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## A multi-scale coevolutionary approach to predict protein-protein interactions

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### A Input data

The starting point of our analysis is the phylogenetic profile matrix (PPM): a binary matrix  $(n_i^a)_{i=1, \dots, N}^{a=1, \dots, M}$  whose entries capture the presence ( $n_i^a = 1$ ) or absence ( $n_i^a = 0$ ) of a domain  $i$  in genome  $a$ , with  $a = 1, \dots, M$  ( $M$  being the number of genomes) and  $i = 1, \dots, N$  ( $N$  being the number of domains). As discussed in the main text, the domains (the columns of the PPM) are then

compared with each other to look for functionally related domains. The data we use are extracted from the Pfam 30.0 database (version of July 2016) [2], and assigned to bacterial or eukaryotic species using the Uniprot species list available on (<http://www.uniprot.org/docs/speclist>).

## B Similarity measures

In standard *phylogenetic profiling* [5] the correlations between the columns ( $\mathbf{n}_i, \mathbf{n}_j$ ) describing a pair of domains are usually evaluated via the Hamming distance, Pearson correlation or the p-value of the Fisher’s exact test. We briefly describe each below.

- **Hamming distance:** counts the number of bits which differ between two binary strings  $\underline{n}_i, \underline{n}_j$  divided by the total number of domains, i.e. the number of species containing exactly one of the two domains,

$$d_H(\underline{n}_i, \underline{n}_j) = |\{n_i^a \neq n_j^a, \quad a = 1, \dots, M\}|/M \quad (1)$$

- **Pearson Correlation:** measures the linear dependence between two domains  $\underline{n}_i, \underline{n}_j$ . It is defined as:

$$r(\underline{n}_i, \underline{n}_j) = \frac{\sum_{a=1}^M (n_i^a - \bar{n}_i)(n_j^a - \bar{n}_j)}{\sqrt{\sum_{a=1}^M (n_i^a - \bar{n}_i)^2} \sqrt{\sum_{a=1}^M (n_j^a - \bar{n}_j)^2}} \quad (2)$$

- **p-value of Fisher Test:** for each couple  $(\underline{n}_i, \underline{n}_j)$  we construct an auxiliary  $2 \times 2$  matrix

$$\begin{pmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{pmatrix} \quad (3)$$

with:

$M_{1,1}$  : Number of species that do not have neither domain  $i$  nor  $j$

$M_{1,2}$  : Number of species that have the  $i$  but not  $j$

$M_{2,1}$  : Number of species that do not have  $i$  but have  $j$

$M_{2,2}$  : Number of species that have both  $i$  and  $j$ .

We define  $R_i = \sum_j M_{i,j}$ ,  $C_j = \sum_i M_{i,j}$ , and we have  $M = \sum_{i,j} M_{i,j}$ . We calculate the conditional probability of getting the actual matrix given the particular row and column sums:

$$P_{\text{cutoff}} = \frac{\binom{R_1}{M_{11}} \binom{R_2}{M_{21}}}{\binom{M}{C_1}} = \frac{R_1! R_2! C_1! C_2!}{M! M_{1,1}! M_{1,2}! M_{2,1}! M_{2,2}!} \quad (4)$$

which is a multivariate generalization of the hyper-geometric distribution. Theoretically, we analyse all the matrices of non negative integers consistent with the marginals  $R_i$ ,  $C_j$  and  $M$ , and for each of them we calculate

the p-value Eq.(4). The p-value of the test is the sum of all p-values which are  $P \leq P_{cutoff}$ . Small p-values thus indicate atypical cases related to correlations between the distributions of the two domains across species.

## C Inference techniques

As a simple but meaningful statistical model we consider a pairwise lattice-gas Ising model with the following Hamiltonian:

$$H(\mathbf{n}) = - \sum_{i=1}^N h_i n_i - \sum_{i<j}^N J_{ij} n_i n_j . \quad (5)$$

It shall model the statistical variability of the phylogenetic profile  $\mathbf{n} = (n_1, \dots, n_N)$  of an entire species. The binary variables  $n_i$  ( $i = 1, \dots, N$ , with  $N$  being the total number of domains) can have values  $n_i = 1$  i.e. the domain is present, or  $n_i = 0$  otherwise. Each row of the PPM has this form, and  $H$  can thus be evaluated for each species contained in the PPM.

To infer the Ising model Eq.(5) reproducing the empirical one- and two-column statistics, we assume as working hypothesis that the phylogenetic profile matrix PPM is composed by configuration sampled from the equilibrium Boltzmann-Gibbs distribution (in the inference process we are free to set the inverse temperature  $\beta = 1$ ):

$$P(\mathbf{n}) = \frac{1}{Z} \exp[-H(\mathbf{n})]. \quad (6)$$

In the inference procedure we look for parameters  $\{\mathbf{h}, \mathbf{J}\}$  (respectively the *fields* and the *phyletic couplings*) that maximize the log-likelihood:

$$L(\mathbf{h}, \mathbf{J}) = \frac{1}{M} \sum_m^M w_m \log[P(\mathbf{n}|\mathbf{h}, \mathbf{J})] \quad (7)$$

where  $w_m$  are reweighting coefficients defined below in Section C.1. Nevertheless the exact maximization procedure requires the determination of the marginals of  $P(\mathbf{n})$  for single variables and pairs, which is highly computationally expensive. In the last years [9] a variety of efficient and accurate approximation methods have been proposed: in this work we use the Mean Field (MF) and pseudo-likelihood maximization (PLM) approximations, which, as showed in Figure 2 of the main text, give similar results on our datasets. We briefly describe each one below.

We also mention that we developed an implementation of Phyletic-Coupling Analysis *PhyDCA*, in Julia, a new open-source language [15]. The package can be downloaded at <https://github.com/GiancarloCroce/PhyCA>.

## C.1 Pre-processing the data

In order to reduce sampling biases, a simple correction is to associate to each phylogenetic profile of a species  $\mathbf{n}^a$  the weight  $w_a = 1/m_a$  with  $m_a$  the number of similar profiles (including  $a$  itself):

$$m_a = |\{\mathbf{n}^b | 1 \leq b \leq M, d_H(\mathbf{n}^a, \mathbf{n}^b) < \theta\}| \quad (8)$$

with  $d_H(\mathbf{n}^a, \mathbf{n}^b)$  being the Hamming distance and  $\theta$  a similarity threshold. It has been empirically observed that setting  $\theta = 0.8$  slightly improves the PPV. The sum of all the weights  $M_{eff} = \sum_{a=1}^M w_a$ , represents the effective number of phylogenetic profiles counted as independent in the analysis.

## C.2 Mean Field (MF)

The Mean Field approximation is currently the fastest approximate inference scheme even if not accurate when the number of samples is small nor when the interactions are strong [9]. The phyletic couplings  $J_{ij}$  are inferred from the empirical correlation matrix:  $J_{ij} = -(C^{-1})_{ij}$  for  $i \neq j$ , with

$$C_{ij} = (f_{ij} - f_i f_j) \quad (9)$$

$$f_i = \frac{\lambda}{2} + (1 - \lambda) \left( \frac{1}{M_{eff}} \sum_{a=1}^M w_a n_i^a \right) \quad (10)$$

$$f_{ij} = \frac{\lambda}{4} + (1 - \lambda) \left( \frac{1}{M_{eff}} \sum_{a=1}^M w_a n_i^a n_j^a \right) \quad (11)$$

where  $\lambda$  is a pseudocount that we set to  $\lambda = \frac{1}{4}$  which we empirically found to produce the optimal PPV.

## C.3 Pseudolikelihood (PLM)

Adapted in [4] to protein sequences, the approximation consists in replacing the Boltzmann-Gibbs measure with the following conditional probability distribution:

$$P(n_i | \mathbf{n}_{\setminus i}) = \frac{\exp \left\{ n_i \left( h_i + \sum_{j \neq i} J_{ij} n_j \right) \right\}}{1 + \exp \left\{ h_i + \sum_{j \neq i} J_{ij} n_j \right\}}, \quad (12)$$

with  $\mathbf{n}_{\setminus i} = (n_1, \dots, n_{i-1}, n_{i+1}, \dots, n_N)$  being the phylogenetic profile of all domains different from  $i$ . For a given data PPM, the likelihood with respect to the distribution Eq.(12) – usually called *pseudo-likelihood* –

$$L_i(J_{i, \setminus i}, h_i) = \frac{1}{M} \sum_{a=1}^M w_a \log P(n_i^a | \mathbf{n}_{\setminus i}^a) \quad (13)$$

can be easily maximized as a function of  $(J_{i, \setminus i}, h_i)$ . As is customary in inference problems, we add an  $\ell_2$  regularization term  $\gamma_h \sum_i h_i^2 + \gamma_J \sum_{i \neq j} J_{ij}^2$  to mitigate the effects of overfitting by penalizing parameters with large value. The free parameters  $\gamma_J$  and  $\gamma_h$  are set to  $\gamma_J = 0.02$  and  $\gamma_h = 0.05$ . We refer to [4] for the details of the implementation, which was adapted to binary variables.

## C.4 Average product correction (APC)

Another correction, introduced in [13], consists in subtracting from the inferred couplings  $J_{ij}$  a contribution due to the single-site properties of  $i$  and  $j$  (the average over sites is denoted by  $\bullet$ ):

$$J_{ij}^{APC} = J_{ij} - \frac{J_{i\bullet}J_{\bullet j}}{J_{\bullet\bullet}}. \quad (14)$$

Since the APC correction aims to minimize background influences like phylogeny and site entropy, we included it in PhyDCA. While for residue-level DCA, APC-corrected scores have a significantly better prediction accuracy [9], an a posteriori analysis show that the effect is small and almost negligible in PhyDCA (see Figure A).

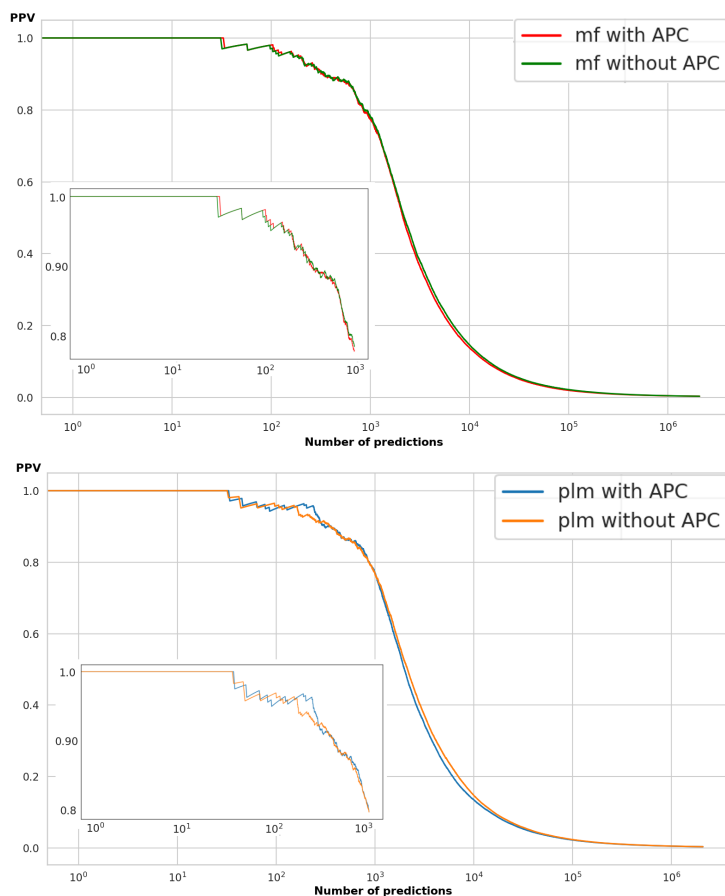


Figure A: **APC correction.** The plots show the PPV curves with and without APC correction for the mean field (upper panel) and pseudo-likelihood (bottom panel) approximations.

## D Results

Figure B shows the histograms of the similarity measures (Hamming distance, Pearson correlation, the P-value of the Fisher’s exact test and phyletic couplings) for all the pairs of domains considered in the main text. Related domains ought to have profiles of high phyletic couplings, high correlations, low p-value of Fisher’s exact test or low Hamming distances, since the first two are similarity, the second two more dissimilarity measures.

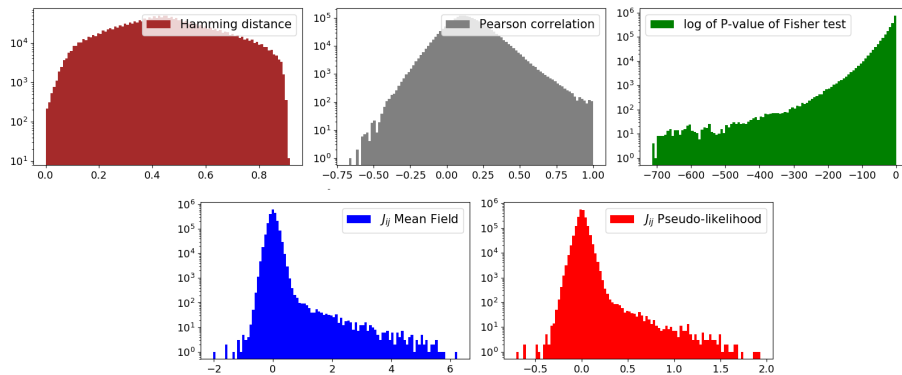


Figure B: **Metrics summary.** The plots show the distribution of the metrics (Hamming distance, Pearson correlation, the P-value of the Fisher’s exact test mean-field and pseudo-likelihood phyletic couplings) for all the couples of domains existing in the *E. coli* K-12 MG1655 strain.

In Figure C we plot the phyletic-couplings  $J_{ij}$  found using the mean-field (MF) and the pseudo-likelihood maximization (PLM) approximations. Predictions are done by sorting all couplings in decreasing order. They are evidently highly similar, but include a partial reordering. To extract Figure 2 of the main text, the blue dots are interpreted as false positives. From Figure C it is evident that these false positives are – even for very large coupling values – similarly distributed for the two approximations in between the true positives (red points), therefore showing that none of the methods has a clear advantage in precision.

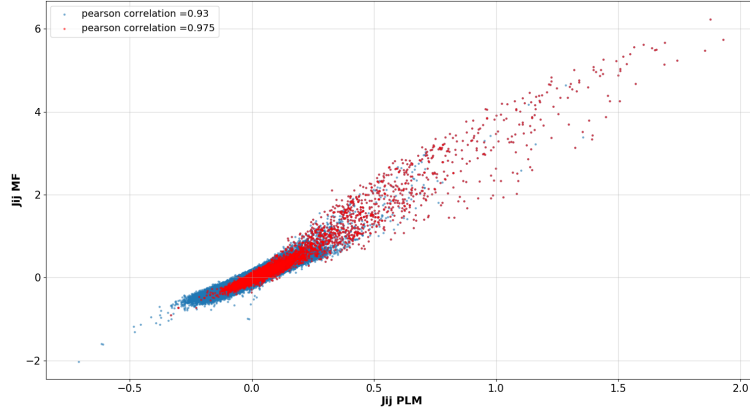


Figure C: **Comparison PLM and MF approximations.** The scatter-plot shows the phyletic couplings found using the MF and the PLM approximations. The blue points are domains pairs are all domain pairs while the red points are those belonging to the positive set of known domain-domain relations (note that the red points form a subset of the blue points). The plot shows that the advantage of PLM over MF (or viceversa) is not visible in the case of domain-domain co-occurrence.

In Figure D we plot the PPV as a function of the couplings (not the cumulative PPV, but PPV per bin of coupling values). It shows that the enrichment of true positive predictions is very high in the tail of large couplings ( $J_{plm} > 0.5$  or  $J_{MF} > 1.5$ ), and remains very limited for smaller couplings ( $J_{plm} < 0.3$  or  $J_{MF} < 0.5$ ).

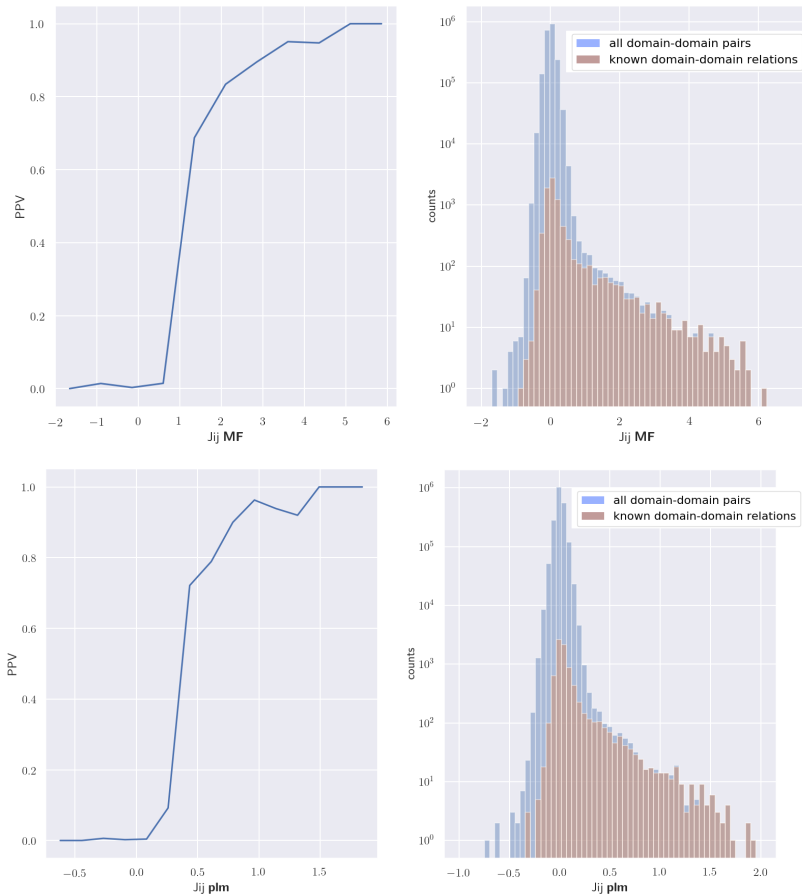


Figure D: **PPV as a function of couplings.** The left figures show the sharp drop of PPV from values close to one for  $J_{plm} > 0.5$  with the PLM approximation (or  $J_{MF} > 1.5$  in the MF approximation) to very low PPV for  $J_{plm} < 0.3$  (or  $J_{MF} < 0.5$ ). The right histograms show the distribution of the phyletic couplings for all domains pairs and for known domain-domain relations. The kink in the histograms between the bulk of small  $J_{ij}$  and the tail of large  $J_{ij}$  is observed to provide a good cutoff value for high-quality predictions. Once again, it is located close to  $J_{plm} = 0.5$  or ( $J_{MF} = 1.5$ ).

## E Progressive paralog matching procedure

As discussed in the main text, a large positive phyletic coupling is a strong indicator of a functional relationship between two domains, but not necessarily of a direct physical interaction. To identify physical interactions we use the procedure introduced in [17], which studies coevolution of domain pairs at the



level of the individual residues. We briefly describe the method here; for a full description and additional details we refer to the original publication [17].

Given a pair of strongly coupled domains and the corresponding multiple sequence alignments (say  $MSA_1$  and  $MSA_2$ ) the problem is to generate a concatenated alignment: we need to find for any sequences belonging to  $MSA_1$  a matching partner in  $MSA_2$ . Only sequences belonging to the same species should be matched. This problem has a trivial solution (which we call *matching by uniqueness*) when there are no paralogs and each species has only one sequence in each MSA. However, many protein domains exist in multiple paralogs across many species. In this case, the method proposed in [17] is based on the idea that the correct matching of interacting paralogs should maximize the inter-domain coevolutionary signal. The matched MSA is then used to identify interacting protein families: an average of the four highest inter-protein residue-residue plmDCA scores larger than 0.2 is a strong indicator for a potential interaction, at least of the joint MSA has an effective size  $M_{eff} > 200$ .

The following procedure was used to create the matched-alignment and compute the DCA scores:

1. download the two PFAM alignments ( $MSA_1$  and  $MSA_2$ );
2. if considering a phylogenetic profile matrix with bacterial [eukaryotic] genomes, select only bacterial [eukaryotic] sequences;
3. run the *progressive paralog-matching (PPM)* to find a locally optimal matching;
4. run plmDCA and compute the DCA score, given by the average of the first four highest interdomain residue-residue plmDCA scores.

In Tables A and B we report the first 100 strongly phyletically coupled domain pairs inside the *E. coli* K-12 MG1655 strain *not* belonging to our list of positive known domain-domain relations (i.e. our predictions for such relations).

Figure E and Figure F show the results of the matching procedure for the domain pairs inside the *E. coli* K-12 MG1655 strain.

|    | pfam ACC dom1 | pfam ACC dom2 | Phyloetic coupling (plm) | Meff joint MSA | paralogues matching | DCA score | domain1 description   | domain2 description   |
|----|---------------|---------------|--------------------------|----------------|---------------------|-----------|---|---|
| 1  | PF03634       | PF04860       | 1.256                    | 203,289        | 0.753               |           | Phage portal protein  | Phage portal protein  |
| 2  | PF02669       | PF03814       | 1.284                    | 320,623        | 0.867               |           | K+-transporting ATPases, c chain                              | Potassium-transporting ATPase A subunit                                       |
| 3  | PF02424       | PF04205       | 1.161                    | 457,627        | 1.063               |           | ABC family  | FMN-binding domain  |
| 4  | PF00950       | PF01297       | 1.813                    | 480,452        | 1.813               |           | ABC 3 transport family  | Zinc-uptake complex component A periplasmic                                   |
| 5  | PF04865       | PF04965       | 1.101                    | 240,503        | 1.286               |           | Baseplate J-like protein                                      | Gene 25-like lysozyme   |
| 6  | PF02670       | PF02702       | 0.975                    | 380,368        | 1.998               |           | K+-transporting ATPases, c chain                              | Osmosensitive K+-channel His kinase sensor domain                             |
| 7  | PF02702       | PF03814       | 0.951                    | 283,446        | 1.658               |           | Osmosensitive K+-channel His kinase sensor domain             | Potassium-transporting ATPase A subunit                                       |
| 8  | PF02702       | PF03814       | 0.951                    | 463,849        | 1.658               |           | Prophage 4-lysozyme protein (AlpA)                            | Domain of unknown function (DUF1102)  |
| 9  | PF06972       | PF13216       | 0.761                    | 340,147        | 1.076               |           | Methyltransferase   | Phage tail fiber  |
| 10 | PF02614       | PF03786       | 0.758                    | 272,834        | 1.247               |           | Glucuronidase   | D-nanonaic acid dehydratase (UxaA)  |
| 11 | PF04203       | PF06798       | 0.711                    | 87,4854        | 1.037               |           | SpoVR like protein  | PhA serine protein kinase C-terminal domain                                   |
| 12 | PF04203       | PF08298       | 0.708                    | 92,2474        | 1.101               |           | SpoVR like protein  | PhA AAA domain  |
| 13 | PF01924       | PF07503       | 0.707                    | 315,084        | 1.462               |           | Hydrogenase formation hypA family                             | HyPF finger   |
| 14 | PF03483       | PF02781       | 0.706                    | 367,212        | 1.170               |           | 6-phosphogluconate dehydrogenase, C-terminal domain           | Glucose-6-phosphate dehydrogenase, C-terminal domain                          |
| 15 | PF03483       | PF00479       | 0.706                    | 260,618        | 1.197               |           | 6-phosphogluconate dehydrogenase, C-terminal domain           | Glucose-6-phosphate dehydrogenase, NAD binding domain                         |
| 16 | PF04285       | PF04293       | 0.702                    | 65,1382        | 1.168               |           | Protein of unknown function (DUF444)                          | SpoVR like protein  |
| 17 | PF06508       | PF14489       | 0.702                    | 259,401        | 1.883               |           | Quiescine biosynthesis protein QacC                           | Quiescine biosynthesis protein QacC   |
| 18 | PF03844       | PF09481       | 0.695                    | 126,589        | 0.745               |           | CT1975-like protein   | CRISPR-associated protein Cse1 (CRISPR_cse1)                                  |
| 19 | PF03965       | PF03965       | 0.687                    | 137,385        | 1.427               |           | like lysozyme   | Type VI secretion system effector; Hcp  |
| 20 | PF03965       | PF03965       | 0.682                    | 171,747        | 1.246               |           | CT1975-like protein   | CRISPR-associated protein Cse2 (CRISPR_cse2)                                  |
| 21 | PF04749       | PF04749       | 0.677                    | 467,551        | 1.246               |           | Baseplate J-like protein                                      | CRISPR-associated protein associated to bacteriophage (DUF2433)               |
| 22 | PF04865       | PF10476       | 0.670                    | 148,469        | 1.651               |           | Baseplate J-like protein                                      | Urecharacterised protein associated to bacteriophage (DUF2433)                |
| 23 | PF02601       | PF02609       | 0.668                    | 784,757        | 2.169               |           | Exonuclease VII large subunit                                 | CRISPR-associated protein Cse1 (CRISPR_cse1)                                  |
| 24 | PF08798       | PF06481       | 0.666                    | 124,257        | 0.758               |           | CRISPR associated protein                                     | CRISPR-associated protein Cse1 (CRISPR_cse1)                                  |
| 25 | PF01455       | PF07503       | 0.663                    | 407,833        | 2.654               |           | HypF/HypC family  | HyPF finger   |
| 26 | PF03374       | PF01924       | 0.660                    | 138,839        | 1.017               |           | Nickel-dependent hydrogenase                                  | Hydrogenase formation hypA family   |
| 27 | PF08798       | PF09485       | 0.655                    | 125,129        | 1.018               |           | CRISPR associated protein                                     | CRISPR-associated protein Cse2 (CRISPR_cse2)                                  |
| 28 | PF02445       | PF02749       | 0.653                    | 448,717        | 1.698               |           | Quinolinate synthetase A protein                              | Quinolinate phosphoribosyl transferase, N-terminal domain                     |
| 29 | PF01242       | PF06508       | 0.650                    | 335,013        | 1.370               |           | 6-pretinoyl tetrahydropterin synthase                         | Quiescine biosynthesis protein QacC   |
| 30 | PF02609       | PF13742       | 0.643                    | 615,725        | 2.052               |           | Exonuclease VII small subunit                                 | OB-fold nucleic acid binding domain   |
| 31 | PF06730       | PF07963       | 0.641                    | 680,942        | 2.564               |           | Bacterial Peptidase A21 N-terminal domain                     | Prokaryotic N-terminal methylation motif                                      |
| 32 | PF03239       | PF13473       | 0.636                    | 130,640        | 1.057               |           | Biotin and thiamin synthase                                   | Alkylamine  |
| 33 | PF03239       | PF03239       | 0.634                    | 324,345        | 1.057               |           | Biotin and thiamin synthase                                   | Alkylamine  |
| 34 | PF02629       | PF06476       | 0.629                    | 324,345        | 1.057               |           | Carlin associated repeat                                      | Alkylamine  |
| 35 | PF07012       | PF10614       | 0.612                    | 171,344        | 0.830               |           | Carlin associated repeat                                      | Alkylamine  |
| 36 | PF05157       | PF06750       | 0.603                    | 496,496        | 58,797              |           | Type II secretion system (T2SS), protein E, N-terminal domain | Type VIII secretion system (T8SS), CseG protein                               |
| 37 | PF11739       | PF13617       | 0.597                    | 71,1033        | 1.694               |           | Dicarbonylate transport                                       | Bacterial Peptidase A24 N-terminal domain                                     |
| 38 | PF06665       | PF01527       | 0.596                    | 503,573        | 1.382               |           | Integrase core domain   | Transposase   |
| 39 | PF00925       | PF00926       | 0.592                    | 706,081        | 1.701               |           | GTP cyclohydrolyase II  | YnfE-like lipoprotein   |
| 40 | PF03374       | PF07503       | 0.588                    | 121,342        | 1.490               |           | Nickel-dependent hydrogenase                                  | 3,4-dihydroxy-2-butanone 4-phosphate synthase                                 |
| 41 | PF08279       | PF13290       | 0.579                    | 780,729        | 1.713               |           | HTH domain  | HyPF finger   |
| 42 | PF02617       | PF03558       | 0.577                    | 425,194        | 2.271               |           | ATP-dependent Clp protease adaptor protein ClpS               | WYL domain  |
| 43 | PF02605       | PF06029       | 0.574                    | 231,377        | 2.313               |           | Metal binding domain of Ada                                   | Leucyl phenylalanyl-HRNA protein transferase                                  |
| 44 | PF01750       | PF02624       | 0.568                    | 369,385        | 1.470               |           | Hydrogenase maturation protease                               | AlkA N-terminal domain  |
| 45 | PF02624       | PF02624       | 0.568                    | 463,849        | 1.470               |           | 4-sulphamoyltransferase                                       | Hydrogenase formation hypA family   |
| 46 | PF02254       | PF02286       | 0.559                    | 507,357        | 1.114               |           | TkK N domain  | Carbohydrate-binding domain 48 (Genomplase N-terminal domain)                 |
| 47 | PF08297       | PF08297       | 0.555                    | 398,553        | 1.105               |           | Methyltetrahydrofolate reductase                              | Carbohydrate-binding domain 48 (Genomplase N-terminal domain)                 |
| 48 | PF04246       | PF13375       | 0.550                    | 305,417        | 1.795               |           | Positive regulator of sigma(E), RseC/MucC                     | Cobalamin-independent synthase, N-terminal domain                             |
| 49 | PF02261       | PF02548       | 0.548                    | 519,478        | 1.736               |           | Aspartate decarboxylase                                       | RatC Barrel sandwich hybrid domain  |
| 50 | PF02504       | PF02504       | 0.544                    | 656,787        | 1.571               |           | Fatty acid synthesis protein                                  | Ketopantoate hydroxymethyltransferase<br>Glycerol-3-phosphate acyltransferase |

Table A: The first 100 strongly coupled domain pairs inside the *E. coli* K-12 MG1655 strain *not* belonging to our list of positives

|     | pfam    | ACC_dom1 | pfam    | ACC_dom2 | Phyletic coupling (phn) | Meff joint MSA | paralogs matching DCA score | domain1 description  | domain2 description                                      |
|-----|---------|----------|---------|----------|-------------------------|----------------|-----------------------------|--|--|
| 51  | PF01155 |          | PF07503 |          | 0.543                   | 411187         | 1095                        | Hydrogenase; urease nickel incorporation, metallohydroperase, hypA |  |
| 52  | PF01844 |          | PF03354 |          | 0.538                   | 143,531        | 0831                        | HNH endonuclease   | Hypr1 finger   |
| 53  | PF05336 |          | PF06134 |          | 0.536                   | 78,9312        | 1221                        | L-rhamnose isomerase   | L-rhamnose isomerase (RhaA)                              |
| 54  | PF04166 |          | PF07005 |          | 0.533                   | 266,819        | 1129                        | Putative sugar-binding N-terminal domain                           | Putative sugar-binding N-terminal domain                 |
| 55  | PF02261 |          | PF02569 |          | 0.532                   | 678,229        | 1897                        | Aspartate decarboxylase  | Putative beta-alanine ligase                             |
| 56  | PF04166 |          | PF17042 |          | 0.531                   | 279,134        | 1185                        | Pyridoxal phosphate biosynthetic protein PdxA                      | Putative nucleotide-binding of sugar-metabolising enzyme |
| 57  | PF04349 |          | PF13632 |          | 0.528                   | 177,038        | 0944                        | Periplasmic glucan biosynthesis protein, MdoG                      | Glycosyl transferase family group 2                      |
| 58  | PF06485 |          | PF09707 |          | 0.526                   | 109,756        | 1593                        | CRISPR-associated protein Cas2 (CRISPR_cas2)                       | CRISPR-associated protein (Cas_Cas2CT1978)               |
| 59  | PF06794 |          | PF14698 |          | 0.525                   | 249,501        | 1703                        | Argininosuccinate lyase  | Argininosuccinate lyase C-terminal                       |
| 60  | PF00677 |          | PF08855 |          | 0.522                   | 665,871        | 1708                        | Lumazine binding domain  | 6,7-dimethyl-8-ribityllumazine synthase                  |
| 61  | PF00075 |          | PF13389 |          | 0.522                   | 449,200        | 1101                        | Glyoxyltransferase family 9 (thiopyruvate synthase)                |  |
| 62  | PF00075 |          | PF06454 |          | 0.517                   | 469,505        | 1708                        | SNP2 family N-terminal domain                                      | SNP2 zinc finger   |
| 63  | PF00075 |          | PF06454 |          | 0.517                   | 469,505        | 1708                        | SNP2 family N-terminal domain                                      | SNP2 zinc finger   |
| 64  | PF00677 |          | PF09026 |          | 0.513                   | 89,820         | 1766                        | dTDP-4-dehydrothymine 3,5-epimerase                                | GDP-mannose 4,6 dehydratase                              |
| 65  | PF00681 |          | PF03363 |          | 0.510                   | 701,524        | 1193                        | CRISPR-associated protein Cse1 (CRISPR_cse1)                       | CRISPR-associated protein (Cas_Cas2CT1978)               |
| 66  | PF00885 |          | PF09707 |          | 0.508                   | 78,0705        | 0802                        | CRISPR-associated protein Cse1 (CRISPR_cse1)                       | GTP cyclohydrolase II                                    |
| 67  | PF07027 |          | PF11739 |          | 0.506                   | 561,098        | 1615                        | Protein of unknown function (DUF1318)                              | Dicarboxylate transport                                  |
| 68  | PF07503 |          | PF07503 |          | 0.504                   | 78,5197        | 1112                        | Hydrogenase maturation protease                                    | Hypr1 finger   |
| 69  | PF06485 |          | PF09704 |          | 0.498                   | 308,601        | 2008                        | CRISPR-associated protein Cse1 (CRISPR_cse1)                       | CRISPR-associated protein (Cas_Cas5)                     |
| 70  | PF06885 |          | PF00926 |          | 0.497                   | 112,612        | 0737                        | Hydrogenase maturation protease                                    | CRISPR-associated protein (Cas_Cas5)                     |
| 71  | PF00677 |          | PF00925 |          | 0.494                   | 590,538        | 2166                        | 6,7-dimethyl-8-ribityllumazine synthase                            | 3,4-dihydroxy-2-butanone 4-phosphate synthase            |
| 72  | PF03773 |          | PF13192 |          | 0.484                   | 757,052        | 1534                        | Lumazine binding domain  | GTP cyclohydrolase II                                    |
| 73  | PF00668 |          | PF01783 |          | 0.483                   | 132,523        | 1292                        | Predicted permease   | Thioredoxin domain                                       |
| 74  | PF02228 |          | PF05872 |          | 0.481                   | 407,211        | 3312                        | Ribosomal protein L34  | Ribosomal L22p protein family                            |
| 75  | PF02227 |          | PF02152 |          | 0.476                   | 267,316        | 1161                        | Putative neutral zinc metalloprotease                              | Bacterial protein of unknown function (DUF553)           |
| 76  | PF00370 |          | PF02152 |          | 0.461                   | 496,776        | 1692                        | GTP cyclohydrolase I   | Dihydrocouperin aldolase                                 |
| 77  | PF00370 |          | PF02152 |          | 0.460                   | 421,729        | 1283                        | EGGY family of carbohydrate kinases, N-terminal domain             | C-terminal domain of alpha-glycerophosphate oxidase      |
| 78  | PF00370 |          | PF02152 |          | 0.459                   | 421,729        | 1283                        | EGGY family of carbohydrate kinases, N-terminal domain             | C-terminal domain of alpha-glycerophosphate oxidase      |
| 79  | PF00444 |          | PF04068 |          | 0.453                   | 229,191        | 1439                        | aspartate aminotransferase family 136                              | NiFe/NiFeS hydrogenase subunit C-terminal                |
| 80  | PF03742 |          | PF04068 |          | 0.452                   | 228,453        | 3186                        | Ribosomal protein L36  | Ribosomal protein L34                                    |
| 81  | PF01326 |          | PF17191 |          | 0.451                   | 632,988        | 1555                        | OB-fold nucleic acid binding domain                                | RecG wedge domain  |
| 82  | PF02283 |          | PF03618 |          | 0.451                   | 103,633        | 1298                        | Pyruvate phosphate dikinase, PEP pyruvate binding domain           | Kinase/pyrophosphorylase                                 |
| 83  | PF04973 |          | PF02572 |          | 0.450                   | 468,079        | 1683                        | Cobamide kinase / cobamide phosphate guanylyltransferase           | ATP-corrinoid adenosyltransferase BtuB (CoA)/CobP        |
| 84  | PF04257 |          | PF13521 |          | 0.450                   | 169,274        | 1138                        | Nicotinamide mononucleotide transporter                            | AAA domain   |
| 85  | PF06485 |          | PF13538 |          | 0.448                   | 134,063        | 0906                        | Exodeoxyribonuclease V, gamma subunit                              | UvrD-like helicase C-terminal domain                     |
| 86  | PF00128 |          | PF02146 |          | 0.447                   | 150,926        | 1319                        | associated protein Cas2 (CRISPR_cas2)                              | CRISPR-associated protein (Cas_Cas5)                     |
| 87  | PF04333 |          | PF05494 |          | 0.444                   | 351,543        | 1027                        | Alpha amylase, catalytic domain                                    | 4-alpha-glucanotransferase                               |
| 88  | PF04865 |          | PF07484 |          | 0.440                   | 296,656        | 1396                        | MxaA lipoprotein   | MitC protein   |
| 89  | PF13891 |          | PF07484 |          | 0.432                   | 168,580        | 1083                        | Baseplate J-like protein   | Phage Tail Collar Domain                                 |
| 90  | PF02675 |          | PF13396 |          | 0.428                   | 514,594        | 1711                        | PLD-like domain  | Phospholipase_D-nuclease N-terminal                      |
| 91  | PF00626 |          | PF03994 |          | 0.423                   | 132,222        | 1567                        | S-adenosylmethionine decarboxylase                                 | Domain of Unknown Function (DUF390)                      |
| 92  | PF00626 |          | PF03872 |          | 0.422                   | 888,552        | 1607                        | 4-alpha-glucanotransferase   | Alpha amylase, C-terminal alpha-beta domain              |
| 93  | PF00626 |          | PF03872 |          | 0.422                   | 888,552        | 1607                        | 4-alpha-glucanotransferase   | Domain of Unknown Function (DUF390)                      |
| 94  | PF02540 |          | PF14730 |          | 0.420                   | 132,633        | 1845                        | Hypr1 finger   | NiFe/NiFeS hydrogenase subunit C-terminal                |
| 95  | PF03783 |          | PF08349 |          | 0.419                   | 139,183        | 1480                        | Thiamine biosynthesis protein (ThiI)                               | Protein of unknown function (DUF1722)                    |
| 96  | PF03453 |          | PF07012 |          | 0.418                   | 56,7289        | 1223                        | MocA N-terminal region (domain I and II)                           | Carlin associated repeat                                 |
| 97  | PF08402 |          | PF06463 |          | 0.418                   | 752,101        | 2050                        | TOBE family  | Molybdenum Cofactor Synthesis C                          |
| 98  | PF01967 |          | PF13343 |          | 0.418                   | 547,287        | 1188                        | Bacterial extracellular solute-binding protein                     | Bacterial extracellular solute-binding protein           |
| 99  | PF02401 |          | PF03453 |          | 0.418                   | 859,202        | 2297                        | MocB family  | MocA N-terminal region (domain I and II)                 |
| 100 | PF00925 |          | PF04551 |          | 0.414                   | 476,398        | 1403                        | LytB protein   | GepE protein   |
| 101 | PF02589 |          | PF01872 |          | 0.414                   | 780,584        | 1436                        | GTP cyclohydrolase II  | RibD C-terminal domain                                   |
|     |         |          | PF02652 |          | 0.412                   | 111,010        | 0974                        | LUD domain   | L-lactate permease                                       |

Table B: The first 100 strongly coupled domain pairs inside the *E. coli* K-12 MG1655 strain *not* belonging to our list of positives

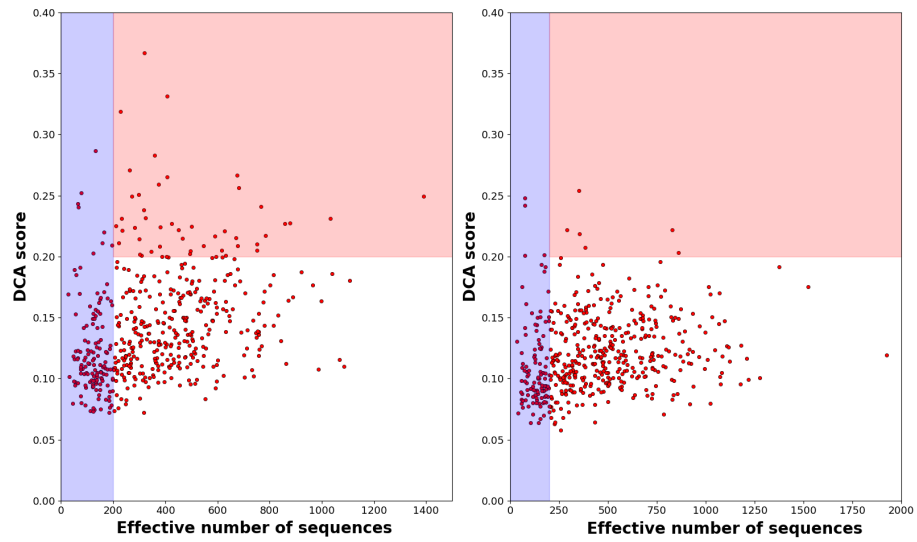


Figure E: **Matching procedure for *E. coli***. The panel on the left shows the result of the matching procedure for the 500 most significant predictions for domain families existing inside the K12 strain of *E. coli* (the list can be found on the [Github](#) page at `results/ECOLI_matching_results.dat`). On the right, as a comparison, a random matching for the same domain pairs.

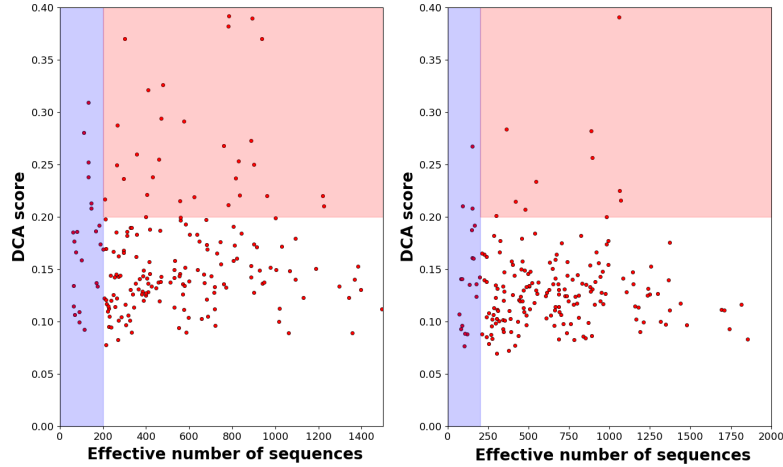


Figure F: **Matching procedure for *E. coli* domains in iPfam**  
 The panel on the left shows the result of the matching procedure for the 200 pairs of highest phylogenetic couplings belonging to the iPfam database. The complete list can be found on the [Github](#) page at `results/ECOLI_matching_iPfam_results.dat`. On the right, as a comparison, a random matching for the same domain pairs.

Figure G show the results of residue-level DCA for the 200 domain pairs of strongest phyletic couplings, which are co-localized in one protein in *E. coli*. Due to the co-localization, the generation of a joint MSA is trivial in this case; the paralogs-matching can be avoided. Note that domains can co-occur in the same protein without direct physical interactions. Out of the 200 pairs, 144 of these domain pairs are also listed in iPfam, meaning that a direct physical interaction is structurally known.

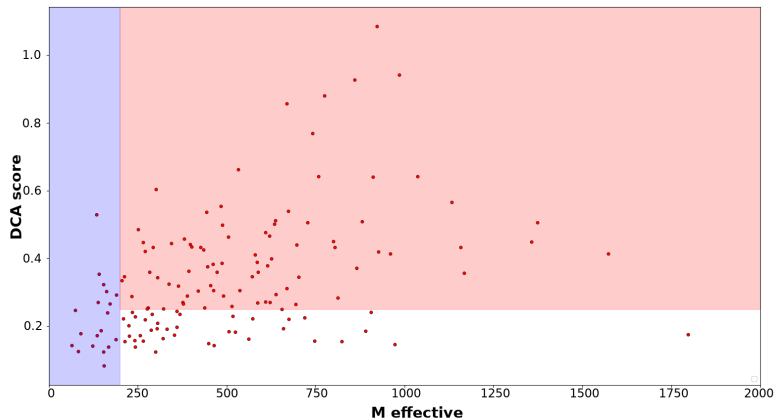


Figure G: **Results of residue-level DCA for domains co-localized in the same protein.** The panel shows the results of residue-level DCA for the 200 domain pairs of strongest phyletic couplings. Co-localization does not necessarily imply physical interaction. However, out of 200 pairs, 144 are also listed in iPfam database as being in physical contact in experimentally resolved PDB structures. Due to their co-localization, the generation of a joint MSA is trivial in this case and the paralogs-matching can be avoided.

## E.1 Network analysis

As stated in the main text, CoPAP and PhyDCA treat very different confounding factors of coevolutionary analysis – phylogenetic biases and indirect correlations. Nevertheless from Figure 3 of the main text, it appears that almost none of the correlated pairs strongly coupled in PhyDCA, are actually discarded by CoPAP. But are the correlations of pairs, which are retained by CoPAP as non phyletically coupled, but discarded by PhyDCA, really an indirect network effect of the PhyDCA couplings?

To answer this question, we first introduce in Fig. H two scatter plots of the phyletic couplings vs. Pearson correlations between domain pairs, in the first case for the 3611 domain pairs of highest CoPAP score, in the second case for all domain pairs. In both cases, we see a clear triangular shape, indicating that large couplings lead to large correlations, but large correlations can exist between weakly coupled pairs. Since our PhyDCA model reproduces correlations using couplings, the latter case must result from indirect correlations. Also as a consequence, the phyletic coupling network is substantially sparser than the correlation network.

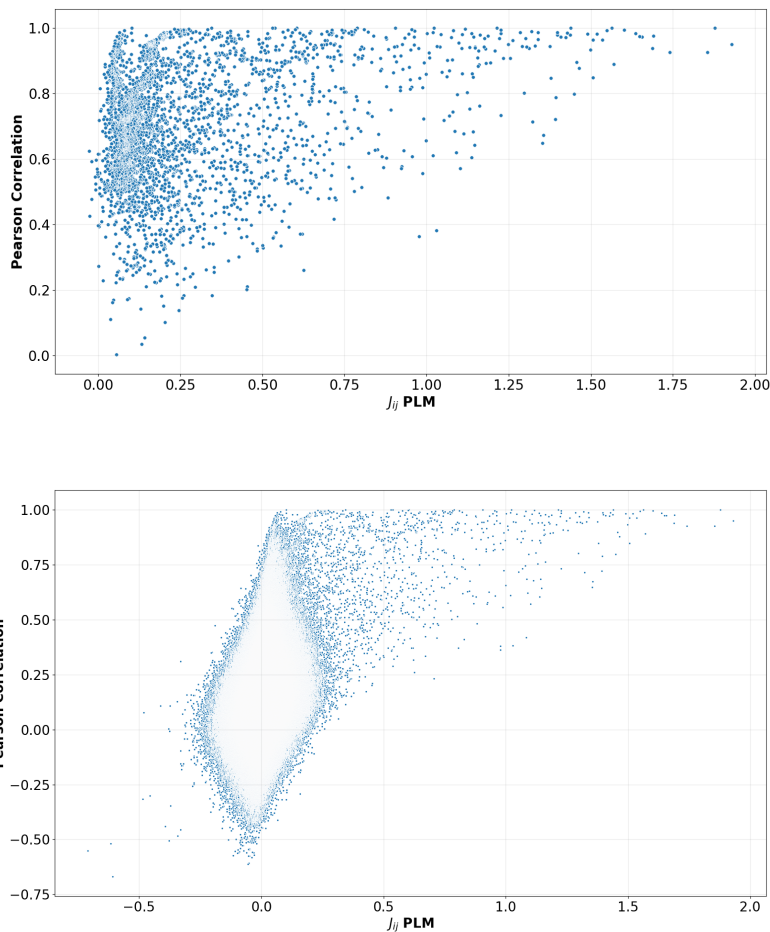


Figure H: **Couplings vs. correlations.** The figure shows a scatter plot of PhyDCA couplings vs. Pearson correlations, for the 3611 domain pairs of highest CoPAP score in the upper panel, and for all domain pairs in the lower one.

To corroborate this, in Fig. I, we consider the network of the 1000 strongest phyletic couplings and study the correlations as a function of the shortest-path distance between domains along this network. Correlations decrease with distance until they saturate at a low but non-zero level. This is coherent with the idea that empirical correlations found in the data have at least three contributions – direct correlations induced by direct couplings (at distance 1), indirect couplings induced by coupling chains, and a ground level of correlations, which possibly result from phylogenetic correlations between the species and other sampling effects.

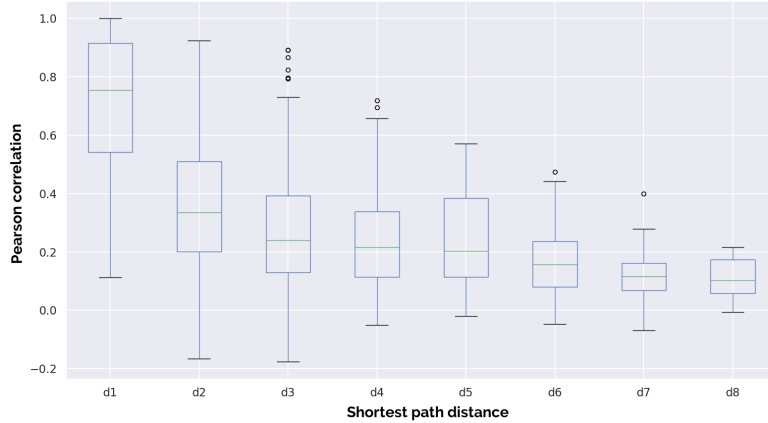


Figure I: **Correlation decay on PhyDCA network.** The figure shows the decay of empirical correlations between pairs of domain belonging to the network of the first 1000 strongest phyletic couplings as a function of their shortest-path distance on this network.

If we take alternatively the network induced by the 1613 pairs, which have large Pearson correlations and are preserved by CoPAP (the intersection of the red and green circles in Fig. 3A in the main text), we also find a correlation decrease (as to be expected in any sparsely connected graphical model), cf. Fig. J. However, the decay is slower than on the PhyDCA network, even if the network is denser. Pairs in the PhyDCA network are thus less correlated than pairs at the same distance in the correlation network, which shows that the phyletic coupling network more parsimoniously explains the connectivity patterns present in the data.



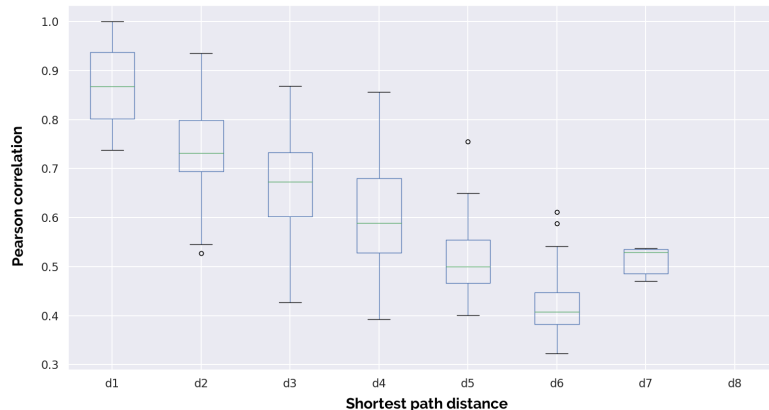


Figure J: **Correlation decay on CoPAP-Pearson network.** The figure shows the decay of empirical correlations between pairs of domain belonging to the network of the 1613 domain pairs of strongest CoPAP-preserved Pearson correlations (the intersection of the red and green circles in Fig. 3A in the main text) as a function of their shortest-path distance on this network.

## F All bacteria

In the main text we use the model organism *Escherichia coli* as reference genome in order to have a large set of known domain-domain relationships. In this section we consider a broader selection of genomes by applying the same methodology to all 9,358 Pfam domains appearing in bacteria. To access the accuracy of our prediction we compile a number of known domain-domain relationships: **intra-protein localization** (out of 2,972,104 proteins 866,591 contain multiple domains, giving rise to 26,381 distinct domain-domain relations), **domain-domain contacts in 3d structures** (from the iPfam database, for a total of 545 known relationships), **protein-protein interaction** (from the IntAct database, obtaining 67,409 domain pairs). This leads to a total of 92,428 known relationship (cf. Figure K, Panel A).

We then select the couplings between domains which are only present in *E. coli* genome ( cf. Figure K, Panel B and C) finding 96% correlation with the couplings inferred in the main text, thus proving the robustness of the results with respect to the selection of domains.

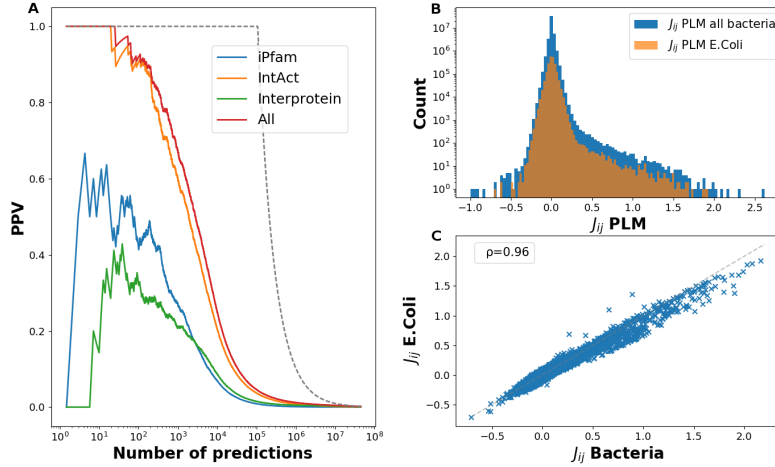


Figure K: **Phylogenetic couplings** Panel A shows the PPV of the phyletic couplings of all bacterial domains for predicting domain-domain relationships (including protein architecture, iPfam and IntAct entries). Panel B shows a histogram of couplings  $J_{ij}$ , as inferred by PLM, for the domains present in all bacteria and for those appearing only in *E. coli*. In Panel C we retain from the bacterial phyletic couplings only the couplings between domains present in *E. coli*. Then we compare them with the couplings found by the procedure described in the main text, finding a correlation of 96% between the two.

We have applied the paralog-matching analysis to the 200 most coupled bacterial domain pairs (see Figure L ). A list of the domain pairs, their phylogenetic coupling and the DCA score can be found on the [Github](#) page at `results/ALLBACTERIA_matching_results.dat`).

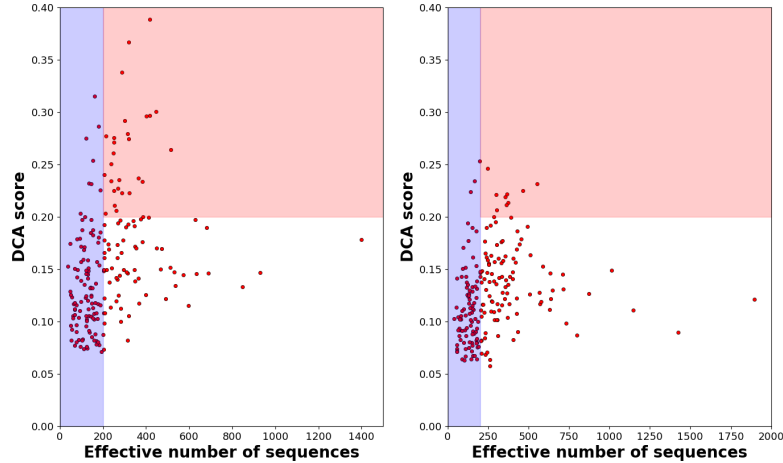


Figure L: **Matching procedure for bacteria:** Panel A shows the effective sequence number and the DCA scores for the 200 most significant PhydCA predictions. Panel B shows a random matching for the same domain pairs.

### F.1 Negative phyletic couplings

As discussed in the main text, a negative phyletic coupling disfavors the joint presence of two domains in the same genome and thereby highlights alternative solutions for the same functionality. In Table D we report the 100 most strongly negatively coupled domain pairs with their PFAM description.

## G Eukaryotic genomes: human as reference species

The complete list of results of our analysis applied on eukaryotic genomes using human as reference species can be found on the [Github](#) page at [results/HUMAN\\_matching\\_results](#).

|    | pfam ACC dom1 | pfam ACC dom2 | Phyletic coupling | domain1 description                                     | domain2 description                                     |
|----|---------------|---------------|-------------------|---|---|
| 1  | PF00303       | PF02511       | -0.9978           | Thymidylate synthase                                    | Thymidylate synthase complementing protein              |
| 2  | PF01220       | PF01487       | -0.9277           | Dehydroquinase class II                                 | Type I 3-dehydroquinase                                 |
| 3  | PF02834       | PF13563       | -0.9075           | LigT like Phosphoesterase                               | 2'-5' RNA ligase superfamily                            |
| 4  | PF00406       | PF13207       | -0.8258           | Adenylation kinase                                      | AAA domain  |
| 5  | PF01205       | PF02594       | -0.7077           | Uncharacterized protein family UPF0029                  | Uncharacterised ACR, YggU family COG1872                |
| 6  | PF13623       | PF13624       | -0.7051           | SurA N-terminal domain                                  | SurA N-terminal domain                                  |
| 7  | PF04816       | PF12847       | -0.6316           | tRNA (adenine(22)-N(1))-methyltransferase               | Methyltransferase domain                                |
| 8  | PF00636       | PF14622       | -0.6281           | Ribonuclease III domain                                 | Ribonuclease-III-like                                   |
| 9  | PF00186       | PF02511       | -0.6281           | Dihydrofolate reductase                                 | Thymidylate synthase complementing protein              |
| 10 | PF01227       | PF02649       | -0.6118           | GTP cyclohydrolase I                                    | Type I GTP cyclohydrolase folE2                         |
| 11 | PF06745       | PF13481       | -0.5844           | KaKc  | AAA domain  |
| 12 | PF02677       | PF08331       | -0.581            | Uncharacterized BCR, COG1636                            | Domain of unknown function (DUF1730)                    |
| 13 | PF02696       | PF03190       | -0.5651           | Uncharacterized ACR, YgiU/UPF0061 family                | Protein of unknown function, DUF255                     |
| 14 | PF00311       | PF02436       | -0.5432           | Phosphoenolpyruvate carboxylase                         | Conserved carboxylase domain                            |
| 15 | PF02502       | PF06026       | -0.5371           | Ribose/Galactose isomerase                              | Ribose 5-phosphate isomerase A (phosphoriboisomerase A) |
| 16 | PF00245       | PF05787       | -0.5333           | Alkaline phosphatase                                    | Bacterial protein of unknown function (DUF839)          |
| 17 | PF00075       | PF13456       | -0.5317           | RNase H   | Reverse transcriptase-like                              |
| 18 | PF01169       | PF02659       | -0.5294           | Uncharacterized protein family UPF0016                  | Putative manganese efflux pump                          |
| 19 | PF01321       | PF05195       | -0.5165           | Creatinase/Prolidase N-terminal domain                  | Aminopeptidase P, N-terminal domain                     |
| 20 | PF02594       | PF09186       | -0.5139           | Uncharacterised ACR, YggU family COG1872                | Domain of unknown function (DUF1949)                    |
| 21 | PF02595       | PF13660       | -0.5071           | Glycerate kinase family                                 | Domain of unknown function (DUF4147)                    |
| 22 | PF02595       | PF05161       | -0.4983           | Glycerate kinase family                                 | MOFRL family  |
| 23 | PF00491       | PF04371       | -0.4956           | Arginase family   | Pomphyrmonas-type peptidyl-arginine deiminase           |
| 24 | PF02664       | PF05221       | -0.4583           | S-Ribosylhomocysteinease (LuxS)                         | S-adenosyl-L-homocysteine hydrolase                     |
| 25 | PF00719       | PF02833       | -0.453            | Inorganic pyrophosphatase                               | DHHA2 domain  |
| 26 | PF00670       | PF02664       | -0.446            | S-adenosyl-L-homocysteine hydrolase, NAD binding domain | S-Ribosylhomocysteinease (LuxS)                         |
| 27 | PF04306       | PF07264       | -0.445            | Protein of unknown function (DUF456)                    | Etoposide-induced protein 2.4 (EIT24)                   |
| 28 | PF00141       | PF06628       | -0.4378           | Peroxidase  | Catalase-related immune-responsive                      |
| 29 | PF13441       | PF13488       | -0.4371           | YMGG-like Gly-zipper                                    | Glycine zipper  |
| 30 | PF08328       | PF10397       | -0.4304           | Adenylosuccinate lyase C-terminal                       | Adenylosuccinate lyase C-terminus                       |
| 31 | PF00821       | PF01293       | -0.4195           | Phosphoenolpyruvate carboxylase                         | Phosphoenolpyruvate carboxylase                         |
| 32 | PF03100       | PF05140       | -0.4192           | CcmE  | ResB-like family  |
| 33 | PF01027       | PF12811       | -0.4075           | Inhibitor of apoptosis-promoting Bax1                   | Bax inhibitor 1 like                                    |
| 34 | PF03379       | PF05140       | -0.4022           | CcmB protein  | ResB-like family  |
| 35 | PF01863       | PF10263       | -0.3981           | Protein of unknown function DUF45                       | SprT-like family  |
| 36 | PF01458       | PF07743       | -0.396            | Uncharacterized protein family (UPF0051)                | HSCB C-terminal oligomerisation domain                  |
| 37 | PF05140       | PF16327       | -0.3919           | ResB-like family  | Cytochrome c-type biogenesis protein CcmF C-terminal    |
| 38 | PF01878       | PF03883       | -0.3849           | EVE domain  | Peroxide stress protein YaaA                            |
| 39 | PF03352       | PF08713       | -0.3776           | Methyladenine glycosylase                               | DNA alkylation repair enzyme                            |
| 40 | PF01592       | PF02657       | -0.3774           | NifU-like N terminal domain                             | Fe-S metabolism associated domain                       |
| 41 | PF04011       | PF13421       | -0.3727           | LemA family   | SPFH domain-Band 7 family                               |
| 42 | PF00274       | PF01116       | -0.3667           | Fructose-bisphosphate aldolase class-I                  | Fructose-bisphosphate aldolase class-II                 |
| 43 | PF01817       | PF07736       | -0.364            | Chorismate mutase type II                               | Chorismate mutase type I                                |
| 44 | PF01070       | PF02589       | -0.3613           | FMN-dependent dehydrogenase                             | LUD domain  |
| 45 | PF01943       | PF13440       | -0.3613           | Polysaccharide biosynthesis protein                     | Polysaccharide biosynthesis protein                     |
| 46 | PF00282       | PF13086       | -0.355            | Pyridoxal-dependent decarboxylase conserved domain      | AAA domain  |
| 47 | PF00141       | PF00199       | -0.3527           | Peroxidase  | Catalase  |
| 48 | PF01458       | PF01491       | -0.3511           | Uncharacterized protein family (UPF0051)                | Fratxinin-like domain                                   |
| 49 | PF00368       | PF02542       | -0.3461           | Hydroxymethylglutaryl-coenzyme A reductase              | YggB family   |

Table C: The 100 most strongly negative coupled domain pairs. Negative phyletic couplings identify alternative solutions for the same functionality.

|     | pfam    | ACC | dom1    | pfam | ACC     | dom2 | Phyletic coupling   | domain1 description  | domain2 description |
|-----|---------|-----|---------|------|---------|------|---|--|---------------------|
| 50  | PF01523 |     | PF08367 |      | -0.3405 |      | Putative modulator of DNA gyrase                            | Peptidase Mf6C associated                                  |                     |
| 51  | PF02386 |     | PF02705 |      | -0.3401 |      | Cation transport protein                                    | K+ potassium transporter                                   |                     |
| 52  | PF00368 |     | PF04551 |      | -0.3398 |      | Hydroxymethylglutaryl-coenzyme A reductase                  | GcpE protein   |                     |
| 53  | PF00368 |     | PF02401 |      | -0.3385 |      | Hydroxymethylglutaryl-coenzyme A reductase                  | IyfB protein   |                     |
| 54  | PF01977 |     | PF16582 |      | -0.3343 |      | 3-octaprenyl-4-hydroxybenzoate carboxy-lyase                | Middle domain of thiamine pyrophosphate                    |                     |
| 55  | PF13476 |     | PF13514 |      | -0.3331 |      | AAA domain  | AAA domain   |                     |
| 56  | PF00368 |     | PF01128 |      | -0.3326 |      | Hydroxymethylglutaryl-coenzyme A reductase                  | 2-C-methyl-D-erythritol 4-phosphate cytidyltransferase     |                     |
| 57  | PF04204 |     | PF07021 |      | -0.3316 |      | Homoserine O-succinyltransferase                            | Methionine biosynthesis protein MetW                       |                     |
| 58  | PF00145 |     | PF11907 |      | -0.3261 |      | C-5 cytosine-specific DNA methylase                         | Domain of unknown function (DUF3427)                       |                     |
| 59  | PF01458 |     | PF04384 |      | -0.3256 |      | Uncharacterized protein family (UPF0051)                    | Iron-sulphur cluster assembly                              |                     |
| 60  | PF01699 |     | PF03741 |      | -0.3253 |      | Sodium/calcium exchanger protein                            | Integral membrane protein TerC family                      |                     |
| 61  | PF01797 |     | PF13087 |      | -0.3217 |      | Transposase IS200 like                                      | AAA domain   |                     |
| 62  | PF02675 |     | PF16653 |      | -0.3216 |      | S-adenosylmethionine decarboxylase                          | Saccharopine dehydrogenase C-terminal domain               |                     |
| 63  | PF04402 |     | PF13598 |      | -0.3197 |      | Protein of unknown function (DUF541)                        | Domain of unknown function (DUF4139)                       |                     |
| 64  | PF01544 |     | PF01769 |      | -0.3186 |      | CorA-like Mg2+ transporter protein                          | Divalent cation transporter                                |                     |
| 65  | PF13145 |     | PF13616 |      | -0.3184 |      | PP1C-type PPIASE domain                                     | PP1C-type PPIASE domain                                    |                     |
| 66  | PF01070 |     | PF11870 |      | -0.3179 |      | FMN-dependent dehydrogenase                                 | Domain of unknown function (DUF3390)                       |                     |
| 67  | PF01904 |     | PF03649 |      | -0.3159 |      | Protein of unknown function DUF72                           | Uncharacterised protein family (UPF0014)                   |                     |
| 68  | PF01226 |     | PF04657 |      | -0.3145 |      | Formate/nitrite transporter                                 | Putative inner membrane exporter, YdcZ                     |                     |
| 69  | PF03547 |     | PF13579 |      | -0.3136 |      | Membrane transport protein                                  | Glycosyl transferase 4-like domain                         |                     |
| 70  | PF09992 |     | PF13276 |      | -0.3116 |      | Phosphodiester glycosidase                                  | HTH-like domain  |                     |
| 71  | PF00444 |     | PF00872 |      | -0.3092 |      | Ribosomal protein L36                                       | Transposase, Mutator family                                |                     |
| 72  | PF01070 |     | PF06127 |      | -0.3081 |      | FMN-dependent dehydrogenase                                 | Protein of unknown function (DUF962)                       |                     |
| 73  | PF00444 |     | PF13338 |      | -0.3052 |      | Ribosomal protein L36                                       | Transcriptional regulator, AbiE1 autotoxin                 |                     |
| 74  | PF00368 |     | PF02670 |      | -0.3052 |      | Hydroxymethylglutaryl-coenzyme A reductase                  | 1-deoxy-D-xylulose 5-phosphate reductoisomerase            |                     |
| 75  | PF00368 |     | PF08436 |      | -0.3044 |      | Hydroxymethylglutaryl-coenzyme A reductase                  | 1-deoxy-D-xylulose 5-phosphate reductoisomerase C-terminal |                     |
| 76  | PF00368 |     | PF13288 |      | -0.3044 |      | Hydroxymethylglutaryl-coenzyme A reductase                  | DXP reductoisomerase C-terminal domain                     |                     |
| 77  | PF01458 |     | PF01592 |      | -0.3038 |      | Uncharacterized protein family (UPF0051)                    | NifU-like N terminal domain                                |                     |
| 78  | PF14789 |     | PF14805 |      | -0.3036 |      | Tetrahydrodipicolinate N-succinyltransferase middle         | Tetrahydrodipicolinate N-succinyltransferase N-terminal    |                     |
| 79  | PF01042 |     | PF14588 |      | -0.3036 |      | Endoribonuclease L-PSP                                      | YjgF/chloranate mutase-like, putative endoribonuclease     |                     |
| 80  | PF02110 |     | PF05690 |      | -0.302  |      | Hydroxyethylthiazole kinase family                          | Thiazole biosynthesis protein ThiG                         |                     |
| 81  | PF02677 |     | PF13484 |      | -0.3017 |      | Uncharacterized BCR, COG1636                                | 4Fe-4S double cluster binding domain                       |                     |
| 82  | PF13414 |     | PF13738 |      | -0.3002 |      | TPR repeat  | Pyridine nucleotide-disulphide oxidoreductase              |                     |
| 83  | PF03073 |     | PF08212 |      | -0.2999 |      | TspO/MBR family   | Lipocalin-like domain                                      |                     |
| 84  | PF03592 |     | PF13671 |      | -0.2998 |      | Terminase small subunit                                     | AAA domain   |                     |
| 85  | PF01244 |     | PF07784 |      | -0.2997 |      | Membrane dipeptidase (Peptidase family M19)                 | Protein of unknown function (DUF1622)                      |                     |
| 86  | PF02900 |     | PF13384 |      | -0.2991 |      | Catalytic LigB subunit of aromatic ring-opening dioxygenase | Homeodomain-like domain                                    |                     |
| 87  | PF01391 |     | PF05050 |      | -0.2976 |      | Collagen triple helix repeat (20 copies)                    | Methyltransferase FkBM domain                              |                     |
| 88  | PF03413 |     | PF13936 |      | -0.2971 |      | Peptidase propeptide and YPEB domain                        | Helix-turn-helix domain                                    |                     |
| 89  | PF01988 |     | PF16916 |      | -0.2956 |      | VIT family  | Dimerisation domain of Zinc Transporter                    |                     |
| 90  | PF08309 |     | PF13088 |      | -0.2952 |      | IVVD repeat   | BNR repeat-like domain                                     |                     |
| 91  | PF00317 |     | PF08471 |      | -0.2947 |      | Ribonucleotide reductase, all-alpha domain                  | Class II vitamin B12-dependent ribonucleotide reductase    |                     |
| 92  | PF11127 |     | PF14524 |      | -0.2947 |      | Protein of unknown function (DUF2892)                       | Wzt C-terminal domain                                      |                     |
| 93  | PF00565 |     | PF07394 |      | -0.2945 |      | Staphylococcal nuclease homologue                           | Protein of unknown function (DUF1501)                      |                     |
| 94  | PF00954 |     | PF12844 |      | -0.2938 |      | Uncharacterized protein conserved in bacteria (DUF2188)     | Helix-turn-helix domain                                    |                     |
| 95  | PF02635 |     | PF13440 |      | -0.293  |      | DsrE/DsrF-like family                                       | Polysaccharide biosynthesis protein                        |                     |
| 96  | PF01906 |     | PF01987 |      | -0.2929 |      | Putative heavy-metal-binding                                | Mitochondrial biogenesis AIM24                             |                     |
| 97  | PF07582 |     | PF13492 |      | -0.2927 |      | AP endonuclease family 2 C terminus                         | GAF domain   |                     |
| 98  | PF02230 |     | PF13740 |      | -0.2915 |      | Phospholipase/Carboxylesterase                              | ACT domain   |                     |
| 99  | PF02535 |     | PF13274 |      | -0.2907 |      | ZIP Zinc transporter  | Protein of unknown function (DUF4065)                      |                     |
| 100 | PF01680 |     | PF03740 |      | -0.2907 |      | SOR/SNZ family  | SOR/SNZ family   |                     |

Table D: The 100 most strongly negative coupled domain pairs. Negative phyletic couplings identify alternative solutions for the same functionality.

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