain constant during deformation, i.e. deformation in the three directions have to balance. For an incompressible cube having the volume $V_0 = x \times y \times z$, before deformation and the volume $V = (x+\Delta x) \times (y+\Delta y) \times (z+\Delta z)$ after deformation, the two volumes have to be equal; $V_0 = V$ i.e.

$$x \times y \times z = (x + \Delta x) \times (y + \Delta y) \times (z + \Delta z)$$

⇓

$$\frac{(x+\Delta x) \times (y+\Delta y) \times (z+\Delta z)}{x \times y \times z} = \frac{x+\Delta x}{x} \times \frac{y+\Delta y}{y} \times \frac{z+\Delta z}{z} = 1$$

$$\Downarrow$$

$$\left(1 + \frac{\Delta x}{x}\right) \times \left(1 + \frac{\Delta y}{y}\right) \times \left(1 + \frac{\Delta z}{z}\right) = 1$$

$$\Downarrow$$

$$\mathbf{(1 + \varepsilon_x \times (1 + \varepsilon_y \times (1 + \varepsilon_z = 1$$

Changing coordinate system, this is equivalent to:

$$\mathbf{(\varepsilon_L + 1) \times (\varepsilon_C + 1) \times (\varepsilon_T + 1) = 1}$$