# Non-adiabatic molecular dynamics on graphics processing units: performance and application to rotary molecular motors

Laurens D. M. Peters, $^\dagger$  Jörg Kussmann, $^\dagger$  and Christian Ochsenfeld $^{*,\dagger,\ddagger}$ 

†Chair of Theoretical Chemistry, Department of Chemistry, University of Munich (LMU), Butenandtstr. 7, D-81377 München, Germany ‡Max Planck Institute for Solid State Research, Heisenbergstr. 1, D-70569 Stuttgart,

Germany

E-mail: c.ochsenfeld@fkf.mpg.de

## **Contents**



## 1 Structures

All optimized structures and initial conditions (structure and velocity) are available at https://www.cup.uni-muenchen.de/pc/ochsenfeld/download/.

## 2 Performance



Figure S1: Log-log plot of the timings of (left) Coulomb and (right) exchange integral evaluations and their derivatives with respect to the nuclear coordinates of polyethine  $(III_n)$  with  $n = 40, 50, 75, 100$  calculated at PBE0/def2-SVP level of theory on CPUs. The slope of the linear fit is equal to the effective scaling behavior of the routine.



Figure S2: Log-log plot of the timings of (left) Coulomb and (right) exchange integral evaluations and their derivatives with respect to the nuclear coordinates of polyethine  $(III_n)$  with  $n = 40, 50, 75, 100$  calculated at PBE0/def2-SVP level of theory on GPUs. The slope of the linear fit is equal to the effective scaling behavior of the routine.



Figure S3: Log-log plot of the timings of (left) Coulomb and (right) exchange integral evaluations and their derivatives with respect to the nuclear coordinates of dialkylethene  $(\mathbf{IV}_n)$ with  $n = 40, 50, 75, 100$  calculated at PBE0/def2-SVP level of theory on GPUs. The slope of the linear fit is equal to the effective scaling behavior of the routine.

## 3 Illustrative Examples

#### 3.1 Validation

Table 1: Mean absolute errors (MAE; in atomic units) of excited state energies  $(\omega_I)$ , gradients  $(\omega_I^{\xi}$  $\binom{\xi}{I}$ , and non-adiabatic coupling vectors  $(\tau_I^{\xi})$  $\sum_{I \to J}^{k}$  of the four rotary molecular machines (C,  $\mathbf{N}$ ,  $\mathbf{S}$ , and  $\mathbf{O}$ ) calculated at RPA and TDA ( $\omega$ B97/def2-SVP) level of theory on GPUs, comparing two different thresholds for preLink ( $\vartheta_{\rm pre}$ ), the preLink gradient ( $\vartheta_{\rm pre}^{\nabla}$ ), and the TDDFT convergence  $(\vartheta_{\text{TDDFT}})$ .

י $\sim$ 0.1.0.000						
$\vartheta_{\underline{\text{pre}}}$	$10^{-3}$ vs $10^{-4}$					
$\eta$ pre	$10^{-10}$ vs $10^{-11}$					
$\vartheta$ TDDFT	$10^{-5}$ vs $10^{-6}$					
$\mathbf C$						
	<b>RPA</b>	<b>TDA</b>				
$MAE(\omega_1)$	$3.19 \times 10^{-4}$	$6.55 \times 10^{-7}$				
$MAE(\omega_1^{\xi})$	$4.03 \times 10^{-5}$	$4.34 \times 10^{-6}$				
$MAE(\tau_{0\rightarrow 1}^{\xi})$	$9.89 \times 10^{-4}$	$2.71 \times 10^{-5}$				
$\overline{\mathbf{N}}$						
	<b>RPA</b>	<b>TDA</b>				
$MAE(\omega_1)$	$1.98 \times 10^{-4}$	$2.90 \times 10^{-6}$				
$MAE(\omega_1^{\xi})$	$2.42 \times 10^{-5}$	$1.22 \times 10^{-5}$				
$MAE(\tau_{0\rightarrow 1}^{\xi})$	$9.43 \times 10^{-4}$	$6.34 \times 10^{-5}$				
S						
	<b>RPA</b>	<b>TDA</b>				
$MAE(\omega_1)$	$3.91 \times 10^{-4}$	$1.09 \times 10^{-6}$				
$MAE(\omega_1^{\xi})$	$2.84 \times 10^{-5}$	$8.33 \times 10^{-6}$				
$MAE(\tau_{0\rightarrow 1}^{\xi})$	$7.85 \times 10^{-4}$	$4.32 \times 10^{-5}$				
O						
	<b>RPA</b>	<b>TDA</b>				
$MAE(\omega_1)$	$1.94 \times 10^{-4}$	$9.07 \times 10^{-7}$				
$MAE(\omega_1^{\xi})$	$2.82 \times 10^{-5}$	$7.30 \times 10^{-6}$				
$MAE(\tau_{0\rightarrow 1}^{\xi})$	$6.35\ \times 10^{-4}$	$3.59 \times 10^{-5}$				

Screening Thresholds and Convergence Criteria

Table 2: Mean absolute errors (MAE; in atomic units) of excitation energies  $(\omega_I)$ , gradients  $(\omega_I^\xi$  $\binom{\xi}{I}$ , and non-adiabatic coupling vectors  $(\tau_I^{\xi})$  $\sum_{I \to J}^{S}$  of the four rotary molecular machines (C,  $\mathbf{N}, \mathbf{S}, \text{ and } \mathbf{O}$  calculated at RPA and TDA ( $\omega$ B97/def2-SVP) level of theory on GPUs, comparing RPA and TDA.

RPA vs. TDA						
		N				
	MAE $(\omega_1)$ 6.49 $\times 10^{-3}$ 8.17 $\times 10^{-3}$ 5.41 $\times 10^{-3}$ 6.70 $\times 10^{-3}$					
	MAE( $\omega_1^{\xi}$ ) 6.43 × 10 <sup>-4</sup> 5.86 × 10 <sup>-4</sup> 5.28 × 10 <sup>-4</sup> 4.60 × 10 <sup>-4</sup>					
	$MAE(\tau_{0\rightarrow1}^{\xi})$ 8.73 $\times 10^{-3}$ 8.14 $\times 10^{-3}$ 8.81 $\times 10^{-3}$ 8.86 $\times 10^{-3}$					



Figure S4: Excitation energies of the four rotary molecular machines  $(C, N, S, \text{ and } O)$ calculated at (left) RPA and (right) TDA  $(\omega B97/\text{def2-SVP})$  level of theory.



Figure S5: Comparison of relative  $S_0$  and  $S_1$  energies of the four rotary molecular machines  $(C, N, S, \text{ and } O)$  calculated at TDA  $(\omega B97/\text{def2-SVP})$  and  $CASSCF(2,2)/\text{def2-SVP}$  level of theory. The two geometries were obtained from geometry optimizations of the ground state (*opt*) and the S<sub>1</sub> state (close to the conical intersection,  $cx$ ) at TDA ( $\omega$ B97/def2-SVP) level of theory. CASSCF calculations were perfomed with ORCA v4.0 (Neese, F. The ORCA program system, Wiley Interdiscip. Rev.: Comput. Mol. Sci. 2012, 2, 73-78). TDA describes both states at  $cx$  and trends between the different molecular machines remarkably well.

## 3.2 Relaxed Difference Densities



Figure S6: Plots of the relaxed difference densities of the first excited state  $(\mathbf{P}_1)$  of  $(\text{up}) \mathbf{C}$ and (down) N calculated at (left) RPA and (right) TDA  $(\omega B97/\text{def2-SVP})$  level of theory.



Figure S7: Plots of the relaxed difference densities of the first excited state  $(\mathbf{P}_1)$  of  $(\text{up})$  S and (down) O calculated at (left) RPA and (right) TDA  $(\omega B97/\text{def2-SVP})$  level of theory.

## C  $E_0^{\xi} + \omega_1^{\xi}$ 1  $\tau_0^\xi$  $0\rightarrow 1$ RPA TDA RPA TDA N  $E_0^{\xi} + \omega_1^{\xi}$ 1  $\tau_0^\xi$  $0\rightarrow 1$ RPA TDA RPA TDA

## 3.3 Excited State Gradients and Non-adiabatic Coupling Vectors

Figure S8: Excited state gradients of the first excited state  $(E_0^{\xi} + \omega_1^{\xi})$  $_{1}^{\xi}$ ) and non-adiabatic coupling vectors between the ground and the first excited state  $(\tau_0^{\xi})$  $\binom{.5}{0}$  of (up) **C** and (down) N calculated at RPA and TDA  $(\omega B97/\text{def2-SVP})$  level of theory.



Figure S9: Excited state gradients of the first excited state  $(E_0^{\xi} + \omega_1^{\xi})$  $_{1}^{\xi}$ ) and non-adiabatic coupling vectors between the ground and the first excited state  $(\tilde{\tau}_0^{\xi})$  $\binom{.5}{0}$  of (up) **S** and (down) O calculated at RPA and TDA  $(\omega B97/\text{def2-SVP})$  level of theory.

## 3.4 Dihedrals



Figure S10: Dihedrals during the non-adiabatic molecular dynamics simulations of C, N,  $S$ , and  $O$ . Blue lines indicate simulations starting from a dihedral  $> 0$  and red a dihedral  $< 0$ . All rotors show no clear preference towards clockwise  $(< 0)$  or counterclockwise  $(> 0)$ rotations.