Supplementary Materials

1. Survey questions and general population data

Characteristic	Question text	Original source of the question text and general population data		
		U.S.	Germany	S. Korea
1. Not having money for food	Have there been times in the past 12 months when you did not have enough money to buy food you or your family needed? (a)Yes–(b)No	Gallup World Poll (2010)	Gallup World Poll (2010)	Social Integration Status Survey (2011)
2. Donating to charity	In the past month, have you donated money to a charity? (a)Yes–(b)No	Gallup World Poll (2010)	Gallup World Poll (2010)	Social Survey of Welfare (2017)
3. Experiencing theft	Within the past 12 months, have you had money or property stolen from you or another household member? (a)Yes–(b)No	Gallup World Poll (2010)	Gallup World Poll (2010)	–
4. Religion importance	Is religion an important part of your daily life? (a)Yes–(b)No	Gallup World Poll (2010)	Gallup World Poll (2010)	Religion of Koreans by Gallup (1984–2014)
5. Worship attendance	Have you attended a place of worship or a religious service within the past 7 days? (a)Yes–(b)No	Gallup World Poll (2010)	Gallup World Poll (2010)	Religion of Koreans by Gallup (1984–2014)
6. God and morality	Which one of these comes closer to your opinion? (a) It is not necessary to believe in God in order to be moral and have good values. (b) It is necessary to believe in God in order to be moral and have good values.	Pew (2011)	Pew (2011)	–
7. Belief in a god	Do you believe in god or a supreme being? (a)Yes–(b)No	Pew (2011)	Pew (2011)	Religion of Koreans by Gallup (1984–2014)
8. Smoking	These days, are you smoking any tobacco product at least once a day? (Tobacco smoking includes cigarettes, cigars, pipes, and any other form of smoked tobacco).(a)Yes–(b)No	World Health Organization (2010)	World Health Organization (2010)	National Nutrition Survey (2016)
9. Military force	Do you agree that it is sometimes necessary to use military force to maintain order in the world? (a)Yes–(b)No	Pew (2011)	Pew (2011)	–
10. Homosexuality acceptance	Which one of these comes closer to your opinion? (a) Homosexuality is a way of life that should be accepted by society. (b) Homosexuality is a way of life that should not be accepted by society.	Pew (2011)	Pew (2011)	Gallup Daily Opinion (2017)

Table S1. Survey questions about different attributes. Survey questions asking about different attributes in the United States, Germany, and South Korea, including the source of question texts and the objective data for minority and majority group sizes in the general population.

Characteristic	U.S. (%)		Germany (%)		S. Korea (%)	
	(a)	(b)	(a)	(b)	(a)	(b)
1. Not having money for food	19	81	5	95	3	97
2. Donating to charity	57	43	43	57	26.7	73.7
3. Experiencing theft	12	88	9	91	–	–
4. Religion importance	70	30	27	73	52	48
5. Worship attendance	53	47	33	67	44	56
6. God and morality	47	53	33	67	–	–
7. Belief in a god	64	36	38	62	39	61
8. Smoking	15.2	85	21.9	78	23.9	76.1
9. Military force	76.5	24	50	50	–	–
10. Homosexuality	35.5	64	12.1	87.9	34	58

Table S2. Objective size of minority and majority group for each attribute. Objective data for minority and majority group sizes for each attribute in the general populations of the United States, Germany, and South Korea. Texts of answers (a) and (b) correspond to those listed in the Table S1 for each answer.

2. Survey results for South Korea

Survey results for South Korea show similar pattern as those for the United States and Germany, shown in Fig. 2. In the heterophilic networks $(0 \leq h < 0.5)$, the minority (a) underestimates its own size, and the majority (b) overestimates the size of the minority, resembling false uniqueness. In homophilic networks $(0.5 < h \leq 1)$, the minority (a) overestimates its own size and the majority (b) underestimates the size of the minority, resembling false consensus. We calculated the group level perception bias as a relative measure: the perceived size of the minority group divided by the objective minority size obtained in national surveys. The right inset displays the same information on a log scale to make the amount of underestimation and overestimation comparable. The left inset shows the perception bias of small minority, whose bias ranges wider than for other groups. In general, as minority-group size becomes smaller, social perception biases increase. To fit the survey results, we used the curve fit.

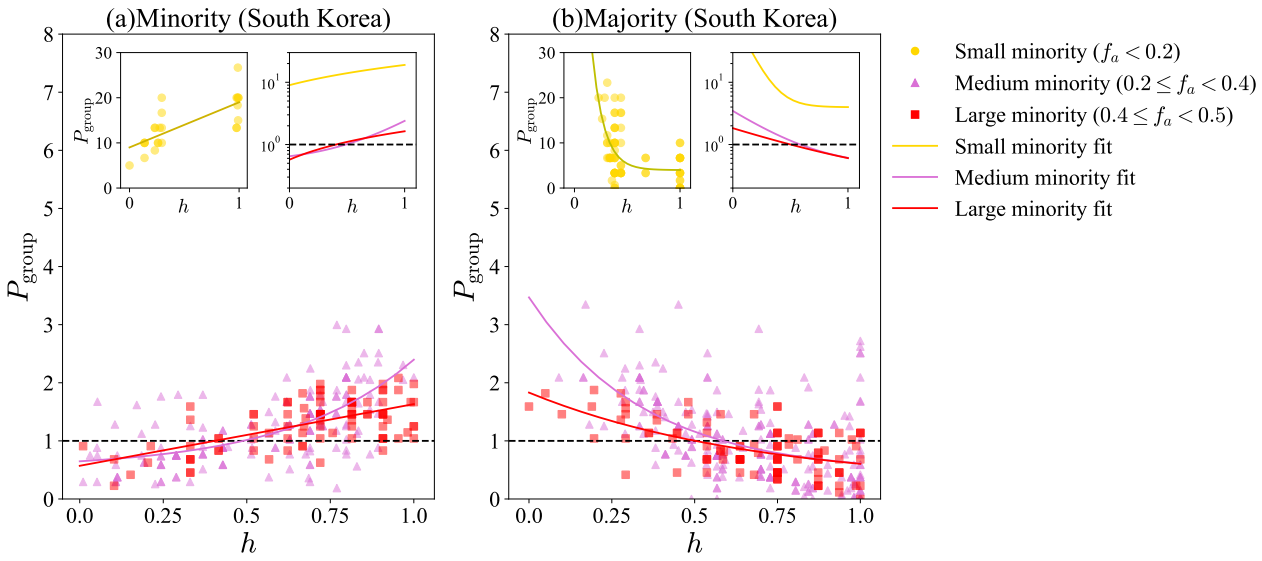

Fig. S1. Bias in perception of minority-group size, for participants whose personal networks exhibit different levels of homophily (h), and for attributes held by a small, medium, or large minority group in South Korea. Different colors distinguish perception biases for attributes that in South Korea are held by a small($f_a < 0.2$), medium ($0.2 \leq f_a < 0.4$), or large ($0.4 \leq f_a < 0.5$) minority group. Each data point represents the perception bias of a group where one individual involved for an attribute. The left inset shows the bias of small minority group, and the right inset displays the same bias on a log scale to make the amount of underestimation and overestimation comparable. The solid lines with colors for the group sizes are drawn by the curve fit.

2. Distribution of individual-level perception bias as a function of degree

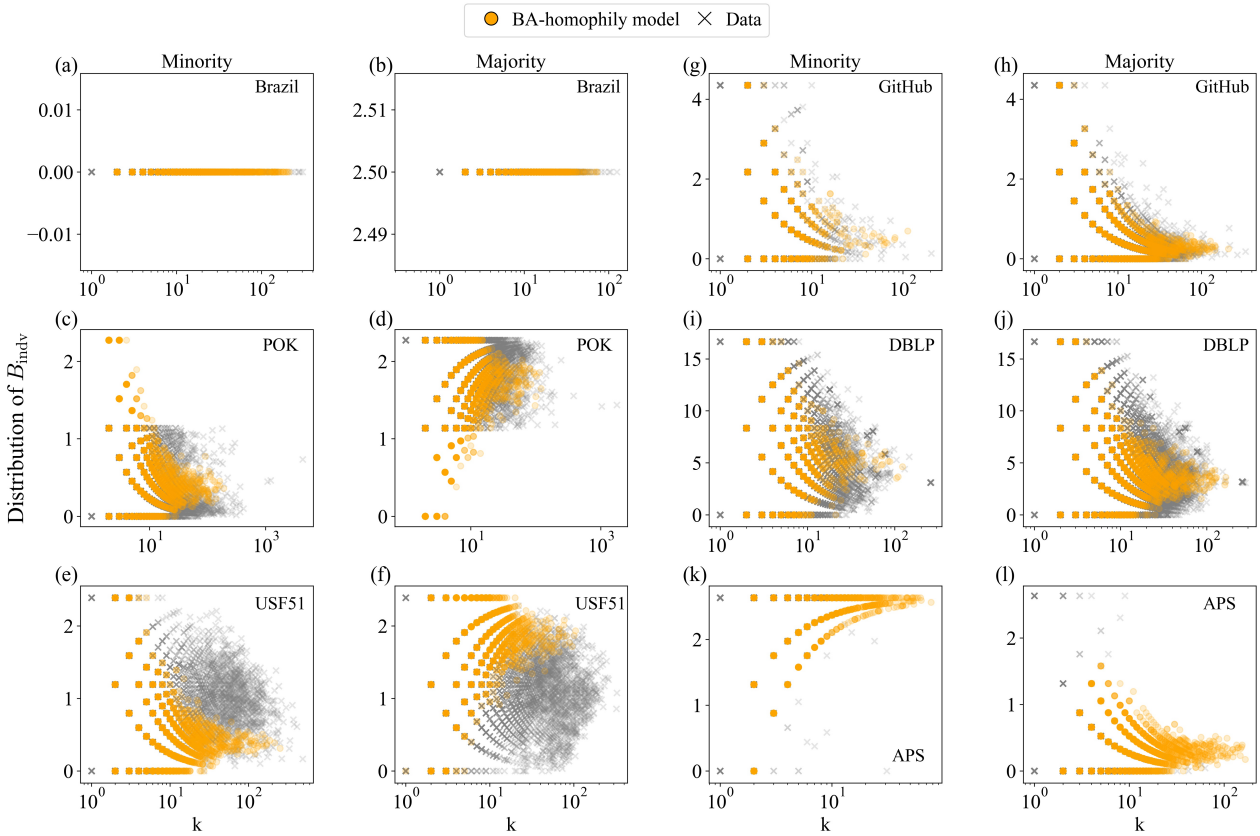

Fig. S2. Distribution of individual-level perception bias ($P_{\text{indv.}}$) as a function of degree (k). Each row represents one empirical network. The left two columns show the distribution of $P_{\text{indv.}}$ for the heterophilic empirical networks [Brazilian sexual contact network (Brazil), Swedish online dating network (POK), Facebook network of an university (USF51)] and the right two columns show the biases for the homophilic empirical networks [GitHub developers' network (GitHub), DBLP developers' network (DBLP), American Physical Society network (APS)]. The gray crosses represent the perception bias of each individual estimated from the empirical network, and orange circles show the perception bias obtained from the BA-homophily model. The model reproduces well the heterogeneous distribution of individual perception bias in most of the networks. The simulation results are aggregated over 50 iterations and the network size is $N = 2,000$. The x axis is shown in log scale.

3. Growth rate (C) in BA-homophily model

On the basis of the derivation provided by Karimi et al.[28], we can derive the exact degree dynamics of the BA-homophily model. Let us assume $K_a(t)$ and $K_b(t)$ as the sum of the degrees of each group a and b, respectively. With the number of links a node has, the total number of links at each time step will be $K(t) = K_a(t) + K_b(t) = 2mt$. One can also describe the evolution of each group's degree as

$$\begin{cases} K_a(t+\Delta t) = K_a(t) + m\Big(f_a\big(1+\dfrac{h_{aa}K_a(t)}{h_{aa}K_a(t)+h_{ab}K_b(t)}\big) + f_b\dfrac{h_{ba}K_a(t)}{h_{bb}K_b(t)+h_{ba}K_a(t)}\Big)\Delta t, \\ K_b(t+\Delta t) = K_b(t) + m\Big(f_b\big(1+\dfrac{h_{bb}K_b(t)}{h_{bb}K_b(t)+h_{ba}K_a(t)}\big) + f_a\dfrac{h_{ab}K_b(t)}{h_{aa}K_a(t)+h_{ab}K_b(t)}\Big)\Delta t. \end{cases} \tag{S1}$$

Here, one can separate the amount of increase of the degree for each group with the limit $\Delta t \to 0$,

$$\begin{cases} \dfrac{dK_a}{dt} = m\Big(f_a\big(1+\dfrac{h_{aa}K_a(t)}{h_{aa}K_a(t)+h_{ab}K_b(t)}\big) + f_b\dfrac{h_{ba}K_a(t)}{h_{bb}K_b(t)+h_{ba}K_a(t)}\Big), \\ \dfrac{dK_b}{dt} = m\Big(f_b\big(1+\dfrac{h_{bb}K_b(t)}{h_{bb}K_b(t)+h_{ba}K_a(t)}\big) + f_a\dfrac{h_{ab}K_b(t)}{h_{aa}K_a(t)+h_{ab}K_b(t)}\Big). \end{cases} \tag{S2}$$

We can assume that $K_a(t)$ ($K_b(t)$) grows as a linear function of time. Given this assumption, we can write that $K_a(t) = Cmt$ ($K_b(t) = (2-C)mt$) based on $K(t) = 2mt$.

$$\begin{cases} \dfrac{dK_a}{dt} = Cm = m\Big(f_a\big(1+\dfrac{h_{aa}Cmt}{h_{aa}Cmt+h_{ab}(2-C)mt}\big) + f_b\dfrac{h_{ba}Cmt}{h_{bb}(2-C)mt+h_{ba}Cmt}\Big), \\ \dfrac{dK_b}{dt} = (2-C)m = m\Big(f_b\big(1+\dfrac{h_{bb}(2-C)mt}{h_{bb}(2-C)mt+h_{ba}Cmt}\big) + f_a\dfrac{h_{ab}(2-C)mt}{h_{aa}Cmt+h_{ab}(2-C)mt}\Big). \end{cases} \tag{S3}$$

Then, we can derive the relation of C with group sizes and edge density in a group (Eq. 9) from Eq. S2 by using p_{aa} (Eq. 5) and p_{ab} (Eq. 6).

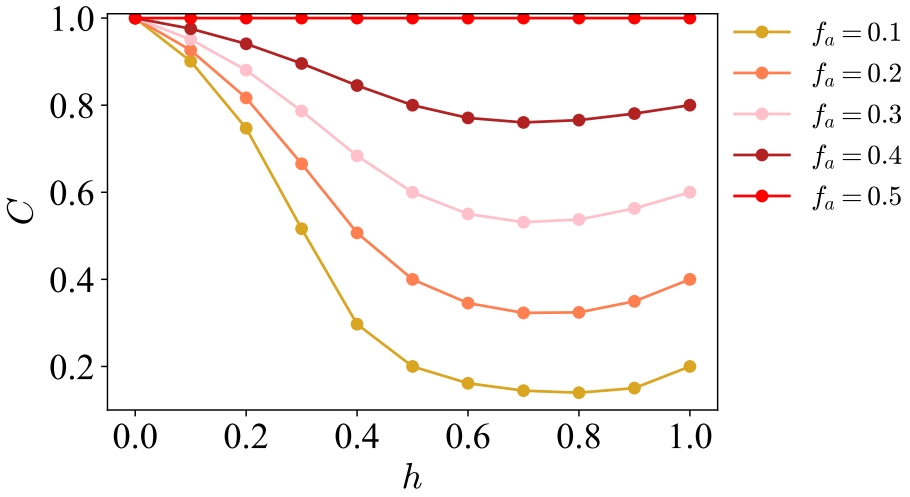

Fig. S3. The relation between the minority's degree growth rate (C) and the symmetric homophily (h). As the minority group's size f_a decreases, the degree growth rate of the minority drastically decreases with increasing symmetric homophily h ($h_{aa} = h_{bb}$). C is a function of h and f_a as described in Eq. 9.

4. Relationship between symmetric homophily (h) and Newman's assortativity (q)

The symmetric homophily is equivalent to Newman's assortativity measure (q)[29]. The latter corresponds directly to the homophily parameter in the BA-homophily model after adjusting for scale in a relation $q = 2h - 1$. In the BA-homophily model, $h = 0$ means complete heterophily ($q = -1$), $h = 0.5$ indicates no relationship between network structure and attributes ($q = 0$), and $h = 1$ indicates complete homophily ($q = 1$). The relationship slightly deviates from the linear function for small minority-group sizes, but is independent of group sizes when $h \approx 0.5$.

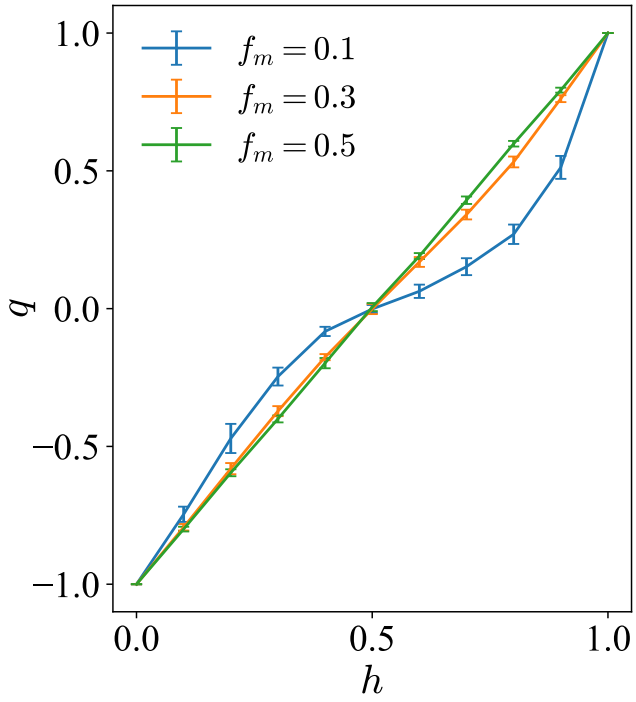

Fig. S4. Relationship between Newman's assortativity (q) and symmetric homophily (h) in the BA-homophily model, for different sizes of minority group (f_a). Newman's assortativity is proportional to h scaled as $2h - 1$. The relationship slightly deviates from the linear function for small minority-group sizes, but is independent of group sizes when $h \approx 0.5$.

5. Bias of individual perceptions aggregated with those of 1-hop neighbors, assuming asymmetric homophily

Here, we investigate to what extent and under what structural conditions individuals can reduce their perception bias for the prevalence of an attribute in a population by asking their friends about their perceptions and integrating those perceptions with their own when homophily is asymmetric. We build on DeGroot's weighted belief formalization by aggregating an individual's perception (*ego*) with the averaged perceptions of the individual's direct neighbors (*1-hop*)[34] with an assumption of asymmetric homophily in the BA-homophily model.

Figure S5 shows a comparison of the average perception bias ($\bar{P}_{\text{indv.}}$) of individuals who are in (a) the minority and (b) the majority, with the bias of their perceptions aggregated with those of their 1-hop neighbors. The minority group size is fixed to 0.1 in the top row, 0.3 in the middle row, and 0.5 in the bottom row. The homophily for the minority group h_{aa} is fixed to $0.1, 0.5, 0.9$ (depicted by lines of different colors), while homophily for the majority group h_{bb} ranges from 0 to 1 along the horizontal axis.

Results for asymmetric homophily (Fig. S5) are generally similar to those for symmetric homophily (Fig. 5): accounting for the opinion of 1-hop neighbors can decrease perception bias when networks are heterogeneous. For the majority, aggregating their own perceptions with those of their 1-hop neighbors leads to a robust improvement in perception accuracy, as described in Fig. S5b, d, and f. For the minority, accounting for 1-hop neighbors also helps decrease the bias, though less than for the majority. However, when minority group is small and homophily is highly asymmetric ($h_{aa} = 0.5$ and $h_{bb} < 0.5$), accounting for 1-hop neighbors can indeed increase the bias see Fig. S5a).

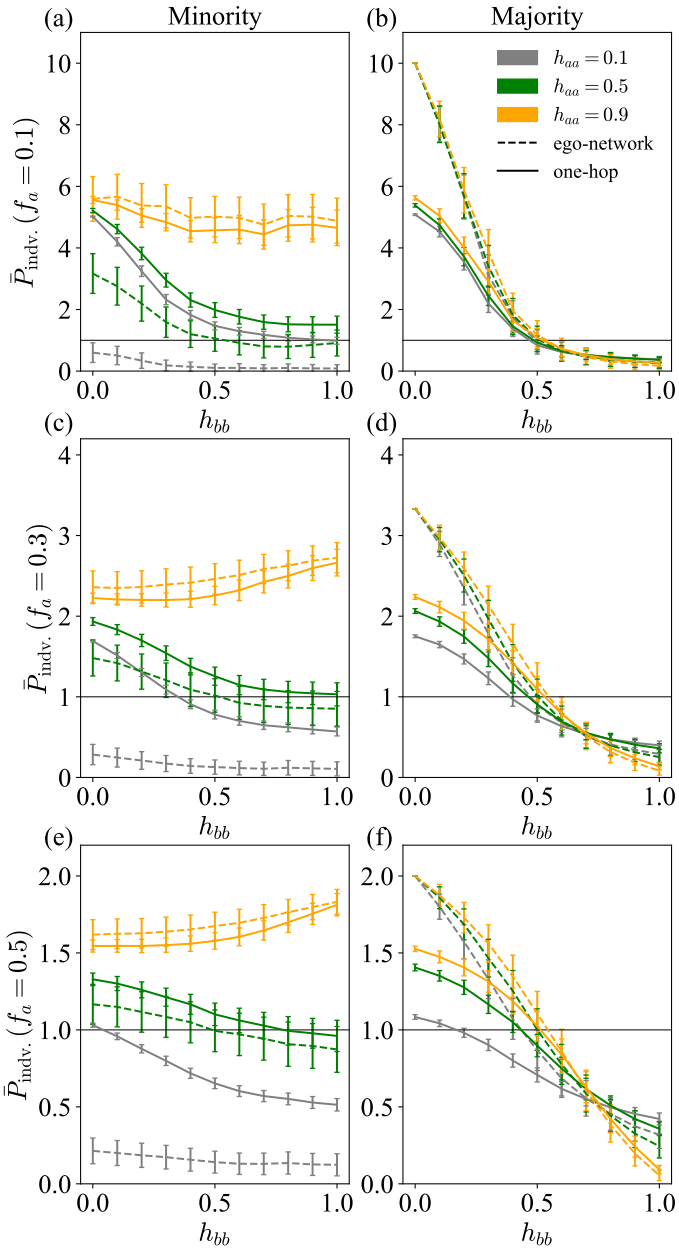

Fig. S5. Individual-level perception biases (dashed lines) compared to the biases of the weighted average of perceptions of individuals and their 1-hop neighbors (solid lines), assuming asymmetric homophily. The minority-group size is fixed at 0.1 in the top row, 0.3 in the middle row, and 0.5 in the bottom row. The homophily for the minority h_{aa} (depicted by lines of different colors) is fixed, while homophily for the majority h_{bb} ranges from 0 to 1 along the horizontal axis.