

Load transfer mechanism and critical length of anchorage zone for anchor bolt  
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<b>Abstract:</b>	The length of anchorage zone of an anchor bolt affects the distribution of axial force and shear stress therein. Based on a shear–displacement model, the load distribution of anchor bolts in the elastic deformation stage was analysed. Moreover, the mechanical response of threaded steel anchor bolts with different anchorage lengths was explored through pull-out test and numerical simulation. The results showed that axial force and shear stress were negatively exponentially distributed within the anchorage zone of anchor bolts in which there were the maximum axial force and shear stress at the beginning of the anchorage zone. In the elastic deformation stage of the anchorage, the longer the anchorage length, the more uniformly the shear stress was distributed within the anchorage zone and the larger the ultimate shear stress; however, there was a critical anchorage length, which, when exceeded, the ultimate shear stress remained unchanged. The calculation formula for the critical anchorage length was deduced and a reasonable anchorage length determined. The research result provides an important theoretical basis for rapid design of support parameters for anchor bolts.
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# 1 **Load transfer mechanism and critical length of anchorage zone for anchor bolt**

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6  
7 **Abstract:** The length of anchorage zone of an anchor bolt affects the distribution of axial force and shear stress  
8 therein. Based on a shear–displacement model, the load distribution of anchor bolts in the elastic deformation  
9 stage was analysed. Moreover, the mechanical response of threaded steel anchor bolts with different anchorage  
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18 **Keywords:** anchorage zone, load transfer, pull-out test, critical length.

## 19 20 **1 Introduction**

21 As a key parameter affecting the design of bolt supports, the length of anchorage zone influences the  
22 anchoring force and support effect of anchor bolts, however, a theoretical basis for such a design remains absent,  
23 resulting in unreasonable anchorage lengths, thus leading to anchor support failure or extra cost[1,2]. Therefore, it  
24 is a challenge to guarantee that anchorage lengths satisfy design requirements while saving cost and therefore it is  
25 necessary to explore the load transfer mechanism and reasonable anchorage length of anchor bolts.

26 The load transfer mechanism of anchor bolts is a research hot-spot. The shear stress on anchor surface in the  
27 pull-out process can be divided into three parts: cohesion, mechanical self-locking force, and friction force[3].  
28 Many mechanical models have been proposed: the shear lag model for an anchoring system based on the  
29 condition of considering bonding conditions of different interfaces[4], the simple trilinear constitutive model that  
30 describe the shear slip of the bonding interface between the anchor cable and grouting body[5], the stick-slip  
31 relationship and the trilinear stick–slip model established through pull-out tests on anchor bolts[6,7], the

32 three-parameter and two-parameter combined-power models of the distribution of axial force within the  
33 anchorage zone[8], the hyperbolic function model of load transfer by using mathematical–mechanical methods[9].  
34 Zhu(2009) derived a function describing the distribution of frictional resistance on anchor bolts in an elastic  
35 homogeneous rock mass[10]. By applying displacement–shear stress theory and finite element analysis (FEA), the  
36 shear stress in the anchorage zone is distributed following a Gaussian function along the anchorage length.  
37 Through various *in situ* and laboratory tests[11], the distribution characteristics of axial force within the anchorage  
38 zone was obtained[12]. Despite the aforementioned research, no consensus has been reached as to the stress  
39 distribution in the anchorage zone.

40 As for research on anchorage length, the failure behaviours of bonded anchorage bodies under a fixed  
41 anchorage length was explored [13,14], the bearing capacity did not significantly increase when the anchorage  
42 length exceeded the critical anchorage length[15]. Huang(2018) proposed a method for calculating the critical  
43 anchorage length of anchor bolts and verified its feasibility through engineering case studies[16]. Based on the  
44 bonding effect, The anchorage length has a serious influence on the bearing capacity of anchor bolts and shear  
45 stress on interfaces under the effect of cyclic load[17-19]. The calculation formula for the critical anchorage  
46 length of anchor bolts can be deduced according to the principle of displacement compatibility between the  
47 anchorage body and surrounding rock[20-22]. Liu(2010) thought that the anchorage length has to exceed 20 times  
48 the diameter of the anchor bolt when applying full-thread GFRP anchor bolts in situ[23]. The aforementioned  
49 research achievements remain mostly hypothetical, and do not take the design requirements of actual parameters  
50 of anchor bolts into account.

51 In the present study, the mechanical properties and stress distribution characteristics of the anchorage zone  
52 under different anchorage lengths were explored to reveal the load transfer mechanism of the anchorage zone and  
53 propose a method for designing a reasonable anchorage length of anchor bolts.

54

## 55 **2 Analysis of mechanical properties of the anchorage zone**

56 An anchoring system comprises: anchor bolts, anchoring agent, surrounding rocks, and parts of the anchor  
57 bolts. An anchor bolt is divided into exposed, free, and anchorage zones (Fig 1) along its length. When the anchor  
58 bolt is subjected to pull-out effects, the axial force in the free zone is transferred to the anchorage zone due to  
59 elastic deformation therein. Based on bonding, friction, and mechanical meshing between the anchor bolt and  
60 anchoring agent, the circular binding body formed by the anchoring agent, and the effect of the borehole wall,



61 load is transferred to the surrounding rock. The anchoring force refers to the binding force between the anchorage  
62 zone of anchor bolts and a rock mass, that is, the constraint force on the anchor bolt from the surrounding rock,  
63 which is frequently considered as an important index with which to measure anchor integrity.

64 Based on the force transfer process of anchoring system, it can be seen that there are three mechanical  
65 interfaces in the anchoring system. When analysing the mechanical properties of the anchorage zone in the elastic  
66 stage, the two interfaces (including anchor bolt–anchoring agent and anchoring agent–borehole wall interfaces)  
67 were explored. When applying pull-out force to an anchor bolt, the shear stress on the anchorage zone depends on  
68 the coupling mechanism between interfaces[24,25]. For grouted anchor bolts, relative displacement occurs  
69 between the anchor bolts and surrounding slurry, thus failing in slip on the anchor bolt–anchoring agent interface.  
70 Then, the shear stress on the interface is lower than the ultimate shear strength of the interface. For a resin anchor  
71 bolt, the anchor bolt is deformed with its anchoring agent, generally failing in slip on the anchoring agent–  
72 borehole wall interface. In this case, the shear stress on the interface is equivalent to the ultimate shear strength.  
73 The latter was explored in the present study.

74 According to different deformation forms of anchoring agent–borehole wall interface, the pull-out process of  
75 anchor bolts into three stages was simplified[5,26], as shown in Fig 2.

76 In Stage I (elastic deformation stage), the shear stress is proportional to the shear displacement of the  
77 interface which is intact. In this case,  $0 \leq \mu \leq \mu_1$  and the relationship between shear stress  $\tau$  and displacement  $\mu$  is  
78 expressed as follows:

$$79 \quad \tau = \frac{\tau_1}{\mu_1} \mu \quad (1)$$

80 where,  $\tau_1$  and  $\mu_1$  refer to the ultimate bonding strength of anchorage body and shear displacement at the ultimate  
81 bonding strength of anchorage zone, respectively.

82 In Stage II (interface softening and damage stage), the interface is partly damaged and therefore shear stress  
83 linearly declines with shear displacement. In this context,  $\mu_1 \leq \mu \leq \mu_2$  and the shear stress can be calculated as  
84 follows:

$$85 \quad \tau = \frac{\tau_1 - \tau_2}{\mu_1 - \mu_2} \mu + \frac{\tau_2 \mu_1 - \tau_1 \mu_2}{\mu_1 - \mu_2} \quad (2)$$

86 where,  $\tau_2$  and  $\mu_2$  are the residual bonding strength of anchorage zone and the minimum shear displacement under  
87 the residual bonding strength of the anchorage zone, respectively.

88 In Stage III (residual strength stage), the interface was completely damaged; in this context,  $\mu \geq \mu_2$  and the

89 shear stress is expressed as follows:

$$90 \quad \tau = \tau_2 \quad (3)$$

91 By modifying the micro-element model[27,28], the distribution equation for axial force in the anchorage  
92 zone is expressed as follows:

$$93 \quad P(x) = \frac{e^{\beta x} - e^{\beta(2L_b - x)}}{(1 - e^{2\beta L_b})} P \quad (4)$$

94 The equation for shear stress distribution of anchoring agent–borehole wall interface is as follows:

$$95 \quad \tau(x) = \frac{e^{\beta x} + e^{\beta(2L_b - x)}}{\pi D(e^{2\beta L_b} - 1)} \beta P \quad (5)$$

96 where,  $D$ ,  $P$ , and  $\beta$  separately denote the diameter of the borehole, pull-out force of an anchor bolt, and a material  
97 parameter given by:

$$98 \quad \beta^2 = \frac{4\tau_1}{\mu_1 D E_a} \quad (6)$$

99 where,  $E_a$  is the elastic modulus of the anchorage zone.

100 According to Equations 4 and 5, the distribution curves of axial force and shear stress in the anchorage zone  
101 are drawn, as shown in Fig 3.

102 The axial force and shear stress of anchorage body monotonically decreased from the beginning to the end of  
103 the anchorage zone while the rate of change thereof gradually declined. At the beginning ( $x = 0$ ) of the anchorage  
104 zone, the axial force and shear stress on the anchorage body were at a maximum and the axial force was  
105 equivalent to that in the free zone of an anchor bolt. On condition of having sufficient pull-out force, relative  
106 displacement and damage first appeared at the beginning of the anchorage zone. Afterwards, damage gradually  
107 extended to the end of the anchorage zone. At the end ( $x = L_b$ ) of the anchorage zone, the axial force was zero  
108 while there was still a residual shear stress present.

109

### 110 **3 The influence of anchorage length on the stress distribution in the anchorage zone**

111 Bolt support is complex and concealed from observers, so it is hard to measure the deformation and stress on  
112 the anchor bolts in field. It is necessary to verify the result obtained through theoretical analysis by conducting  
113 laboratory testing to analyse the load transfer characteristics of an anchoring system.

#### 114 **3.1 Test materials and platform**

115 In the test, the left-handed threaded steel anchor bolts were applied and the thick-walled steel tube and resin

116 cartridge were separately taken as the anchoring matrix and binding material (Fig 4). Considering the binding  
 117 effect of this resin anchoring agent, a seamless steel tube with the inner diameter of 30 mm was used, in which  
 118 threads were processed. The parameters of test materials are shown in Table 1.

119 **Table 1. Parameters of mechanical properties of the test materials.**

Anchor bolt	Types of anchor bolts	Diameter/ mm	Length/ mm	Tensile strength/ MPa	Yield strength/ MPa	Breaking force / kN
	Threaded steel	20	2000	570	400	218.7
Anchoring agent	Type	Characteristic	Length/ mm	Diameter/ mm	Gelation time/ s	Waiting time for installation / s
	Z2350	Intermediate speed	500	23	91~180	480

120 The pull-out test was conducted by applying an LW-1000 horizontal tensile test machine (Fig 4). Before the  
 121 test, the back collet was fixed by using a latch and the end of the anchor bolt with threads was placed into the back  
 122 collet and fixed through pallet nuts. Moreover, the anchor end (seamless steel tube) was fixed using a front collet.  
 123 During the test, the front collet was driven through a piston and a pull rod to move away from the back collet to  
 124 simulate a pull-out force on the anchor bolt. A sensor was used to collect and transfer data (in real time) to a  
 125 computer.

### 126 3.2 Test scheme

127 Strain gauges were distributed in the anchorage zone at 100 mm intervals to measure the stress and strain on  
 128 the anchorage body under the pull-out effect and analyse the change in stress in the anchorage zone. TS3890 static  
 129 resistance strain gauges were used to measure the strain (Fig 5).

130 During the test, the four-level loads (25, 50, 75, and 100 kN) were separately applied to the anchorage zones  
 131 with the anchorage lengths of 500, 1000, and 1500 mm. The load was maintained for 3 s and the mechanical  
 132 response of the anchorage body under different anchorage lengths and pull-out loads analysed.

### 133 3.3 The influence of anchorage length on stress distribution in the anchorage zone

#### 134 3.3.1 Shear stress

135 Based on measured parameters of anchor bolts for mining service and surrounding rocks, the elastic moduli  
 136 of the anchorage body and resin cartridge, diameter of anchor bolt, diameter of borehole, and Poisson's ratio of  
 137 surrounding rocks were 200 GPa, 3 GPa, 20 mm, 30 mm, and 0.24, respectively. On this basis, the curves for  
 138 comparing changes of shear stress are shown in Fig 6.

139 Fig 6 shows the shear stress distributions on interfaces in the anchorage zone for anchorage lengths of 0.5, 1,  
 140 1.5, and 2 m when the pull-out force was 50 kN. It can be seen from the Fig 6 that under the same pull-out force  
 141 and different anchorage lengths, the shear stress on the interfaces did not change linearly but reached a maximum

142 at the beginning of the anchorage zone and gradually reduced to zero with increasing distance from the beginning.  
 143 The interface was mainly stressed close to the end of the free zone. The shorter the anchorage length, the more  
 144 uniformly the shear stress was distributed along the anchorage zone and the higher the maximum shear stress on  
 145 the interfaces. With increasing anchorage zone length, the shear stress on the interfaces decreased and was  
 146 gradually transferred to the section near the end of the anchorage zone. At the end nearest the applied load (near  
 147 end, hereinafter), debonding occurred and the shear stress was gradually transformed into a frictional resistance.  
 148 In this case, the shear stress on the anchorage body was low at a certain distance from the near end. When the  
 149 anchorage length reached a certain level, the distribution curves of shear stress on interfaces gradually coincided,  
 150 implying that further increasing the anchorage length had little significant effect on the maximum shear stress.

151 **A FLAC3D numerical model was established.** During simulation, the anchorage interface in a rock mass was  
 152 simulated by applying interface elements while contact elements were used to simulate the contact interface of  
 153 media effecting force transfer. The interface elements were used for simulation based on the Mohr-Coulomb  
 154 model. **The contact constitutive model for elements was adjusted through parameter setting** to simulate the true  
 155 interface, in which anchor bolt was simulated by using an isotropic elasticity model.

156 **Table 2. Mechanical parameters of materials.**

Performance parameters	Tensile strength /MPa	Yield strength/MPa	Shear modulus/GPa	Bulk modulus/GPa	Cohesion /MPa	Internal friction angle /°
Anchoring agent	15	-	-	-	-	-
Anchor bolt	570	400	-	-	12	32
<b>Surrounding rocks</b>	<b>2.1</b>	<b>0.96</b>	<b>3.3</b>	<b>5.1</b>	<b>4.6</b>	<b>38</b>

157 The model measures 1.0 m × 1.0 m × 1.2 m (length × width × height) and the total length of anchor bolt was  
 158 1.2 m, including an anchorage zone and a free zone of 1.0 m and 0.2 m long, respectively. The anchor bolt, with a  
 159 diameter of 20 mm, was aligned in the centre of the model, with a thickness of anchoring agent of 5 mm simulated.

160 **Fig 7** shows stress distributions in the anchoring agent at anchorage lengths of 0.5 and 1.0 m.

161 Shear stress was mainly distributed within a small zone in the near end and shear stress was exponentially  
 162 distributed and gradually declined from the near end to the far end. The longer the anchorage, the wider the  
 163 distribution of shear stress and the lower the corresponding shear stress; moreover, the longer the anchorage, the  
 164 nearer the shear stress was to zero in the anchorage zone (**it was even negative in places**). The stress distribution  
 165 on the anchorage body in the numerical model shows similarities with analytical solutions based on the shear-slip  
 166 model. In engineering practice, it is necessary to reinforce the vicinity of the interface as much as possible to  
 167 guarantee the strength of surrounding rocks near the interface and also ensure the integrity of anchorage in the  
 168 initial segment.

### 169 3.3.2 Analysis of axial stress

170 The axial stress is given by:

171

$$\sigma_i = \varepsilon_i E_s \quad (7)$$

172 where,  $\sigma_i$  and  $\varepsilon_i$  denote the axial force and strain at point  $i$ , respectively.

173 The axial force at the borehole mouth was equivalent to that in the free zone. With a resin anchoring agent,  
174 the axial force distribution varied and was different from the equivalent distribution in the free zone. The axial  
175 stress gradually decreased from the outer end to the tailing end of the anchor because the cohesion at the near end  
176 of the anchor bolt was gradually overcome with increasing pull-out load and the interface at the tailing end was  
177 constantly driven to resist the pull-out load. Additionally, the axial stress of anchor bolt correspondingly increased.

178 The axial force on the anchor bolt within the anchorage zone also increased with the external load applied to the  
179 anchor bolt.

180 As shown in Fig 8, when applying a pull-out force of 50 kN, the axial force varied quasi-linearly when the  
181 anchorage length was 0.5 m. With increasing anchorage length, the axial force of anchor bolts became less  
182 uniform. When the anchorage length was 1500 mm, the axial force was mainly distributed in the vicinity of the  
183 borehole mouth and decreased with distance therefrom. At a certain anchorage length, the axial force tended to  
184 zero and the peak axial force was unaffected; however, due to the increase in anchorage length, the zone over  
185 which the axial force was distributed expanded and therefore the anchor bolt further from the anchorage interface  
186 was subjected to a small axial force. That is, it exhibited sufficient bearing capacity and can thus bear more load.  
187 The result obtained through numerical simulation was consistent with that obtained by analytic calculation.

### 188 3.4 The influence of pull-out force on the stress distribution in the anchorage zone

#### 189 3.4.1 Distribution of axial stress under different pull-out forces

190 When the anchorage length was 1.0 m, the changes in axial stress of anchorage zone under three-level  
191 pull-out forces (25, 50, and 75 kN) were simulated. In Fig 9, the axial force is seen to be non-linearly distributed  
192 along the anchor. In the elastic stage, anchor bolts showed the same trend of stress distribution with increasing  
193 load, moreover, stress changes were mainly found at the beginning of the anchorage zone where the ultimate  
194 pull-out force was first mobilised. On this basis, it can be inferred that the anchorage body of an anchor bolt was  
195 first damaged at the beginning of its anchorage zone.

#### 196 3.4.2 Distribution of shear stress under different pull-out forces

197 Under low load, the interface between the anchoring agent and the anchor bolt at the borehole mouth was  
198 subjected to elastic deformation. In this case, the anchorage body was undamaged and shear stress within the

199 anchorage zone gradually reduced and was uniformly distributed. With increasing load, the shear stress rapidly  
 200 rose to its peak within a short distance from the borehole mouth: this implied that shear failure started to occur at  
 201 the beginning of the anchorage zone and the failure gradually extended to the deeper anchorage interface with  
 202 increasing load. As the maximum shear stress remained unchanged, the locus of the peak shear stress shifted to the  
 203 deeper anchorage zone. With a large anchorage length, there was a wider response range to external load within  
 204 the anchorage zone, so the anchorage body can bear a larger load, thus improving the bearing capacity of the  
 205 anchorage zone. By analysing Fig 10, it can be found that, within the ultimate bearing range, the larger the  
 206 pull-out force, the less uniform the stress distribution; the longer the anchorage, the more centralised the shear  
 207 stress on the interface at the beginning of the anchorage zone.

208

## 209 **4 Determination of reasonable anchorage length**

### 210 4.1 Determination of critical anchorage length

211 It can be seen from Fig 6(d) and Fig 8(d) that there was a critical length of anchorage zone under the effect of  
 212 pull-out force, beyond which the ultimate bearing capacity of the anchor bolts did not increase. When the external  
 213 load reached a certain level, the anchorage layer changed from one undergoing elastic deformation to  
 214 elasto-plastic deformation and the shear stress on the anchorage interface did not continue to increase. To  
 215 guarantee anchorage body function, the maximum shear stress on the anchorage zone cannot exceed the ultimate  
 216 shear strength of the anchorage body–rock interface, which was taken as the main controlling condition for  
 217 determining the anchorage length. In this context, the resistance at the beginning of the anchorage zone was  
 218 equivalent to the ultimate shear stress  $[\tau]$  on the interface. By simultaneously using Equation 4, the ultimate  
 219 pull-out force of the anchorage zone can be obtained thus:

220

$$P_{\max} = \frac{\pi D[\tau](e^{2\beta L_b} - 1)}{\beta(1 + e^{2\beta L_b})} \quad (8)$$

221 Owing to  $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ , assuming  $x = \beta L_b$ , the following result can be attained:

222

$$P_{\max} = \frac{\pi D[\tau]}{\beta} \tanh(\beta L_b) \quad (9)$$

223 The ultimate bearing capacity of anchoring system increased with increasing anchorage length and shear  
 224 capacity of the anchorage interface. With the constant growth of anchorage length, the bearing capacity of the  
 225 anchoring system increased, then stabilised, as shown in Fig 11.

226 When  $\beta L_b$  was infinite,  $\tanh(\beta L_b)$  tended to unity; however, in practical engineering, it not only needs to be

227 technically satisfactory, but also cost-effective. According to the peak, and incremental, axial force, the  
 228 eigenvalues of the system can be attained (Table 3).

229 **Table 3. A comparison between the peak axial force and  $\beta L_b$  eigenvalues.**

$P_{\max}$	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	0.995
$\beta L_b$	1.48	1.53	1.59	1.66	1.74	1.84	1.95	2.1	2.3	2.65	3
$P_{\max}$ increment / $\times 10^{-5}$	189	173	155	136	117	97	79	59	40	20	10

230 According to the corresponding relationship between  $P_{\max}$  and  $\beta L_b$  in Table 3, it can be seen that the  
 231 increment of  $\beta L_b$  increased with  $P_{\max}$ . This meant that, after reaching a certain critical value, the anchorage length  
 232 needs to be increased by much more when augmenting the axial force on the anchor bolt by the same amount.  
 233 Therefore, there is a certain reasonable length range, in which technical and economic effects can both be satisfied.  
 234 When  $P_{\max} > 0.9$ , it is supposed that  $k$  denotes the increment of  $\beta L_b$  required for the same increase in axial force  
 235 on the anchor bolt, that is, the efficiency of increasing the peak axial force of anchor bolt by increasing the  
 236 anchorage length (Fig 12) can be deduced.

237 As shown in Fig 12, when  $P_{\max} < 0.98$ , the increment in  $\beta L_b$  and  $k$  increased slightly; when  $P_{\max} \geq 0.98$ , the  
 238 increment in  $\beta L_b$  and  $k$  both increased, therefore,  $P_{\max} = 0.98$  can be considered as a criterion for discriminating a  
 239 reasonable anchorage length, with which economic principles are also satisfied on the premise of realising the  
 240 desired technical end. In this case,  $\beta L_b = 2.3$ , so the reasonable anchorage length of such anchor bolts was  $0.435\beta$ ,

241 that is,  $0.87 \sqrt{\frac{\tau_1}{\mu_1 D E_a}}$ .

242

## 243 5 Conclusions

244 (1) Based on the shear–displacement model, the analytical expressions for the distribution of axial force on  
 245 the anchorage body and shear stress on the anchorage body–surrounding rock interface along the anchorage zone  
 246 were attained. Furthermore, based on the shear–displacement model, it was found that the axial force decreased in  
 247 a non-uniform manner along the anchor bolt to the deeper anchorage zone. Moreover, the shear stress on interface  
 248 at the beginning of anchorage zone of the anchor bolts was maximised, then decreased along anchor.

249 (2) The influence of anchorage length on the stress distribution along an anchor bolt was obtained: in the  
 250 elastic deformation stage, the longer the anchorage length, the more uniform the shear stress distribution along the  
 251 anchorage zone and the higher the maximum shear stress on the interface. Beyond a certain critical anchorage  
 252 length, further increases therein caused no significant influence on the maximum shear stress.

253 (3) It was shown that there was a critical anchorage length: as the peak axial force on the anchor bolts

254 exhibited a hyperbolic tangent relationship with the anchorage length, it was determined that the technical and  
255 economic effects of an anchor bolt support system can be realised when the optimal anchorage length was  $0.435\beta$ .  
256

## 257 **Conflict of interest**

258 The authors wish to confirm that there are no known conflicts of interest associated with this publication and  
259 there has been no significant financial support for this work that could have influenced its outcome.

260

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265

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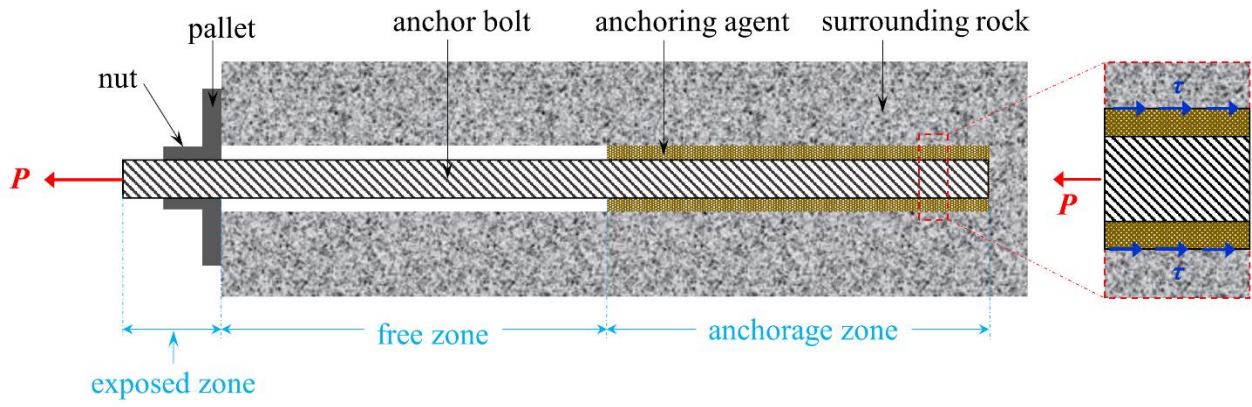
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## 1 **Figures**

- 2 Fig 1. Force transfer mechanism in an anchor bolt.
- 3 Fig 2. Shear stress–displacement relationship on the anchoring agent–borehole wall interface.
- 4 Fig 3. Distributions of axial force and shear stress in the anchorage zone.
- 5 Fig 4. Apparatus for the pull-out test and test materials.
- 6 Fig 5. Distribution of strain gauges.
- 7 Fig 6. Shear stress distribution of anchorage body under a same pull-out force and different anchorage lengths.
- 8 Fig 7. Stress distribution in the anchoring agent at different anchorage lengths.
- 9 Fig 8. Distributions of axial stress in the anchorage zone at different anchorage lengths and a given pull-out force.
- 10 Fig 9. The distribution of axial force in the anchorage zone under different pull-out forces and the same anchorage length.
- 11 Fig 10. Distributions of shear stress in the anchorage zone under different pull-out forces at the same anchorage length.
- 12 Fig 11. Relationships between peak, and incremental, axial force in the anchorage zone with anchorage length.
- 13 Fig 12. Relationships between peak, and incremental, axial forces in the anchorage zone with anchorage length.

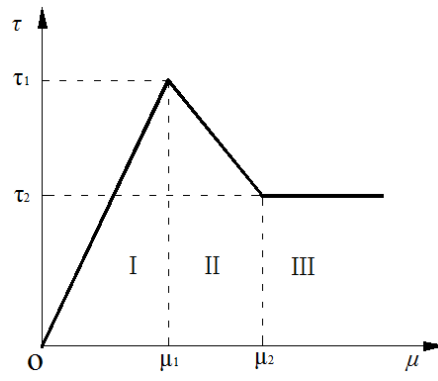
14 Fig 1. Force transfer mechanism in an anchor bolt.

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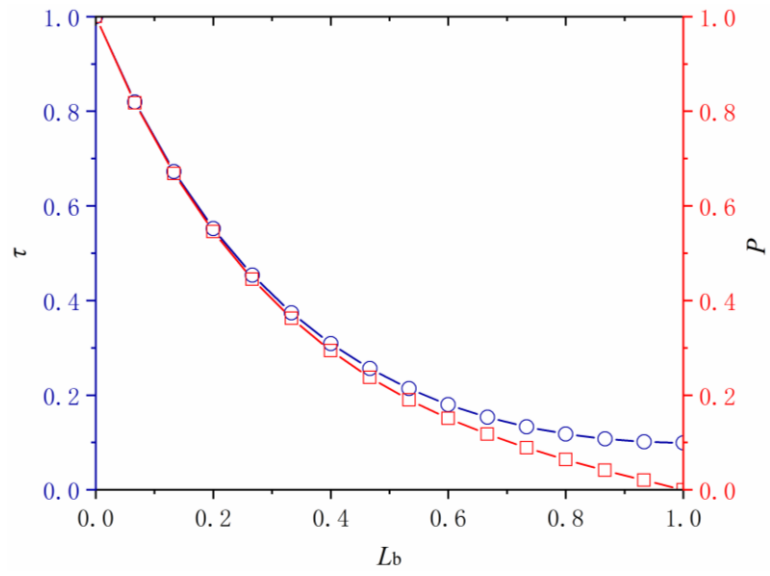
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17 Fig 2. Shear stress–displacement relationship on the anchoring agent–borehole wall interface.



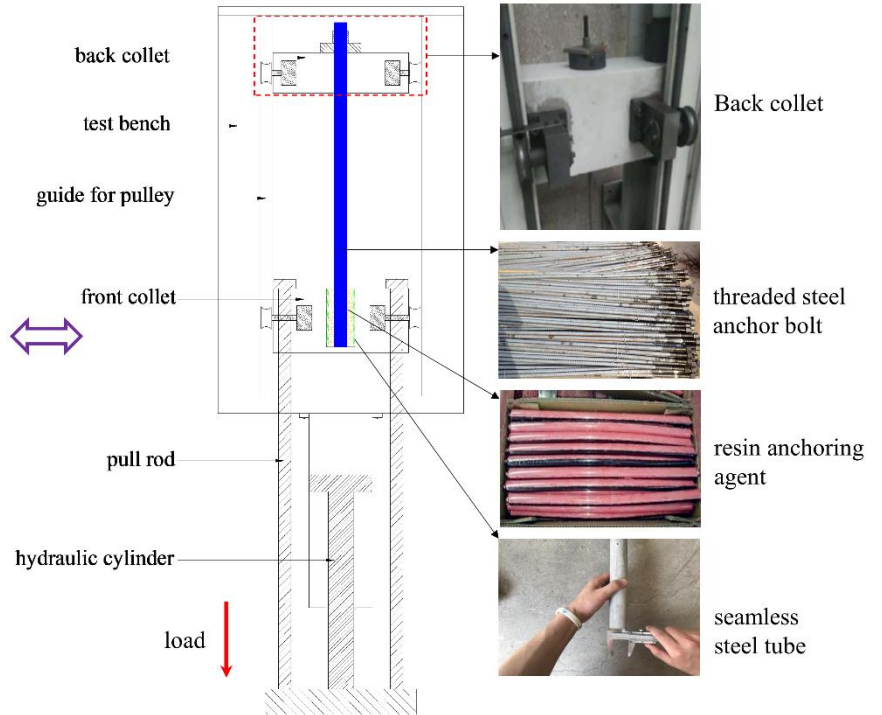
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19 **Fig 3. Distributions of axial force and shear stress in the anchorage zone.**



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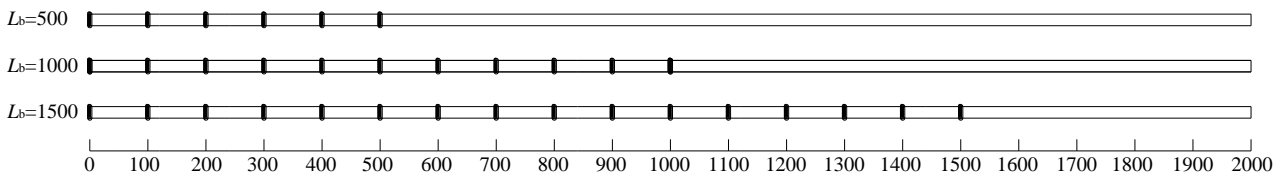
21 Fig 4. Apparatus for the pull-out test and test materials.



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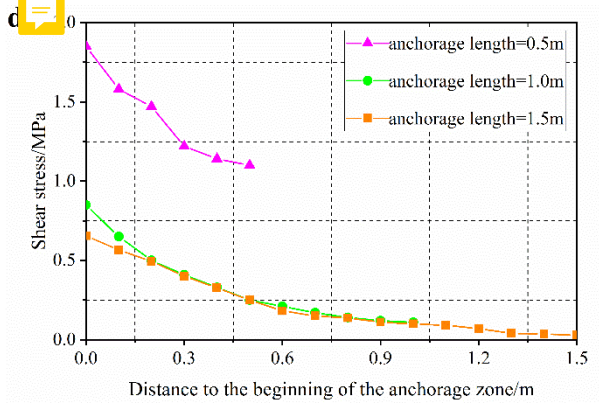
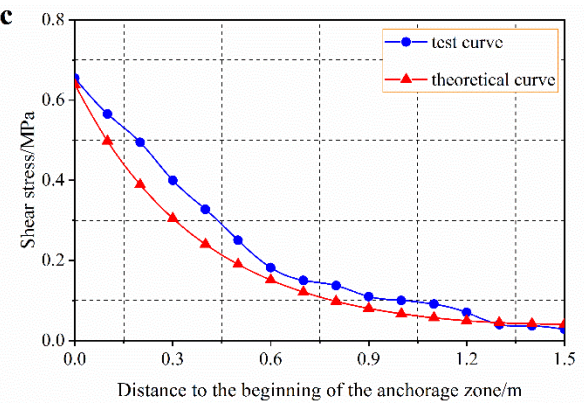
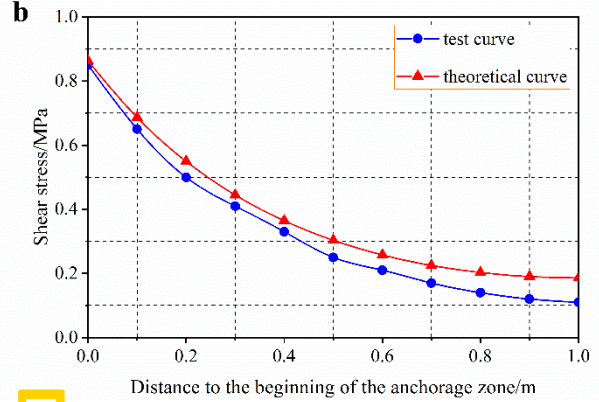
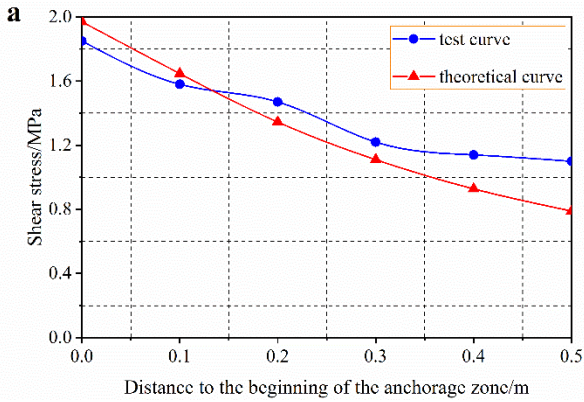
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**Fig 5. Distribution of strain gauges.**



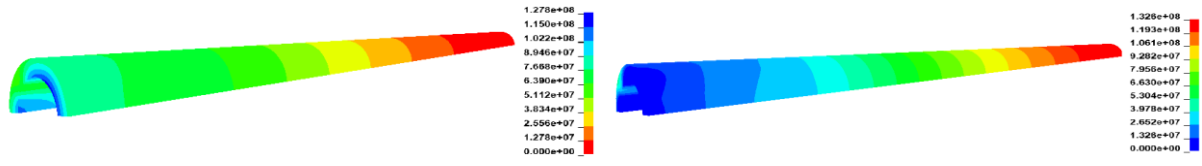
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26 **Fig 6. Shear stress distribution of anchorage body under a same pull-out force and different anchorage lengths.** Anchorage  
 27 lengths of 0.5 m (a), 1.0 m (b), and 1.5 m (c).

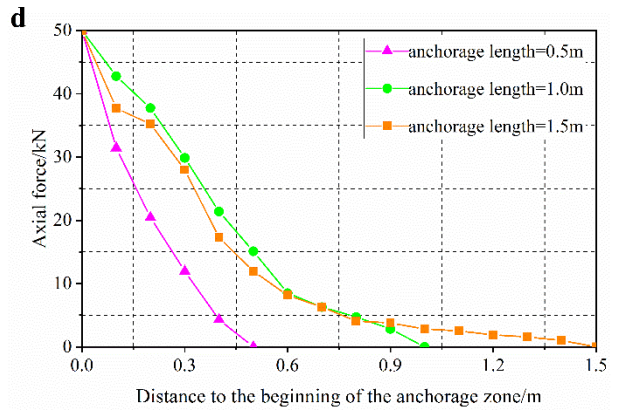
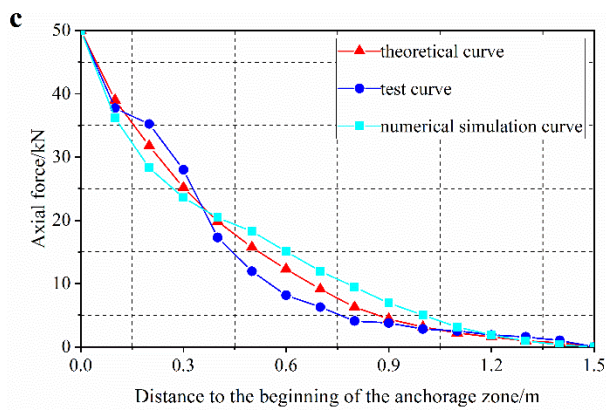
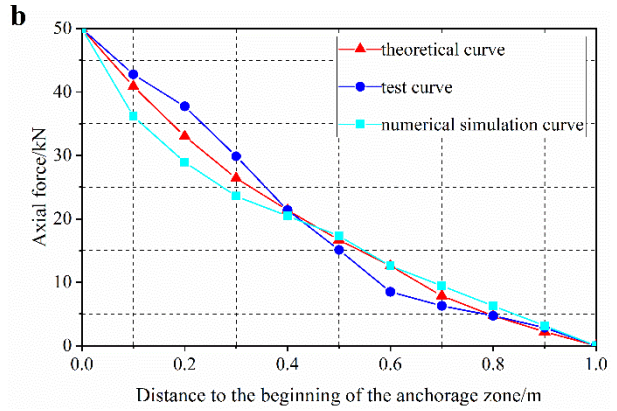
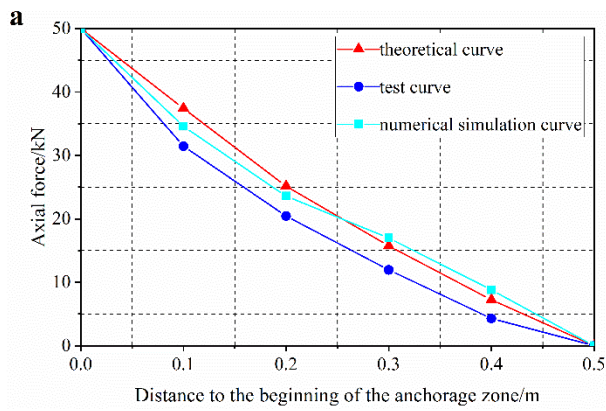




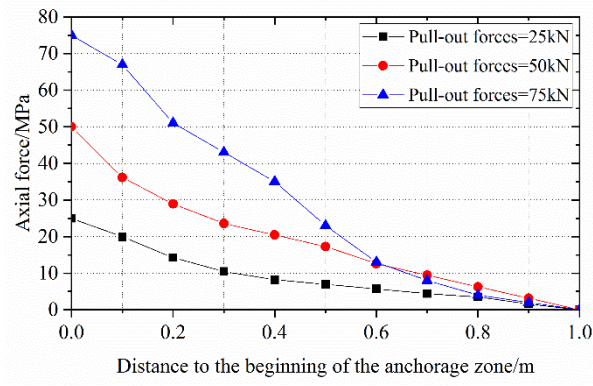
30 Fig 7. Stress distribution in the anchoring agent at different anchorage lengths. (a) 0.5 m; (b) 1.0 m



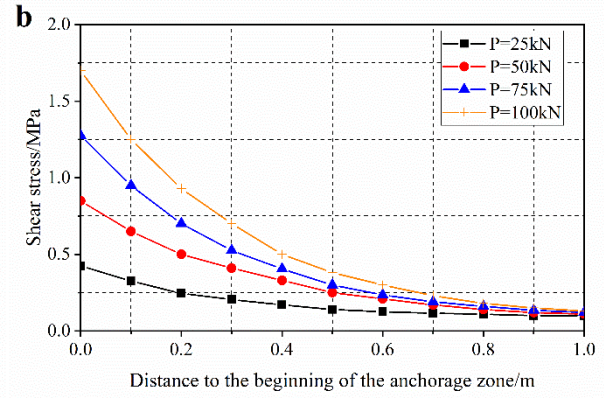
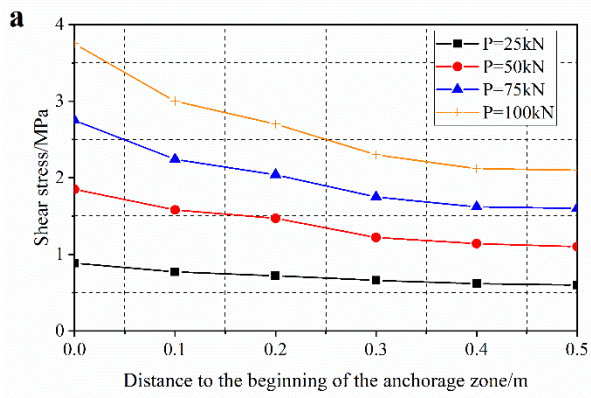
32 **Fig 8. Distributions of axial stress in the anchorage zone at different anchorage lengths and a given pull-out force.** Anchorage  
33 lengths of 0.5 m (a), 1.0 m (b), and 1.5 m (c), (d) is test curves of three length.



36 Fig 9. The distribution of axial force in the anchorage zone under different pull-out forces and the same anchorage length.

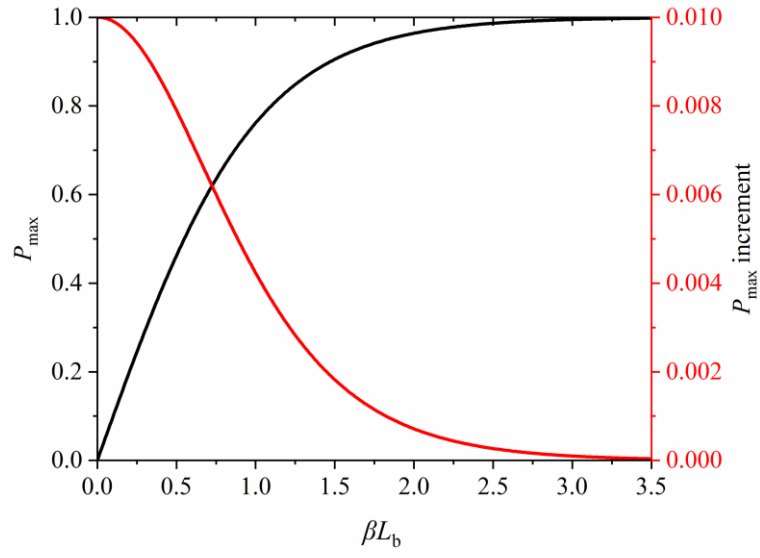


38 **Fig 10. Distributions of shear stress in the anchorage zone under different pull-out forces at the same anchorage length.** (a) 0.5  
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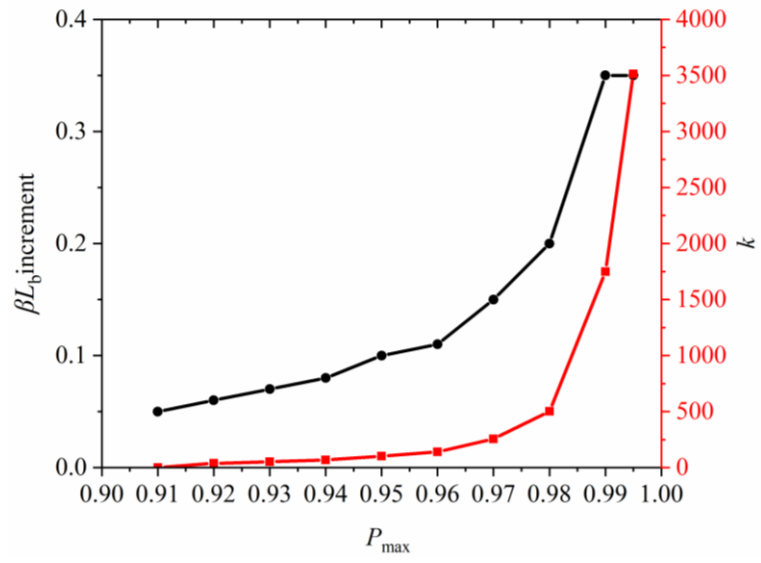


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41 Fig 11. Relationships between peak, and incremental, axial force in the anchorage zone with anchorage length.



43 Fig 12. Relationships between peak, and incremental, axial forces in the anchorage zone with anchorage length.



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