

Supplementary material for “Generating Bessel beams with broad depth-of-field by using phase-only acoustic holograms”

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ABSTRACT

In this document we present the supplementary material of the paper entitled “Generating Bessel beams with broad depth-of-field by using phase-only acoustic holograms”. We show the measurement of the symmetry of the beam for Fraxicon (or flat axicon) and phase-only holograms obtained theoretically, by simulation, and experimentally measured for both, $M = 0$ and $M = 1$.

Symmetry of the beam

In this section we present the supplementary material of the paper entitled “Generating Bessel beams with broad depth-of-field by using phase-only acoustic holograms”. In particular, we show the beam profiles as a function of the polar angle in order to quantify the variations of the amplitude of the side lobes. Figure 1 show the radial variations of a Fraxicon (or flat axicon) and the phase-only hologram obtained theoretically, by simulation, and experimentally measured for both, $M = 0$ and $M = 1$.

For the zero-th order ($M = 0$) Bessel beam (Figs.1(a1-a4)), the Fraxicon produces a very symmetric field, note the lenses also present radial symmetry. When the field is measured at $z = 35$ mm, the experiments maintain such a symmetry indicating the good alignment of the measurement system with respect with the lens and a symmetric vibration pattern produced by the piezoelectric source.

When the zero-th order ($M = 0$) Bessel beam of wide depth-of-field is generated using the hologram (Figs.1(b1-b4)), the side-lobes present strong variations as a function of the polar angle. Since holograms are not symmetrical, neither is the field. However, one can notice that the main lobe maintain the symmetry as it can be observed in Fig.1(b4): the elongated focal spot is circular.

The first-order Bessel beam ($M = 1$) generated using the spiral Fraxicon also presents good cylindrical symmetry. Only weak variations of the beam profile were measured between different angles. Note this apply to the absolute value of the field as the complex field of the vortex beam presents chiral symmetry.

Finally, when the first order Bessel beam is generated using the hologram, the field presents more radial variations. In this case, the variations of the peak pressure amplitude of the side lobes was $0.06 \times p_{\max}$ for the theory, $0.07 \times p_{\max}$ for the simulations and $0.16 \times p_{\max}$ for the experiments. Note in this case since holograms are not symmetrical, neither is the field. However, note that the circular shape of the vortex is maintained, as the distance between peaks remains constant.

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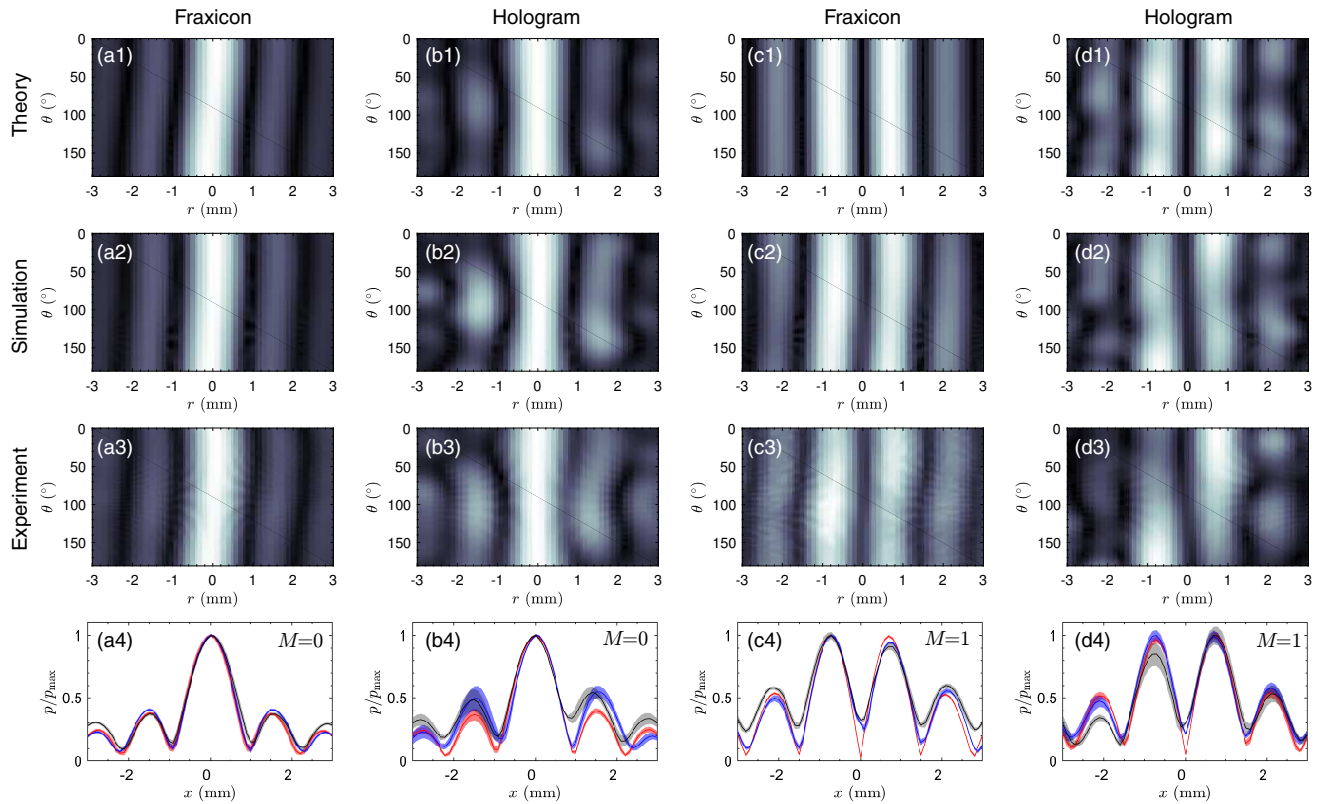


Figure 1. (a-d) Beam profiles as a function of the polar angle measured at $z = 35$ mm. (a1-a3) Theoretical, simulated and experimental beam profile for the fraxicon ($M = 0$). (a4) Radial beam profile calculated as the mean over the azimuthal angle for (red) theory, (blue) simulation and (black) experiment for the fraxicon ($M = 0$). Filled areas show the corresponding standard deviation. (b1-b3) Theoretical, simulated and experimental beam profile for the phase only hologram ($M = 0$). (b4) Radial beam profile calculated as the mean over the azimuthal angle for (red) theory, (blue) simulation and (black) experiment for the phase only hologram ($M = 0$). (c1-c3) Theoretical, simulated and experimental beam profile for the spiral fraxicon ($M = 1$). (c4) Radial beam profile calculated as the mean over the azimuthal angle for (red) theory, (blue) simulation and (black) experiment for the phase only hologram ($M = 1$). (d1-d3) Theoretical, simulated and experimental beam profile for the phase only hologram (vortex, $M = 1$). (d4) Radial beam profile calculated as the mean over the azimuthal angle for (red) theory, (blue) simulation and (black) experiment for the phase-only hologram (vortex, $M = 1$).

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Author contributions statement

NJ and FC conceived the idea and conducted the theoretical modelling; SJG, NJ, and FC performed the numerical simulations and experiments; and NJ, SJG, JMB, FC wrote the manuscript. All authors reviewed the manuscript.

Additional information

Competing interests The authors declare no competing interests.