Spatiotemporal prediction of wildfire size extremes with Bayesian finite sample maxima

Appendix S1: Prior specifications

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Prior distributions were chosen to regularize coefficients on the distribution specific means $\beta^{(\mu)}$ and structural zero parameters $\beta^{(\pi)}$. We used a regularized horseshoe prior on these coefficients, which shrinks irrelevant coefficients towards zero, while regularizing nonzero coefficients (Piironen, Vehtari, and others 2017). For zero-inflated models, we used a multivariate version of the regularized horseshoe (Peltola et al. 2014):

$$\begin{pmatrix} \beta_j^{(\mu)} \\ \beta_j^{(\pi)} \end{pmatrix} \sim \mathrm{N}\left(\mathbf{0}, \begin{pmatrix} \tau_1^2 \tilde{\lambda}_{1,j}^2 & \rho \tau_1 \tau_2 \tilde{\lambda}_{1,j} \tilde{\lambda}_{2,j} \\ \rho \tau_1 \tau_2 \tilde{\lambda}_{1,j} \tilde{\lambda}_{2,j} & \tau_2^2 \tilde{\lambda}_{2,j}^2 \end{pmatrix} \right),$$

$$\tilde{\lambda}_{m,j}^2 = \frac{c_m^2 \lambda_j^2}{c_m^2 + \tau_m^2 \lambda_j^2},$$

for each response dimension $m = 1, 2$ and coefficient $j = 1, ..., p$. Here ρ is a correlation parameter, τ_1 and τ_2 are global variance hyperparameters, c_1 and c_2 are hyperparameters that determine the amount of shrinkage on the largest coefficients, and λ_j is a local scale parameter drawn from a half-Cauchy distribution that control the amount of shrinkage applied to coefficient j (Piironen, Vehtari, and others 2017). With this prior specification, information can be shared across the two response dimensions through the correlation pa-

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rameter ρ, and/or through the local scale parameters λ_j. For count models without structural zeros (the Poisson and negative binomial models), this multivariate prior simplifies to a univariate regularized horseshoe prior.

Spatiotemporal random effects were constructed using a temporally autoregressive, spatially intrinsically autoregressive formulation (Besag and Kooperberg 1995; Banerjee, Carlin, and Gelfand 2014). Temporarily suppressing the superscript that indicates whether the effects are on μ or π, and denoting column t from an $S \times T$ $\mathbf{\Phi}$ as $\boldsymbol{\phi}_t$ we have:

$$\boldsymbol{\phi}_{t=1} \sim \mathrm{N}(\mathbf{0}, (\tau^{(\phi)}(\mathbf{D} - \mathbf{W}))^{-1})$$

$$\boldsymbol{\phi}_t \sim \mathrm{N}(\eta \boldsymbol{\phi}_{t-1}, (\tau^{(\phi)}(\mathbf{D} - \mathbf{W}))^{-1}), \quad t = 2, ..., T$$

where η is a temporal dependence parameter, $\tau^{(\phi)}$ is a precision parameter, $\mathbf{D}$ is an $S \times S$ diagonal matrix with entries corresponding to the number of spatial neighbors for each spatial unit, and $\mathbf{W}$ is an $S \times S$ spatial adjacency matrix with nonzero elements only when spatial unit i is a neighbor of spatial unit j ($w_{i,j} = 1$ if i is a neighbor of j, and $w_{i,j} = 0$ otherwise, including $w_{i,i} = 0$ for all i). $\tau^{(\phi)}$ is a precision parameter. We imposed a soft identifiability constraint that places high prior mass near $\sum_{s=1}^{S} \phi^*_{t,s} = 0$ for all t.

We applied a univariate regularized horseshoe prior to all β coefficients in burned area models (Piironen, Vehtari, and others 2017):

$$\beta_j \sim \mathrm{N}\Big(0, \tau^2 \tilde{\lambda}_j^2\Big), \quad \tilde{\lambda}_j^2 = \frac{c^2 \lambda_j^2}{c^2 + \tau^2 \lambda_j^2},$$

Spatiotemporal random effects were constructed in the same way as for the count models.

Literature cited

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