

**Supplementary Information for**  
**Elastic avalanches reveal marginal behaviour in**  
**amorphous solids**

**Baoshuang Shang, Pengfei Guan, Jean-Louis Barrat**

**Pengfei Guan & Jean-Louis Barrat.**

**E-mail: [pguan@csrc.ac.cn](mailto:pguan@csrc.ac.cn) & [jean-louis.barrat@univ-grenoble-alpes.fr](mailto:jean-louis.barrat@univ-grenoble-alpes.fr)**

**This PDF file includes:**

Supplementary text

Figs. S1 to S9

Tables S1 to S2

## Supporting Information Text

### Fitting parameters

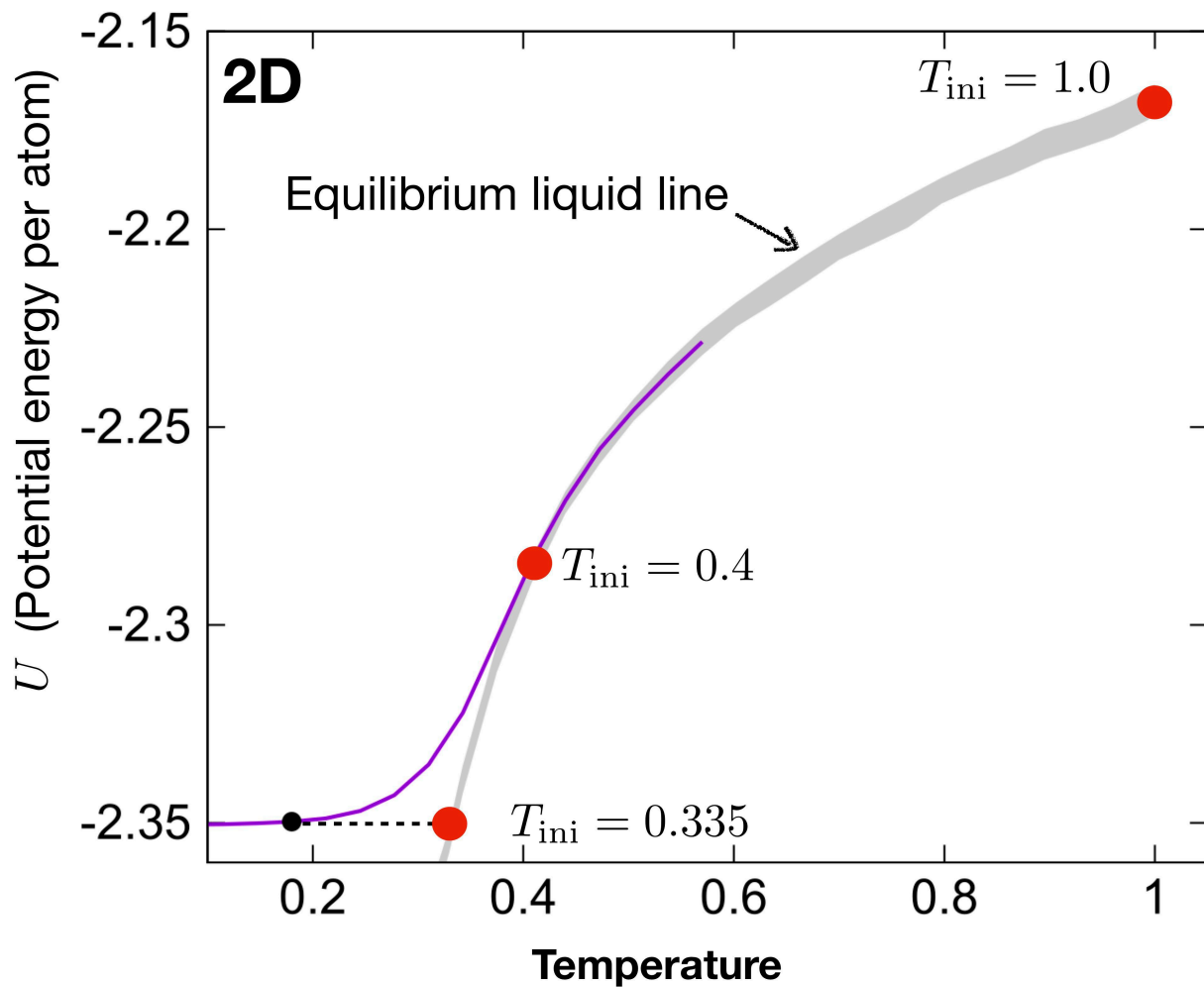
We have summarized all the fitting parameters of scaling collapse in the main text as follows:

**Table S1. Fit parameters for the 2D system, after scaling collapse , the master curve is  $\sim x^{-\tau} f(x/x_c)$ , where avalanche exponent  $\tau = 0.98 \pm 0.01$ .**

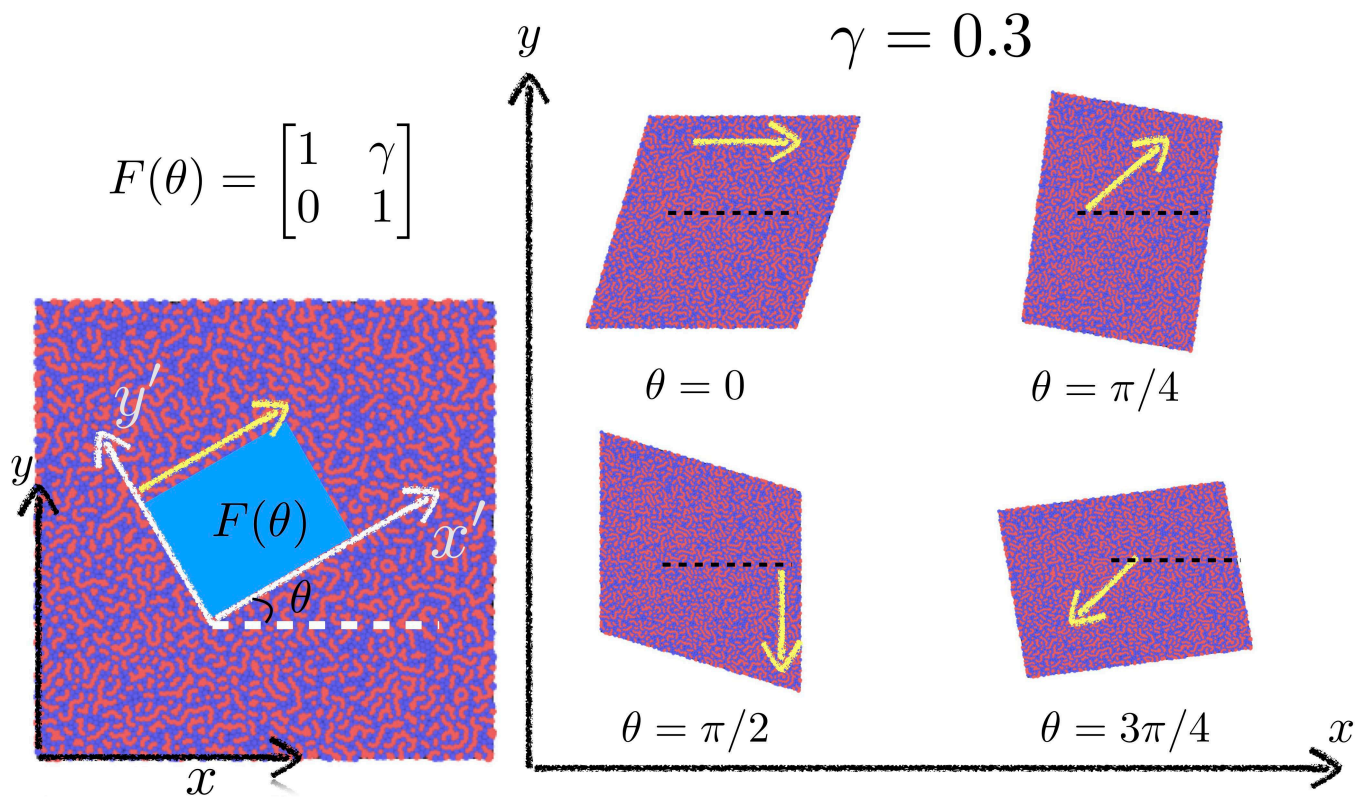
$T_{\text{ini}}$	$d_f/d$	$\alpha$	$\xi_1$	$\xi_1^2 \xi_2$
1.0	0.225(8)	0.19(1)	1.2(1)	0.196(5)
0.4	0.166(5)	0.134(5)	0.84(4)	0.093(5)
0.335	0.050(2)	0.040(8)	0.46(7)	0.013(1)

**Table S2. Fit parameters for the 3D system, after scaling collapse , the master curve is  $\sim x^{-\tau} f(x/x_c)$ , where avalanche exponent  $\tau = 1.01 \pm 0.01$ .**

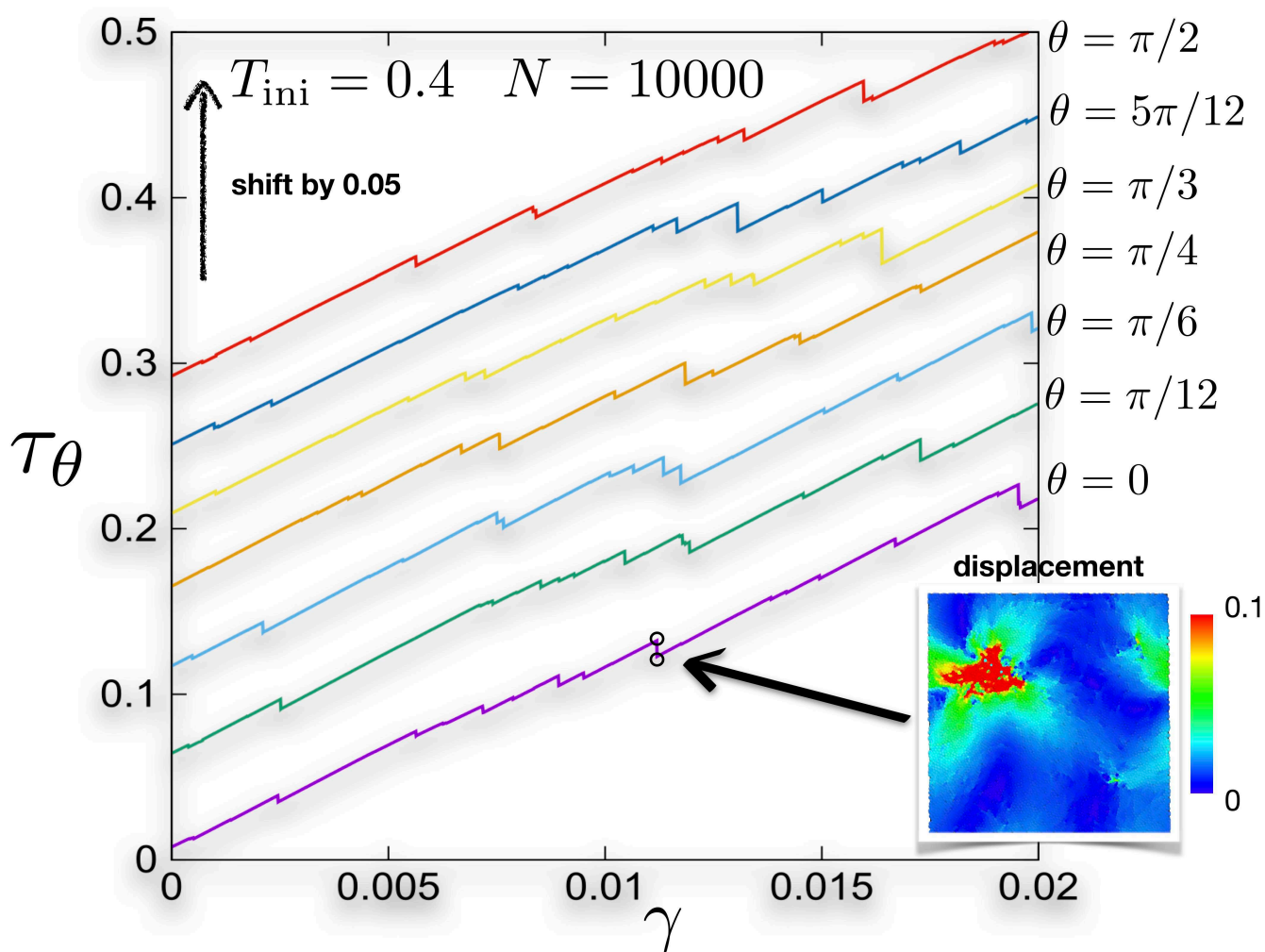
$T_{\text{ini}}$	$d_f/d$	$\alpha$	$\xi_1$	$\xi_1^2 \xi_2$
0.87	0.24(2)	0.172(8)	2.8(6)	0.23(1)
0.61	0.17(2)	0.152(2)	2.6(7)	0.131(4)
0.479	0.090(6)	0.09(1)	1.5(1)	0.040(2)



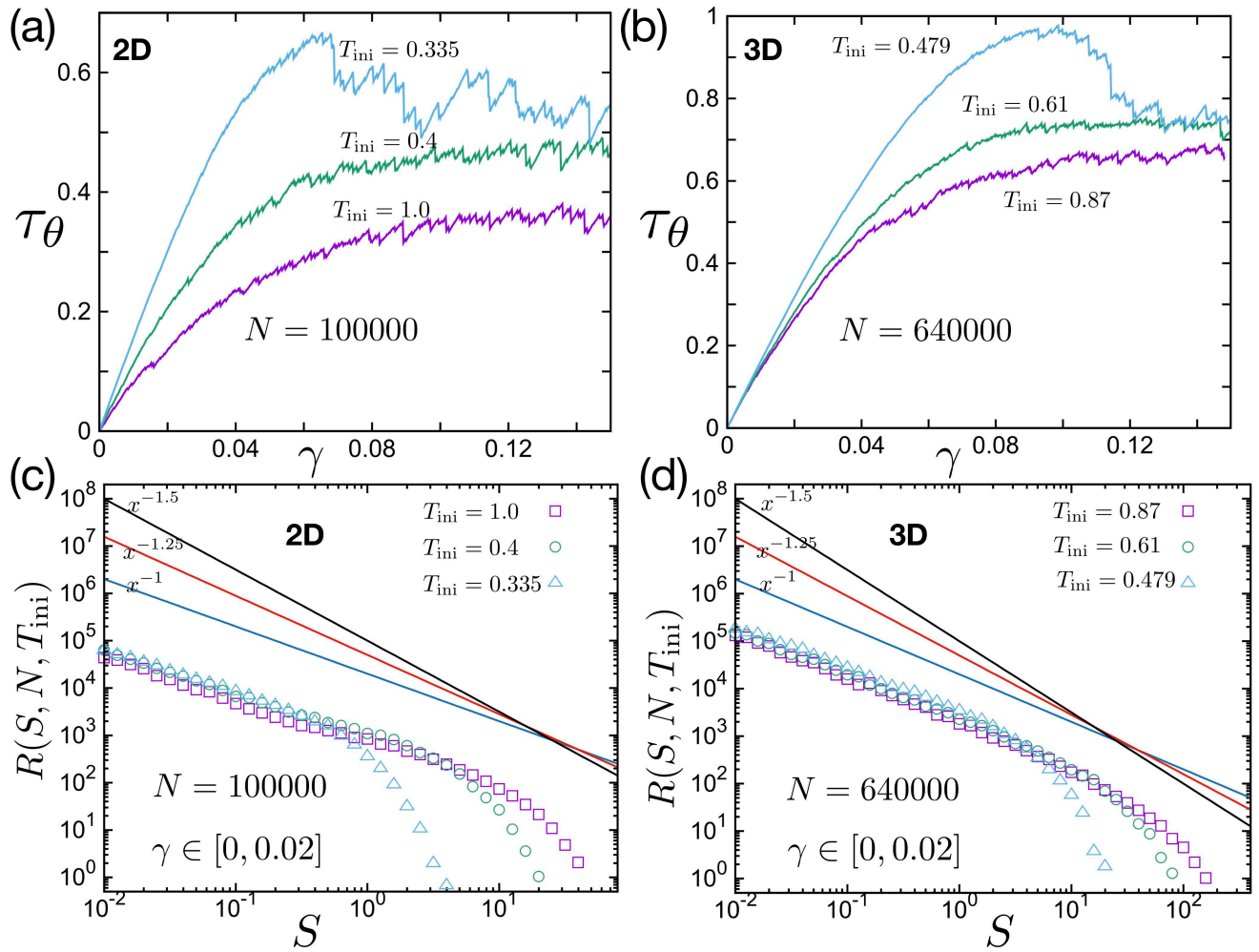
**Fig. S1.** Thermal history of the 2D sample in the temperature and inherent structure potential energy diagram. The grey line is the equilibrium liquid line, and the red solid point is representative thermal history used in the article, the purple line is the liquid to glass transition line at a given quench rate.



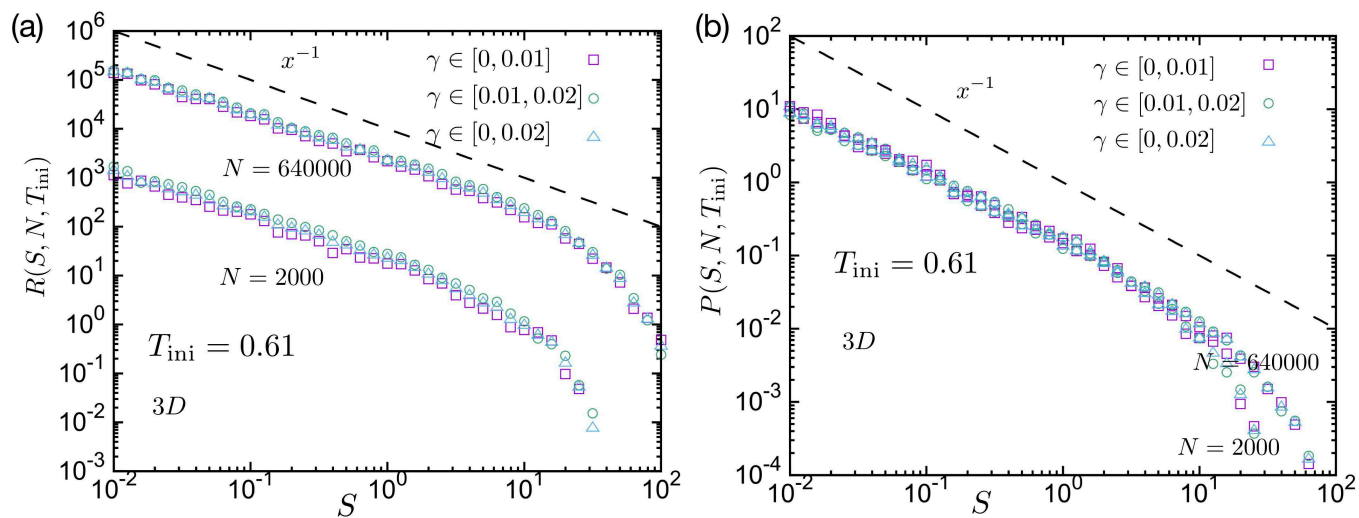
**Fig. S2.** Schematic diagram of the deformations applied in athermal quasistatic shear, left panel: the simple shear deformation gradient in direction  $\theta$ , right panel: typical sample deformation for four different directions at a given strain.



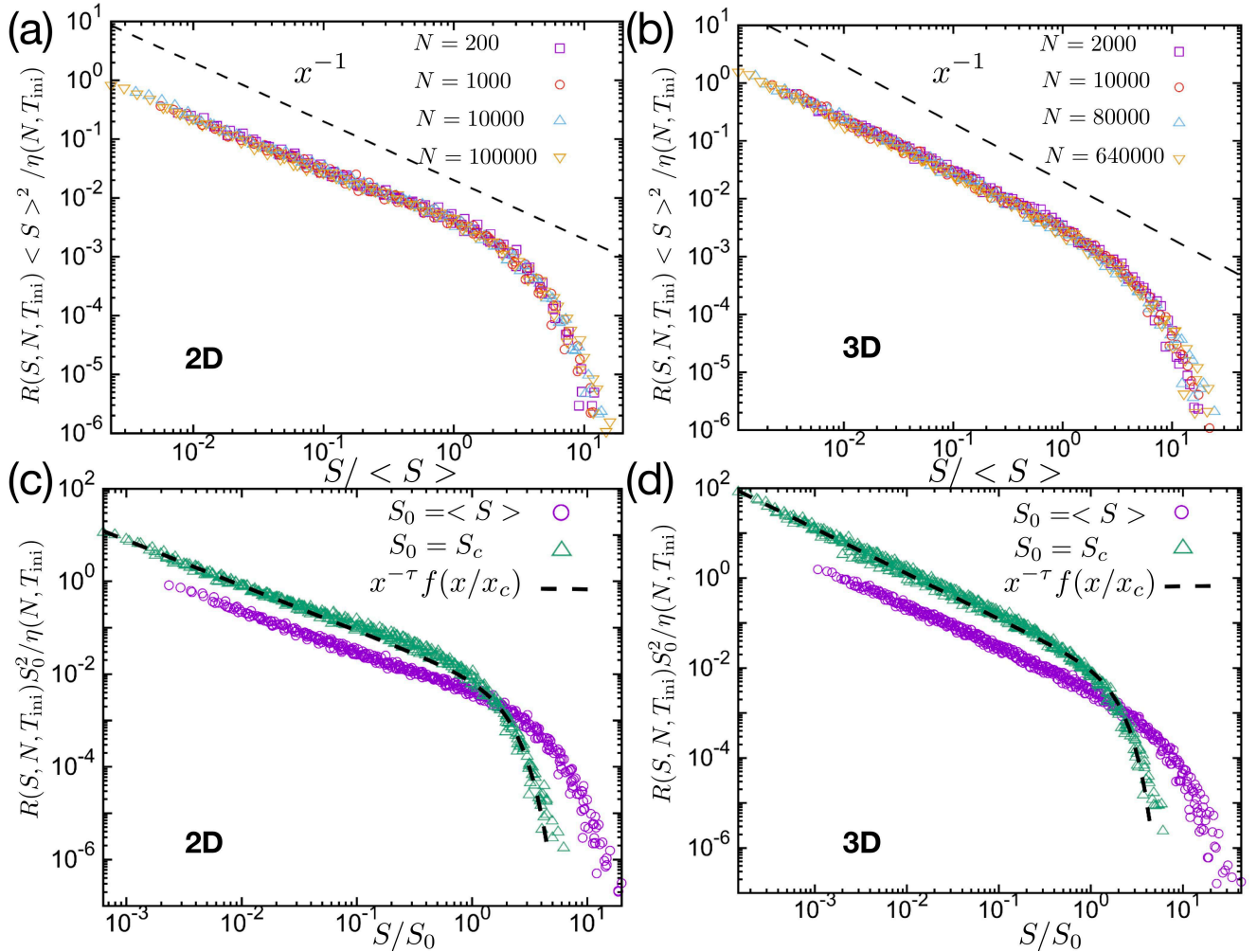
**Fig. S3.** Strain stress curves for different loading directions for  $N = 10000$ ,  $T_{ini} = 0.4$  in the 2D system, each curve from bottom to up is shifted by 0.05. The inset shows a color plot of the displacement field between two configurations separated by the stress drop displayed with hollow circles in the strain stress curve.



**Fig. S4.** Avalanche distribution in the elastic regime for different thermal history: (a),(b) typical stress strain curve of one sample for different thermal history in 2D and 3D system, respectively. (c),(d) The avalanche number distribution within  $\gamma \in [0, 0.02]$  for different thermal histories in 2D and 3D. The solid lines show three possible avalanche exponents.

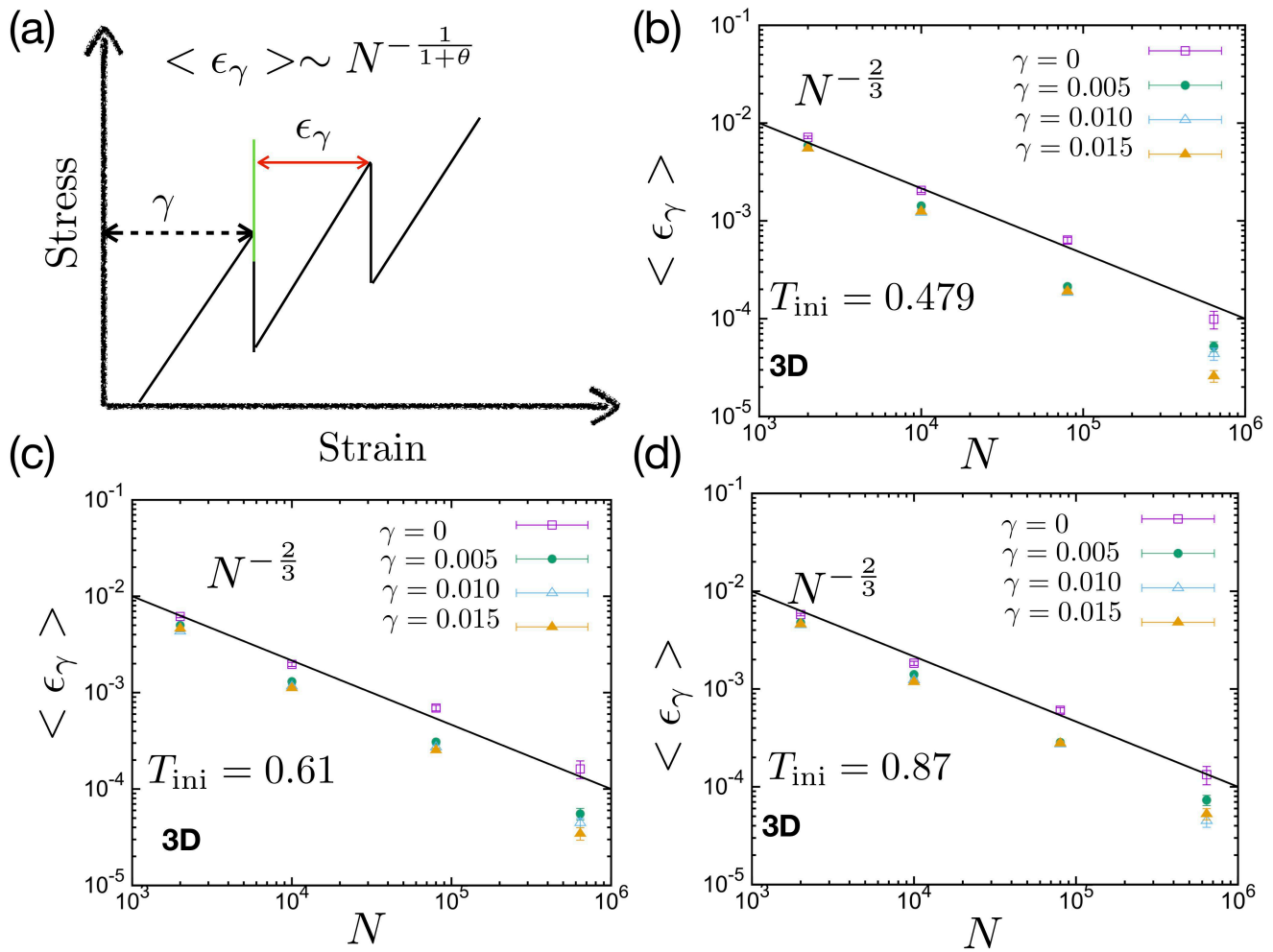


**Fig. S5.** (a) Avalanche number density for different strain intervals, for the  $N = 2000$  and  $N = 640000$  3D system. (b) Probability distribution of avalanche sizes for the same parameters as in (a), the dashed line has a slope  $-1$ .

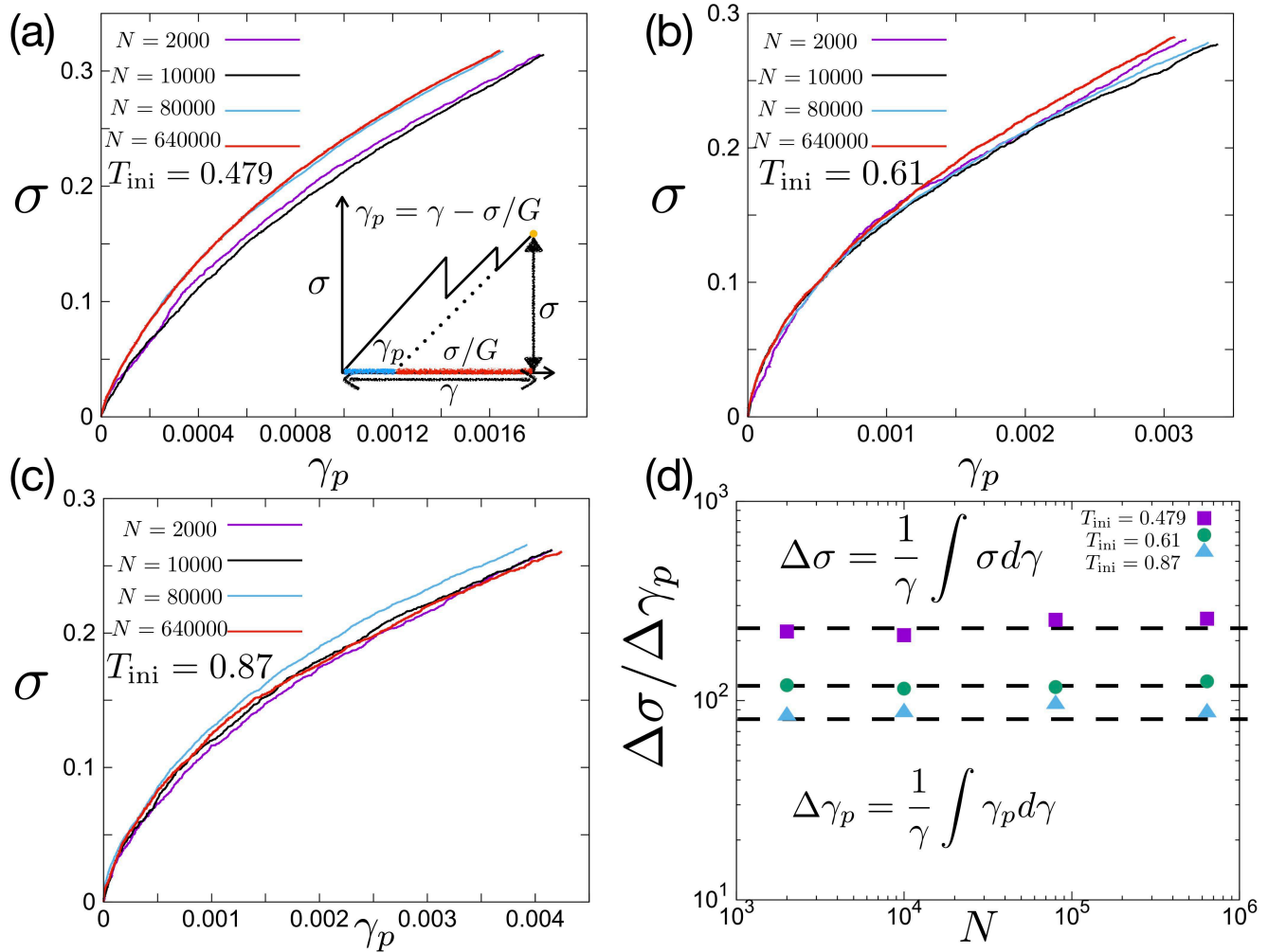


**Fig. S6.** (a),(b) Scaling collapse of the avalanche distribution using the average avalanche size in 2D and 3D systems, respectively. (c),(d) comparison between the scaling collapse obtained using average avalanche size  $\langle S \rangle$  or cutoff value  $S_c$ . The dashed line is a fit using the function  $Ax^{-\tau} f(x/x_c)$ , where  $f(x/x_c) = e^{-x^2/x_c^2}$ ,  $\tau = 0.98 \pm 0.01, 1.01 \pm 0.01$  for 2D,3D systems, respectively.

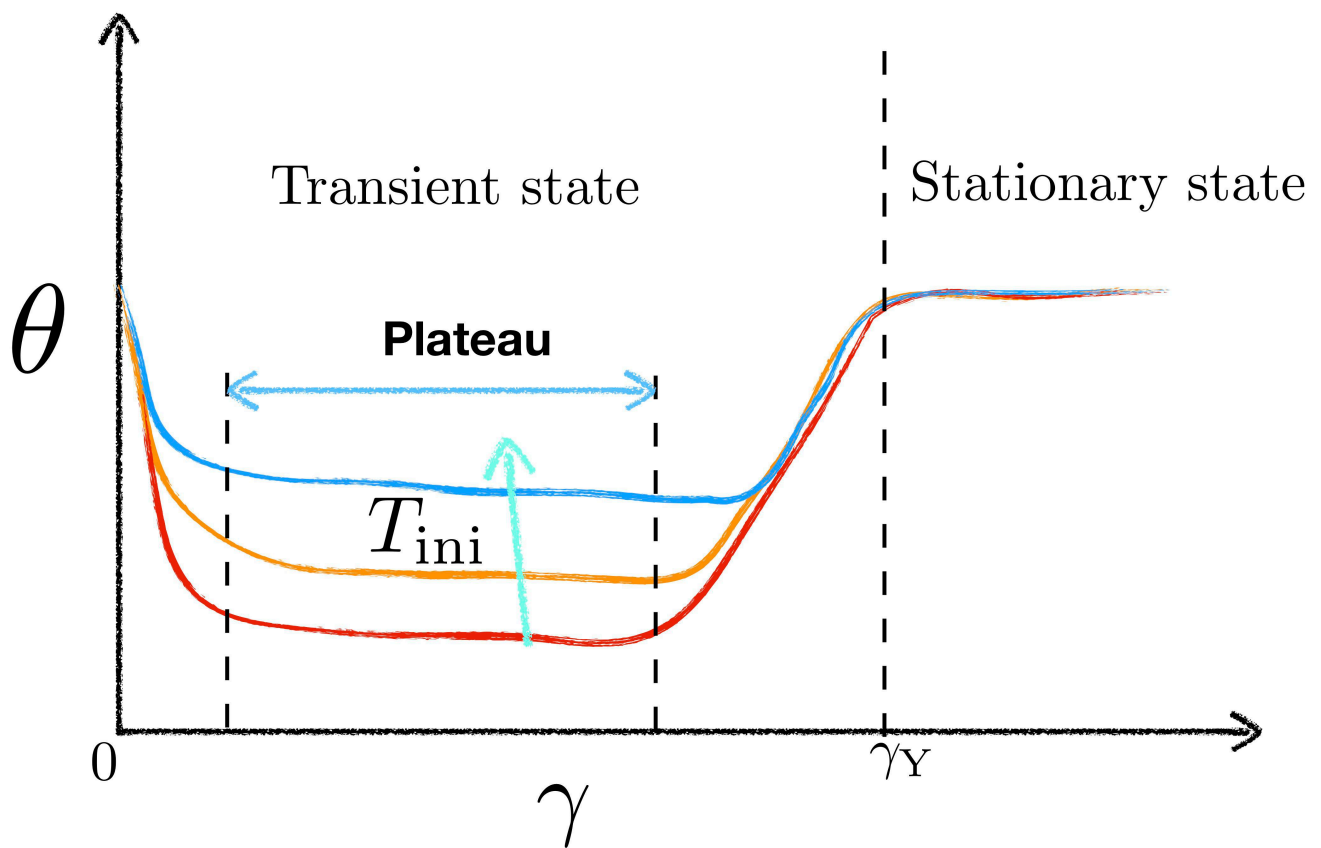




**Fig. S7. Strain at the first plastic event following a previous strain, for different previous strain intervals.** (a) Schematic representation of the strain at the first plastic event,  $\epsilon_\gamma$ , for a given previous strain  $\gamma$ . (b),(c),(d) Mean value of  $\epsilon_\gamma$  versus system size  $N$  for different previous strain intervals  $\gamma$  for  $T_{\text{ini}} = 0.479, 0.61, 0.87$ , respectively, the solid line is  $N^{-\frac{2}{3}}$ .



**Fig. S8.** (a),(b),(c) plastic strain versus stress for different system sizes and thermal histories. The inset in panel (a) shows how the plastic strain  $\gamma_p$  is obtained from the strain-stress curves,  $\gamma_p = \gamma - \sigma/G$ , where  $\gamma$  is the total shear strain,  $\sigma$  is the mean value of shear stress, and  $G$  is the elastic shear modulus. (d) the ratio of average stress and average plastic strain  $\Delta\sigma/\Delta\gamma_p$  versus system sizes for different thermal histories. The horizontal dashed line is the mean value of the ratio  $\Delta\sigma/\Delta\gamma_p$  for each thermal history. The strain interval is  $\gamma \in [0, 0.02]$ .



**Fig. S9.** Schematic representation of the non monotonic evolution of  $\theta$  with strain  $\gamma$  in the transient state and stationary state for different thermal histories.