

Web Appendix for “Design and analysis considerations for cohort stepped wedge cluster randomized trials with a decay correlation structure”

Fan Li*

*fan.f.li@yale.edu

WEB APPENDIX A: CLOSED-FORM SOLUTIONS FOR α_0, α_1 FROM FIRST-STAGE QLS ESTIMATING EQUATIONS

We provide the closed-form solutions to α_0, α_1 based on the first-stage QLS estimating equations. These expressions were first introduced by Shults and Morrow¹, and we review them in our notations for cohort stepped wedge designs. Specifically, the first-stage estimator for α_0 is the solution to the scalar estimating equation

$$\sum_{i=1}^I \frac{\partial}{\partial \alpha_0} [r'_i(\theta) \{G_i^{-1}(\alpha_0) \otimes F^{-1}(\alpha_1)\} r_i(\theta)] = 0 \quad (1)$$

This is equivalent to solving for $f(\alpha_0) = 0$, where

$$f(\alpha_0) = \sum_{i=1}^I \left\{ [\alpha_0^2(N_i - 2)(N_i - 1) + 2\alpha_0(N_i - 1)] \sum_{j=1}^{N_i} r'_{ij}(\theta) F^{-1}(\alpha_1) r_{ij}(\theta) - 2[1 + \alpha_0^2(N_i - 1)] \sum_{j=1}^{N_i-1} \sum_{j'=j+1}^{N_i} r'_{ij}(\theta) F^{-1}(\alpha_1) r_{ij'}(\theta) \right\} / \left\{ (1 - \alpha_0)^2 [1 + (N_i - 1)\alpha_0]^2 \right\}$$

For balanced cohort size such that $N_i = N$, a closed-form expression for α_0 is

$$\alpha_0 = \frac{-(N - 1)a_1 + \sqrt{(N - 1)[(N - 1)a_1 - 2a_2](a_1 + 2a_2)}}{(N - 1)[a_1(N - 2) - 2a_2]}$$

where $a_1 = \sum_{i=1}^I \sum_{j=1}^{N_i} r'_{ij}(\theta) F^{-1}(\alpha_1) r_{ij}(\theta)$, and $a_2 = \sum_{i=1}^I \sum_{j=1}^{N_i-1} \sum_{j'=j+1}^{N_i} r'_{ij}(\theta) F^{-1}(\alpha_1) r_{ij'}(\theta)$. Similarly, the first-stage estimating equation for α_1

$$\sum_{i=1}^I \frac{\partial}{\partial \alpha_1} [r'_i(\theta) \{G_i^{-1}(\alpha_0) \otimes F^{-1}(\alpha_1)\} r_i(\theta)] = 0. \quad (2)$$

is equivalent to the following expression

$$\alpha_1 = \frac{b_m - \sqrt{b_m^2 - 4a_m^2}}{2a_m}$$

where

$$\begin{aligned} a_m &= \sum_{i=1}^I \left[q_{i1} \sum_{j=1}^{N_i} \sum_{t=1}^{T-1} r_{ijt} r_{ij,t+1} + q_{i2} \sum_{j=1}^{N_i-1} \sum_{j'=j+1}^{N_i} \sum_{t=1}^{T-1} (r_{ijt} r_{ij',t+1} + r_{ij't} r_{ij,t+1}) \right] \\ b_m &= \sum_{i=1}^I \left[q_{i1} \sum_{j=1}^{N_i} \sum_{t=1}^{T-1} (r_{ijt}^2 + r_{ij,t+1}^2) + 2q_{i2} \sum_{j=1}^{N_i-1} \sum_{j'=j+1}^{N_i} \sum_{t=1}^{T-1} (r_{ijt} r_{ij't} + r_{ij,t+1} r_{ij',t+1}) \right] \\ q_{i1} &= \frac{1 + (N_i - 2)\alpha_0}{(1 - \alpha_0)[1 + (N_i - 1)\alpha_0]} \\ q_{i2} &= \frac{-\alpha_0}{(1 - \alpha_0)[1 + (N_i - 1)\alpha_0]} \end{aligned}$$

WEB APPENDIX B: MATRIX-ADJUSTED QLS ESTIMATING EQUATIONS

We justify the proposed matrix-adjusted correlation estimates as follows. Recall that the first-stage estimating equation (1) is equivalent to

$$0 = \sum_{i=1}^I \frac{\partial}{\partial \alpha_0} \text{tr} [\{G_i^{-1}(\alpha_0) \otimes F^{-1}(\alpha_1)\} r_i(\theta) r_i'(\theta)] \propto \sum_{i=1}^I \frac{\partial}{\partial \alpha_0} \text{tr} [\{G_i^{-1}(\alpha_0) \otimes F^{-1}(\alpha_1)\} \phi^{-1} r_i(\theta) r_i'(\theta)] \quad (3)$$

and $\phi^{-1} r_i(\theta) r_i'(\theta)$ is the nonparametric moment estimator for the correlation matrix. Specifically when I is small, the estimated residual $y_i - \hat{\mu}_i$ tends to be biased towards zero, and following Preisser et al.², we have $E[(y_i - \hat{\mu}_i)(y_i - \hat{\mu}_i)'] \approx (I - H_i) \text{cov}(y_i) = \phi(I - H_i) A_i^{1/2} \text{corr}(y_i) A_i^{1/2}$ and therefore $E[r_i(\hat{\theta}) r_i'(\hat{\theta})] \approx \phi A_i^{-1/2} (I - H_i) A_i^{1/2} \text{corr}(y_i)$. This last equation suggests that $\phi^{-1} A_i^{-1/2} (I - H_i)^{-1} A_i^{1/2} r_i(\hat{\theta}) r_i'(\hat{\theta})$ is a better estimator for $\text{corr}(y_i)$ compared to the simple cross-product $\phi^{-1} r_i(\hat{\theta}) r_i'(\hat{\theta})$, since the former accounts for finite-sample bias in a multiplicative fashion. Then we define the ‘‘matrix-adjusted’’ estimator $\tilde{R}_i(\theta) = A_i^{-1/2} (I - H_i)^{-1} A_i^{1/2} r_i(\theta) r_i'(\theta)$ for the correlation matrix to replace the simple cross-product $r_i(\theta) r_i'(\theta)$ in (3). See also the Web Appendix B of Li et al.³ for a full exposition on related technical details. In other words, estimating equation (3) could be replaced by

$$\sum_{i=1}^I \frac{\partial}{\partial \alpha_0} \text{tr} [\{G_i^{-1}(\alpha_0) \otimes F^{-1}(\alpha_1)\} \tilde{R}_i(\theta)] = 0. \quad (4)$$

When the true correlation is the proportional decay structure, we provide the closed-form updates for the matrix-adjusted estimating equations. Write $\tilde{R}_i(\theta)$ as a block matrix with each block \tilde{R}_{ijj} corresponding to the repeated measurements for the same individual

$$\tilde{R}_i(\theta) = \begin{pmatrix} \tilde{R}_{i11} & \tilde{R}_{i12} & \dots & \tilde{R}_{i1N_i} \\ \tilde{R}_{i21} & \tilde{R}_{i22} & \dots & \tilde{R}_{i2N_i} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{iN_i1} & \tilde{R}_{iN_i2} & \dots & \tilde{R}_{iN_iN_i} \end{pmatrix}.$$

After some simplification algebra, we could express α_0 as the solution to $g(\alpha_0) = 0$, where

$$g(\alpha_0) = \sum_{i=1}^I \left\{ [\alpha_0^2 (N_i - 2)(N_i - 1) + 2\alpha_0 (N_i - 1)] \sum_{j=1}^{N_i} \text{tr} (F^{-1}(\alpha_1) \tilde{R}_{ijj}) - [1 + \alpha_0^2 (N_i - 1)] \sum_{j \neq j'} \text{tr} (F^{-1}(\alpha_1) \tilde{R}_{ijj'}) \right\} / \left\{ (1 - \alpha_0)^2 [1 + (N_i - 1)\alpha_0]^2 \right\}$$

Similarly, we solve the following bias-corrected estimating equation for α_1

$$\sum_{i=1}^I \frac{\partial}{\partial \alpha_1} \text{tr} [\{G_i^{-1}(\alpha_0) \otimes F^{-1}(\alpha_1)\} \tilde{R}_i(\theta)] = 0. \quad (5)$$

Recall that $F^{-1}(\rho) = (1 - \rho^2)^{-1} \{I + \rho^2 C_2 - \rho C_1\}$, $C_2 = \text{diag}(0, 1, \dots, 1, 0)$ and C_1 is a tridiagonal matrix with zeros on the main diagonal and ones on the two sub-diagonals. After some simplification algebra, we could express α_1 as

$$\alpha_1 = \frac{e_m - \sqrt{e_m^2 - d_m^2}}{d_m}$$

where

$$d_m = \sum_{i=1}^I \left[q_{i1} \sum_{j=1}^{N_i} \text{tr} (C_1 \tilde{R}_{ijj}) + q_{i2} \sum_{j \neq j'} \text{tr} (C_1 \tilde{R}_{ijj'}) \right]$$

$$e_m = \sum_{i=1}^I \left[q_{i1} \sum_{j=1}^{N_i} \text{tr} (\tilde{R}_{ijj}) + q_{i1} \sum_{j=1}^{N_i} \text{tr} (C_2 \tilde{R}_{ijj}) + q_{i2} \sum_{j \neq j'} \text{tr} (\tilde{R}_{ijj'}) + q_{i2} \sum_{j \neq j'} \text{tr} (C_2 \tilde{R}_{ijj'}) \right]$$

WEB APPENDIX C: CLOSED-FORM VARIANCE OF THE INTERVENTION EFFECT UNDER THE PROPORTIONAL DECAY MODEL

Recall that the $TN \times (T + 1)$ design matrix corresponding to cluster i as $Z_i = 1_N \otimes (I_T, X_i)$, where 1_N is a N -vector of ones. The variance of the intervention effect $\text{var}(\hat{\delta})$ equals to the lower-right element of $\phi \left\{ \sum_{i=1}^I Z_i' R_i^{-1}(\tau, \rho) Z_i \right\}^{-1}$, where ϕ is the marginal variance or dispersion. Notice that

$$\sum_{i=1}^I Z_i' R_i^{-1} Z_i = \sum_{i=1}^I \left[1_N' \otimes \begin{pmatrix} I_T \\ X_i' \end{pmatrix} \right] (G_i^{-1}(\tau) \otimes F^{-1}(\rho)) [1_N \otimes (I_T \ X_i)] = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix},$$

where Λ_{11} is of dimension $T \times T$, $\Lambda_{12} = \Lambda_{21}'$ is of dimension $T \times 1$, and Λ_{22} is a scalar. Define $c = (1 - \tau)^{-1}$ and $d = \tau / [(1 - \tau)\{1 + (N - 1)\tau\}]$, and after some algebra, we obtain

$$\begin{aligned} \Lambda_{11} &= I(1_N' 1_N) \otimes (cF^{-1}) + I(1_N' J 1_N) \otimes (-dF^{-1}) = IN(c - Nd)F^{-1}, \\ \Lambda_{12} &= I(1_N' 1_N) \otimes \sum_{i=1}^I (cF^{-1})X_i + (1_N' J 1_N) \otimes \sum_{i=1}^I (-dF^{-1})X_i = N(c - Nd) \sum_{i=1}^I F^{-1} X_i, \end{aligned}$$

and $\Lambda_{22} = N(c - Nd) \sum_{i=1}^I X_i' F^{-1} X_i$. The variance of the intervention effect then becomes

$$\begin{aligned} \text{var}(\hat{\delta}) &= \phi(\Lambda_{22} - \Lambda_{21}\Lambda_{11}^{-1}\Lambda_{12})^{-1} \\ &= \phi \left\{ N(c - Nd) \sum_{i=1}^I X_i' F^{-1} X_i - I^{-1}N(c - Nd) \left(\sum_{i=1}^I X_i' F^{-1} \right) F \left(\sum_{i=1}^I F^{-1} X_i \right) \right\}^{-1} \\ &= \phi IN^{-1}(c - Nd)^{-1} \left\{ I \sum_{i=1}^I X_i' F^{-1} X_i - \left(\sum_{i=1}^I X_i' \right) F^{-1} \left(\sum_{i=1}^I X_i \right) \right\}^{-1}. \end{aligned}$$

The key terms are further expanded as

$$\begin{aligned} I \sum_{i=1}^I X_i' F^{-1} X_i &= (1 - \rho^2)^{-1} \left\{ I \sum_{i=1}^I (X_{i1} + X_{iT}) + I(1 + \rho^2) \sum_{i=2}^{T-2} X_{ii} - 2\rho IV \right\} \\ &= (1 - \rho^2)^{-1} \left\{ -I\rho^2 \sum_{i=1}^I (X_{i1} + X_{iT}) + (1 + \rho^2)IU - 2\rho IV \right\} \\ &= (1 - \rho^2)^{-1} \left\{ -I^2\rho^2 + (1 + \rho^2)IU - 2\rho IV \right\}, \end{aligned}$$

where $U = \sum_{i=1}^I \sum_{t=1}^T X_{it}$ and $V = \sum_{i=1}^I \sum_{t=1}^{T-1} X_{it} X_{i,t+1}$ are design constants that only depend on the stepped wedge assignment layout. The last equality holds because $X_{i1} = 0$, $X_{iT} = 1$ for all i in stepped wedge designs. In particular, the inner summation in V reflects the between-period autoregressive structure to the first order. Next,

$$\begin{aligned} \left(\sum_{i=1}^I X_i' \right) F^{-1} \left(\sum_{i=1}^I X_i \right) &= (1 - \rho^2)^{-1} \left\{ \left(\sum_{i=1}^I X_{i1} \right)^2 + \left(\sum_{i=1}^I X_{iT} \right)^2 + (1 + \rho^2) \sum_{i=2}^{T-1} \left(\sum_{i=1}^I X_{ii} \right)^2 - 2\rho Q \right\} \\ &= (1 - \rho^2)^{-1} \left\{ -\rho^2 \left(\sum_{i=1}^I X_{i1} \right)^2 - \rho^2 \left(\sum_{i=1}^I X_{iT} \right)^2 + (1 + \rho^2)W - 2\rho Q \right\} \\ &= (1 - \rho^2)^{-1} \left\{ -I^2\rho^2 + (1 + \rho^2)W - 2\rho Q \right\}, \end{aligned}$$

where the design constants $W = \sum_{i=1}^I \left(\sum_{t=1}^T X_{it} \right)^2$ and $Q = \sum_{i=1}^{T-1} \left(\sum_{i=1}^I X_{it} \right) \left(\sum_{i=1}^I X_{i,t+1} \right)$. Again, the outer summation in Q reflects the between-period autoregressive structure to the first order. Further note

$$c - Nd = \frac{1}{1 - \tau} - \frac{N\tau}{(1 - \tau)\{1 + (N - 1)\tau\}} = \frac{1}{1 + (N - 1)\tau}$$

Some further algebra leads to the following closed-form variance expression in Section 4.1 of the main text,

$$\text{var}(\hat{\delta}) = \frac{(\phi I/N)(1 - \rho^2)\{1 + (N - 1)\tau\}}{(IU - W)(1 + \rho^2) - 2(IV - Q)\rho}.$$

WEB APPENDIX D: WEB TABLES

Web Table 1 Convergence rates (out of 10000) for quasi-least squares (QLS) and matrix-adjusted quasi-least squares (MAQLS) analyses of simulated outcome data when the treatment effect is zero.

Correlations		Effect size	Design resource			Convergence (%)	
τ	ρ	δ	I	N	T	QLS	MAQLS
0.03	0.2	0.3	18	10	7	100.0	98.3
0.03	0.2	0.3	18	24	4	100.0	99.4
0.03	0.2	0.3	20	14	5	100.0	98.9
0.03	0.2	0.4	21	8	4	100.0	100.0
0.03	0.2	0.5	15	8	4	100.0	100.0
0.03	0.8	0.2	16	12	5	99.9	99.2
0.03	0.8	0.2	24	7	5	100.0	99.8
0.03	0.8	0.3	12	8	5	99.8	98.2
0.03	0.8	0.4	12	5	4	99.8	98.0
0.03	0.8	0.5	10	5	3	99.8	98.4
0.10	0.2	0.3	21	11	8	100.0	99.8
0.10	0.2	0.3	24	11	7	100.0	99.9
0.10	0.2	0.4	15	16	6	100.0	98.4
0.10	0.2	0.4	18	8	7	100.0	100.0
0.10	0.2	0.5	16	7	5	100.0	100.0
0.10	0.8	0.2	20	18	5	100.0	99.7
0.10	0.8	0.3	15	9	4	100.0	100.0
0.10	0.8	0.4	10	20	3	100.0	97.0
0.10	0.8	0.4	12	5	5	100.0	99.5
0.10	0.8	0.5	9	7	4	100.0	99.5

Web Table 2 Convergence rates (out of 10000) for quasi-least squares (QLS) and matrix-adjusted quasi-least squares (MAQLS) analyses of simulated outcome data when the treatment effect is nonzero.

Correlations		Effect size	Design resource			Convergence (%)	
τ	ρ	δ	I	N	T	QLS	MAQLS
0.03	0.2	0.3	18	10	7	100.0	98.3
0.03	0.2	0.3	18	24	4	100.0	99.4
0.03	0.2	0.3	20	14	5	100.0	98.9
0.03	0.2	0.4	21	8	4	100.0	100.0
0.03	0.2	0.5	15	8	4	100.0	100.0
0.03	0.8	0.2	16	12	5	99.9	99.2
0.03	0.8	0.2	24	7	5	100.0	99.8
0.03	0.8	0.3	12	8	5	99.8	98.2
0.03	0.8	0.4	12	5	4	99.8	98.0
0.03	0.8	0.5	10	5	3	99.8	98.4
0.10	0.2	0.3	21	11	8	100.0	99.8
0.10	0.2	0.3	24	11	7	100.0	99.9
0.10	0.2	0.4	15	16	6	100.0	98.4
0.10	0.2	0.4	18	8	7	100.0	100.0
0.10	0.2	0.5	16	7	5	100.0	100.0
0.10	0.8	0.2	20	18	5	100.0	99.7
0.10	0.8	0.3	15	9	4	100.0	100.0
0.10	0.8	0.4	10	20	3	100.0	97.0
0.10	0.8	0.4	12	5	5	100.0	99.5
0.10	0.8	0.5	9	7	4	100.0	99.5

Web Table 3 Percent relative bias of the correlation parameters based on quasi-least squares (QLS) and matrix-adjusted quasi-least squares (MAQLS) for each simulation scenario when the treatment effect is nonzero.

Correlations		Effect size	Design resource			Percent bias for τ		Percent bias for ρ	
τ	ρ	δ	I	N	T	QLS	MAQLS	QLS	MAQLS
0.03	0.2	0.3	18	10	7	-26.6	3.6	-0.6	0.2
0.03	0.2	0.3	18	24	4	-16.0	5.5	-0.3	0.0
0.03	0.2	0.3	20	14	5	-20.2	3.4	-0.5	0.1
0.03	0.2	0.4	21	8	4	-29.4	3.2	-0.4	0.2
0.03	0.2	0.5	15	8	4	-41.3	4.7	-0.7	0.3
0.03	0.8	0.2	16	12	5	-26.2	4.4	-0.6	0.0
0.03	0.8	0.2	24	7	5	-27.6	0.7	-0.6	-0.1
0.03	0.8	0.3	12	8	5	-49.4	8.4	-2.1	-0.3
0.03	0.8	0.4	12	5	4	-74.4	6.4	-2.3	-0.5
0.03	0.8	0.5	10	5	3	-92.2	-0.6	-1.6	-0.5
0.10	0.2	0.3	21	11	8	-9.2	3.9	-0.6	0.6
0.10	0.2	0.3	24	11	7	-8.1	3.4	-0.4	0.6
0.10	0.2	0.4	15	16	6	-11.8	7.1	-0.6	1.1
0.10	0.2	0.4	18	8	7	-12.7	3.9	-0.8	0.7
0.10	0.2	0.5	16	7	5	-16.7	3.7	-0.7	1.0
0.10	0.8	0.2	20	18	5	-8.0	4.9	-0.1	0.2
0.10	0.8	0.3	15	9	4	-15.2	3.1	-0.2	0.2
0.10	0.8	0.4	10	20	3	-16.9	6.4	-0.1	0.3
0.10	0.8	0.4	12	5	5	-27.0	2.6	-1.5	0.0
0.10	0.8	0.5	9	7	4	-29.7	3.9	-1.0	0.1

Web Table 4 Simulation scenarios, nominal size, along with the empirical type I error rates corresponding to the QLS z-tests and MAQLS z-test with different variance estimators. Empirical type I error rates between 0.045 and 0.055 are considered acceptable based on the margin of error from a binomial model with 10000 Monte Carlo replications.

τ	ρ	δ	I	N	T	Pred ^a	MB ^b			BC0 ^c			BC1 ^d			BC2 ^e			BC3 ^f				
							QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS
0.03	0.2	0	18	10	7	0.050	0.056	0.048	0.084	0.084	0.084	0.068	0.068	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.070	0.070	0.071
0.03	0.2	0	18	24	4	0.050	0.061	0.049	0.081	0.081	0.081	0.066	0.065	0.051	0.051	0.050	0.050	0.050	0.050	0.050	0.066	0.066	0.065
0.03	0.2	0	20	14	5	0.050	0.061	0.051	0.083	0.083	0.083	0.069	0.069	0.058	0.058	0.058	0.058	0.058	0.058	0.070	0.070	0.070	
0.03	0.2	0	21	8	4	0.050	0.061	0.052	0.079	0.079	0.079	0.066	0.066	0.053	0.053	0.053	0.053	0.053	0.053	0.066	0.066	0.066	
0.03	0.2	0	15	8	4	0.050	0.070	0.057	0.096	0.096	0.096	0.077	0.077	0.058	0.058	0.058	0.058	0.058	0.058	0.077	0.077	0.077	
0.03	0.8	0	16	12	5	0.050	0.066	0.056	0.091	0.091	0.091	0.074	0.073	0.056	0.056	0.056	0.056	0.056	0.056	0.074	0.074	0.073	
0.03	0.8	0	24	7	5	0.050	0.059	0.053	0.078	0.078	0.078	0.066	0.066	0.056	0.056	0.055	0.055	0.055	0.055	0.067	0.067	0.066	
0.03	0.8	0	12	8	5	0.050	0.071	0.060	0.109	0.109	0.106	0.081	0.079	0.059	0.058	0.058	0.058	0.058	0.058	0.082	0.082	0.079	
0.03	0.8	0	12	5	4	0.050	0.067	0.057	0.104	0.104	0.103	0.079	0.078	0.058	0.057	0.057	0.057	0.057	0.057	0.076	0.076	0.075	
0.03	0.8	0	10	5	3	0.050	0.074	0.063	0.113	0.113	0.112	0.083	0.082	0.057	0.056	0.056	0.056	0.056	0.056	0.072	0.072	0.071	
0.10	0.2	0	21	11	8	0.050	0.060	0.050	0.081	0.081	0.081	0.069	0.069	0.057	0.057	0.057	0.057	0.057	0.057	0.070	0.070	0.070	
0.10	0.2	0	24	11	7	0.050	0.060	0.051	0.076	0.076	0.077	0.064	0.064	0.053	0.053	0.053	0.053	0.053	0.053	0.065	0.065	0.065	
0.10	0.2	0	15	16	6	0.050	0.062	0.048	0.098	0.098	0.097	0.075	0.075	0.058	0.057	0.057	0.057	0.057	0.057	0.078	0.078	0.078	
0.10	0.2	0	18	8	7	0.050	0.059	0.048	0.087	0.087	0.088	0.071	0.071	0.056	0.056	0.056	0.056	0.056	0.056	0.073	0.073	0.073	
0.10	0.2	0	16	7	5	0.050	0.065	0.052	0.091	0.091	0.091	0.072	0.072	0.055	0.055	0.055	0.055	0.055	0.055	0.073	0.073	0.073	
0.10	0.8	0	20	18	5	0.050	0.062	0.053	0.079	0.079	0.079	0.064	0.064	0.052	0.052	0.052	0.052	0.052	0.052	0.065	0.065	0.065	
0.10	0.8	0	15	9	4	0.050	0.065	0.055	0.091	0.091	0.091	0.072	0.072	0.053	0.053	0.053	0.053	0.053	0.053	0.070	0.070	0.070	
0.10	0.8	0	10	20	3	0.050	0.076	0.056	0.116	0.116	0.117	0.084	0.085	0.060	0.060	0.060	0.060	0.060	0.060	0.073	0.073	0.074	
0.10	0.8	0	12	5	5	0.050	0.069	0.058	0.105	0.105	0.105	0.079	0.079	0.058	0.057	0.057	0.057	0.057	0.057	0.080	0.080	0.079	
0.10	0.8	0	9	7	4	0.050	0.076	0.060	0.122	0.122	0.120	0.088	0.086	0.058	0.057	0.057	0.057	0.057	0.057	0.084	0.084	0.083	

^a Pred: Nominal type I error rate.

^b MB: Model-based variance.

^c BC0: Uncorrected sandwich variance of Liang and Zeger (1986).

^d BC1: Bias-corrected sandwich variance of Kauermann and Carroll (2001).

^e BC2: Bias-corrected sandwich variance of Mancl and DeRouen (2001).

^f BC3: Bias-corrected sandwich variance of Fay and Graubard (2001).

Web Table 5 Simulation scenarios, nominal size, along with the empirical type I error rates corresponding to the QLS t -tests and MAQLS t -test with different variance estimators. The degrees of freedom is set to be $\text{DoF} = I - 2$. Empirical type I error rates between 0.045 and 0.055 are considered acceptable based on the margin of error from a binomial model with 10000 Monte Carlo replications.

τ	ρ	δ	I	N	T	Pred ^a	MB ^b			BC0 ^c			BC1 ^d			BC2 ^e			BC3 ^f				
							QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS
0.03	0.2	0	18	10	7	0.050	0.041	0.034	0.063	0.063	0.063	0.047	0.047	0.038	0.038	0.038	0.038	0.038	0.038	0.038	0.049	0.049	0.049
0.03	0.2	0	18	24	4	0.050	0.043	0.034	0.061	0.060	0.060	0.048	0.047	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.047	0.047	0.047
0.03	0.2	0	20	14	5	0.050	0.044	0.038	0.065	0.065	0.065	0.054	0.054	0.044	0.044	0.044	0.044	0.044	0.044	0.055	0.055	0.055	
0.03	0.2	0	21	8	4	0.050	0.045	0.038	0.061	0.061	0.061	0.050	0.050	0.040	0.040	0.040	0.040	0.040	0.040	0.049	0.049	0.049	
0.03	0.2	0	15	8	4	0.050	0.045	0.037	0.070	0.070	0.070	0.053	0.053	0.041	0.041	0.041	0.041	0.041	0.041	0.053	0.053	0.053	
0.03	0.8	0	16	12	5	0.050	0.044	0.038	0.068	0.067	0.067	0.051	0.051	0.038	0.038	0.038	0.038	0.038	0.038	0.051	0.051	0.051	
0.03	0.8	0	24	7	5	0.050	0.047	0.043	0.063	0.062	0.062	0.053	0.052	0.043	0.043	0.043	0.043	0.043	0.043	0.053	0.053	0.053	
0.03	0.8	0	12	8	5	0.050	0.043	0.035	0.072	0.070	0.070	0.051	0.049	0.034	0.032	0.032	0.032	0.032	0.032	0.050	0.050	0.050	
0.03	0.8	0	12	5	4	0.050	0.040	0.034	0.070	0.069	0.069	0.051	0.050	0.036	0.035	0.035	0.035	0.035	0.035	0.049	0.049	0.049	
0.03	0.8	0	10	5	3	0.050	0.038	0.030	0.070	0.070	0.070	0.047	0.046	0.032	0.031	0.031	0.031	0.031	0.031	0.038	0.038	0.038	
0.10	0.2	0	21	11	8	0.050	0.044	0.038	0.066	0.066	0.066	0.053	0.053	0.043	0.044	0.044	0.044	0.044	0.044	0.056	0.056	0.056	
0.10	0.2	0	24	11	7	0.050	0.044	0.038	0.061	0.061	0.061	0.051	0.051	0.043	0.043	0.043	0.043	0.043	0.043	0.052	0.052	0.052	
0.10	0.2	0	15	16	6	0.050	0.041	0.031	0.069	0.068	0.068	0.052	0.052	0.040	0.040	0.040	0.040	0.040	0.040	0.053	0.053	0.053	
0.10	0.2	0	18	8	7	0.050	0.040	0.034	0.067	0.067	0.067	0.053	0.052	0.040	0.040	0.040	0.040	0.040	0.040	0.054	0.054	0.054	
0.10	0.2	0	16	7	5	0.050	0.044	0.034	0.065	0.066	0.066	0.051	0.051	0.038	0.038	0.038	0.038	0.038	0.038	0.052	0.052	0.052	
0.10	0.8	0	20	18	5	0.050	0.046	0.038	0.061	0.061	0.061	0.048	0.048	0.039	0.039	0.039	0.039	0.039	0.039	0.048	0.048	0.048	
0.10	0.8	0	15	9	4	0.050	0.044	0.037	0.066	0.066	0.066	0.048	0.048	0.037	0.037	0.037	0.037	0.037	0.037	0.046	0.046	0.046	
0.10	0.8	0	10	20	3	0.050	0.038	0.028	0.072	0.073	0.073	0.049	0.049	0.030	0.030	0.030	0.030	0.030	0.030	0.040	0.040	0.040	
0.10	0.8	0	12	5	5	0.050	0.041	0.033	0.070	0.069	0.069	0.050	0.049	0.034	0.033	0.033	0.033	0.033	0.033	0.049	0.049	0.049	
0.10	0.8	0	9	7	4	0.050	0.038	0.028	0.074	0.072	0.072	0.048	0.047	0.030	0.029	0.029	0.029	0.029	0.029	0.046	0.046	0.046	

^a Pred: Nominal type I error rate.

^b MB: Model-based variance.

^c BC0: Uncorrected sandwich variance of Liang and Zeger (1986).

^d BC1: Bias-corrected sandwich variance of Kauermann and Carroll (2001).

^e BC2: Bias-corrected sandwich variance of Mancl and DeRouen (2001).

^f BC3: Bias-corrected sandwich variance of Fay and Graubard (2001).

Web Table 6 Simulation scenarios, nominal size, along with the empirical type I error rates corresponding to the QLS t -tests and MAQLS t -test with different variance estimators. The degrees of freedom is set to be $\text{DoF} = I - (T + 1)$. Empirical type I error rates between 0.045 and 0.055 are considered acceptable based on the margin of error from a binomial model with 10000 Monte Carlo replications.

τ	ρ	δ	I	N	T	Pred ^a	MB ^b			BC0 ^c			BC1 ^d			BC2 ^e			BC3 ^f				
							QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS	MAQLS	QLS	MAQLS
0.03	0.2	0	18	10	7	0.050	0.032	0.026	0.049	0.050	0.040	0.040	0.032	0.032	0.041	0.041	0.041	0.032	0.032	0.032	0.041	0.041	0.041
0.03	0.2	0	18	24	4	0.050	0.040	0.031	0.056	0.055	0.043	0.043	0.034	0.034	0.043	0.043	0.043	0.034	0.034	0.034	0.043	0.043	0.043
0.03	0.2	0	20	14	5	0.050	0.040	0.033	0.062	0.062	0.050	0.051	0.040	0.040	0.051	0.051	0.040	0.040	0.040	0.051	0.051	0.051	
0.03	0.2	0	21	8	4	0.050	0.042	0.035	0.058	0.058	0.047	0.047	0.037	0.037	0.047	0.047	0.037	0.037	0.037	0.047	0.047	0.047	
0.03	0.2	0	15	8	4	0.050	0.039	0.030	0.063	0.063	0.048	0.048	0.036	0.036	0.047	0.047	0.036	0.036	0.036	0.047	0.047	0.047	
0.03	0.8	0	16	12	5	0.050	0.038	0.031	0.058	0.058	0.044	0.044	0.034	0.034	0.044	0.044	0.034	0.034	0.033	0.044	0.044	0.044	
0.03	0.8	0	24	7	5	0.050	0.044	0.041	0.060	0.059	0.050	0.049	0.041	0.041	0.049	0.049	0.041	0.041	0.041	0.049	0.049	0.049	
0.03	0.8	0	12	8	5	0.050	0.028	0.021	0.051	0.049	0.034	0.032	0.023	0.022	0.032	0.032	0.023	0.022	0.022	0.032	0.032	0.032	
0.03	0.8	0	12	5	4	0.050	0.031	0.026	0.056	0.055	0.042	0.041	0.028	0.027	0.040	0.040	0.028	0.027	0.027	0.039	0.039	0.039	
0.03	0.8	0	10	5	3	0.050	0.028	0.022	0.057	0.057	0.038	0.037	0.025	0.025	0.033	0.033	0.025	0.025	0.025	0.032	0.032	0.032	
0.10	0.2	0	21	11	8	0.050	0.037	0.032	0.056	0.056	0.046	0.046	0.035	0.035	0.046	0.046	0.035	0.035	0.035	0.046	0.046	0.046	
0.10	0.2	0	24	11	7	0.050	0.041	0.034	0.056	0.057	0.048	0.048	0.039	0.039	0.048	0.048	0.039	0.039	0.039	0.048	0.048	0.048	
0.10	0.2	0	15	16	6	0.050	0.031	0.021	0.055	0.054	0.042	0.041	0.030	0.030	0.041	0.041	0.030	0.030	0.030	0.042	0.042	0.042	
0.10	0.2	0	18	8	7	0.050	0.033	0.025	0.055	0.055	0.042	0.042	0.032	0.032	0.042	0.042	0.032	0.032	0.032	0.044	0.044	0.044	
0.10	0.2	0	16	7	5	0.050	0.036	0.027	0.057	0.058	0.043	0.043	0.031	0.032	0.043	0.043	0.031	0.032	0.032	0.044	0.044	0.044	
0.10	0.8	0	20	18	5	0.050	0.042	0.034	0.056	0.056	0.044	0.044	0.035	0.035	0.044	0.044	0.035	0.035	0.035	0.044	0.044	0.044	
0.10	0.8	0	15	9	4	0.050	0.038	0.032	0.057	0.057	0.042	0.042	0.032	0.032	0.042	0.042	0.032	0.032	0.032	0.041	0.041	0.041	
0.10	0.8	0	10	20	3	0.050	0.029	0.022	0.060	0.061	0.037	0.038	0.024	0.024	0.031	0.031	0.024	0.024	0.024	0.031	0.031	0.031	
0.10	0.8	0	12	5	5	0.050	0.026	0.021	0.049	0.049	0.034	0.033	0.023	0.022	0.033	0.033	0.023	0.022	0.022	0.033	0.033	0.033	
0.10	0.8	0	9	7	4	0.050	0.017	0.011	0.042	0.041	0.026	0.025	0.015	0.015	0.025	0.025	0.015	0.015	0.015	0.025	0.025	0.024	

^a Pred: Nominal type I error rate.

^b MB: Model-based variance.

^c BC0: Uncorrected sandwich variance of Liang and Zeger (1986).

^d BC1: Bias-corrected sandwich variance of Kauermann and Carroll (2001).

^e BC2: Bias-corrected sandwich variance of Mancl and DeRouen (2001).

^f BC3: Bias-corrected sandwich variance of Fay and Graubard (2001).

Web Table 7 Simulation scenarios, predicted power, along with the empirical power corresponding to the QLS z-tests and MAQLS z-test with different variance estimators. Differences from the prediction within 0.008 are considered acceptable based on the margin of error from a binomial model with 10000 Monte Carlo replications.

τ	ρ	δ	I	N	T	Pred ^a	MB ^b			BC0 ^c			BC1 ^d			BC2 ^e			BC3 ^f		
							QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS
0.03	0.2	0.3	18	10	7	0.899	0.907	0.894	0.910	0.910	0.910	0.889	0.889	0.865	0.866	0.890	0.890				
0.03	0.2	0.3	18	24	4	0.886	0.898	0.882	0.901	0.901	0.882	0.882	0.856	0.857	0.881	0.881					
0.03	0.2	0.3	20	14	5	0.897	0.908	0.896	0.911	0.912	0.891	0.892	0.870	0.870	0.893	0.893					
0.03	0.2	0.4	21	8	4	0.875	0.882	0.870	0.887	0.887	0.866	0.866	0.840	0.840	0.866	0.866					
0.03	0.2	0.5	15	8	4	0.907	0.910	0.896	0.917	0.917	0.895	0.896	0.868	0.868	0.896	0.896					
0.03	0.8	0.2	16	12	5	0.886	0.893	0.881	0.899	0.899	0.873	0.873	0.842	0.842	0.873	0.873					
0.03	0.8	0.2	24	7	5	0.882	0.885	0.877	0.891	0.892	0.875	0.875	0.857	0.858	0.874	0.875					
0.03	0.8	0.3	12	8	5	0.941	0.938	0.930	0.943	0.945	0.920	0.923	0.892	0.895	0.919	0.922					
0.03	0.8	0.4	12	5	4	0.952	0.949	0.943	0.957	0.958	0.938	0.940	0.912	0.914	0.936	0.937					
0.03	0.8	0.5	10	5	3	0.946	0.950	0.938	0.957	0.957	0.935	0.935	0.898	0.898	0.922	0.922					
0.10	0.2	0.3	21	11	8	0.878	0.885	0.872	0.893	0.893	0.874	0.874	0.850	0.850	0.877	0.877					
0.10	0.2	0.3	24	11	7	0.878	0.887	0.872	0.890	0.890	0.874	0.874	0.852	0.852	0.876	0.877					
0.10	0.2	0.4	15	16	6	0.906	0.917	0.894	0.919	0.919	0.897	0.896	0.864	0.864	0.899	0.898					
0.10	0.2	0.4	18	8	7	0.916	0.927	0.914	0.930	0.930	0.911	0.911	0.889	0.889	0.914	0.914					
0.10	0.2	0.5	16	7	5	0.886	0.898	0.878	0.905	0.905	0.880	0.880	0.850	0.851	0.882	0.882					
0.10	0.8	0.2	20	18	5	0.861	0.873	0.856	0.879	0.879	0.855	0.855	0.831	0.831	0.855	0.855					
0.10	0.8	0.3	15	9	4	0.895	0.901	0.885	0.905	0.905	0.880	0.880	0.849	0.849	0.877	0.877					
0.10	0.8	0.4	10	20	3	0.944	0.954	0.937	0.959	0.959	0.935	0.936	0.899	0.900	0.922	0.922					
0.10	0.8	0.4	12	5	5	0.932	0.934	0.926	0.940	0.941	0.917	0.919	0.886	0.888	0.916	0.918					
0.10	0.8	0.5	9	7	4	0.973	0.974	0.967	0.976	0.976	0.961	0.961	0.930	0.931	0.959	0.959					

^a Pred: Predicted power.

^b MB: Model-based variance.

^c BC0: Uncorrected sandwich variance of Liang and Zeger (1986).

^d BC1: Bias-corrected sandwich variance of Kauermann and Carroll (2001).

^e BC2: Bias-corrected sandwich variance of Mancl and DeRouen (2001).

^f BC3: Bias-corrected sandwich variance of Fay and Graubard (2001).

Web Table 8 Simulation scenarios, predicted power, along with the empirical power corresponding to the QLS t -tests and MAQLS t -test with different variance estimators. The degrees of freedom is set to be $\text{DoF} = I - 2$. Differences from the prediction within 0.008 are considered acceptable based on the margin of error from a binomial model with 10000 Monte Carlo replications.

τ	ρ	δ	I	N	T	Pred ^a	MB ^b			BC0 ^c			BC1 ^d			BC2 ^e			BC3 ^f		
							QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS
0.03	0.2	0.3	18	10	7	0.860	0.879	0.862	0.883	0.884	0.859	0.859	0.825	0.826	0.861	0.862					
0.03	0.2	0.3	18	24	4	0.844	0.869	0.849	0.876	0.876	0.849	0.849	0.819	0.819	0.848	0.848					
0.03	0.2	0.3	20	14	5	0.862	0.883	0.865	0.887	0.887	0.863	0.864	0.836	0.837	0.865	0.865					
0.03	0.2	0.4	21	8	4	0.839	0.855	0.838	0.860	0.860	0.834	0.834	0.807	0.807	0.833	0.833					
0.03	0.2	0.5	15	8	4	0.859	0.875	0.857	0.890	0.890	0.857	0.857	0.816	0.817	0.856	0.856					
0.03	0.8	0.2	16	12	5	0.838	0.860	0.839	0.863	0.863	0.831	0.832	0.794	0.794	0.832	0.832					
0.03	0.8	0.2	24	7	5	0.852	0.862	0.854	0.869	0.870	0.851	0.852	0.827	0.829	0.851	0.852					
0.03	0.8	0.3	12	8	5	0.887	0.901	0.891	0.912	0.914	0.880	0.884	0.838	0.840	0.882	0.884					
0.03	0.8	0.4	12	5	4	0.903	0.918	0.908	0.930	0.931	0.898	0.900	0.862	0.863	0.895	0.897					
0.03	0.8	0.5	10	5	3	0.878	0.905	0.887	0.921	0.921	0.875	0.875	0.818	0.819	0.855	0.854					
0.10	0.2	0.3	21	11	8	0.843	0.859	0.841	0.869	0.869	0.844	0.844	0.815	0.815	0.847	0.847					
0.10	0.2	0.3	24	11	7	0.848	0.863	0.848	0.868	0.868	0.849	0.849	0.826	0.826	0.851	0.851					
0.10	0.2	0.4	15	16	6	0.858	0.881	0.851	0.888	0.888	0.855	0.855	0.817	0.817	0.859	0.859					
0.10	0.2	0.4	18	8	7	0.879	0.902	0.883	0.906	0.906	0.882	0.883	0.854	0.854	0.885	0.886					
0.10	0.2	0.5	16	7	5	0.838	0.861	0.837	0.873	0.872	0.840	0.841	0.802	0.803	0.843	0.844					
0.10	0.8	0.2	20	18	5	0.821	0.841	0.818	0.848	0.847	0.821	0.821	0.788	0.788	0.820	0.820					
0.10	0.8	0.3	15	9	4	0.845	0.864	0.843	0.870	0.870	0.836	0.837	0.796	0.796	0.833	0.833					
0.10	0.8	0.4	10	20	3	0.875	0.912	0.879	0.920	0.921	0.881	0.881	0.820	0.821	0.856	0.857					
0.10	0.8	0.4	12	5	5	0.875	0.897	0.882	0.905	0.907	0.871	0.872	0.825	0.828	0.871	0.872					
0.10	0.8	0.5	9	7	4	0.914	0.942	0.924	0.948	0.949	0.910	0.911	0.855	0.856	0.906	0.907					

^a Pred: Predicted power.

^b MB: Model-based variance.

^c BC0: Uncorrected sandwich variance of Liang and Zeger (1986).

^d BC1: Bias-corrected sandwich variance of Kauermann and Carroll (2001).

^e BC2: Bias-corrected sandwich variance of Mancl and DeRouen (2001).

^f BC3: Bias-corrected sandwich variance of Fay and Graubard (2001).

Web Table 9 Simulation scenarios, predicted power, along with the empirical power corresponding to the QLS t -tests and MAQLS t -test with different variance estimators. The degrees of freedom is set to be $\text{DoF} = I - (T + 1)$. Differences from the prediction within 0.008 are considered acceptable based on the margin of error from a binomial model with 10000 Monte Carlo replications.

τ	ρ	δ	I	N	T	Pred ^a	MB ^b			BC0 ^c			BC1 ^d			BC2 ^e			BC3 ^f		
							QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS	MAQLS	QLS
0.03	0.2	0.3	18	10	7	0.832	0.857	0.836	0.864	0.864	0.833	0.833	0.798	0.799	0.837	0.837	0.837	0.837	0.837	0.837	
0.03	0.2	0.3	18	24	4	0.833	0.863	0.840	0.868	0.869	0.840	0.840	0.806	0.807	0.839	0.839	0.839	0.839	0.839		
0.03	0.2	0.3	20	14	5	0.850	0.873	0.854	0.879	0.880	0.854	0.854	0.825	0.825	0.856	0.856	0.856	0.856	0.856		
0.03	0.2	0.4	21	8	4	0.831	0.848	0.832	0.855	0.855	0.827	0.827	0.799	0.799	0.827	0.827	0.827	0.827	0.827		
0.03	0.2	0.5	15	8	4	0.842	0.864	0.842	0.877	0.877	0.841	0.841	0.800	0.800	0.840	0.840	0.840	0.840	0.840		
0.03	0.8	0.2	16	12	5	0.815	0.838	0.817	0.845	0.845	0.812	0.811	0.768	0.768	0.811	0.811	0.811	0.811	0.811		
0.03	0.8	0.2	24	7	5	0.845	0.857	0.849	0.864	0.865	0.844	0.845	0.820	0.820	0.844	0.844	0.844	0.844	0.844		
0.03	0.8	0.3	12	8	5	0.838	0.864	0.847	0.880	0.883	0.836	0.838	0.784	0.786	0.837	0.840	0.837	0.840	0.840		
0.03	0.8	0.4	12	5	4	0.876	0.899	0.883	0.911	0.913	0.876	0.878	0.827	0.828	0.873	0.875	0.873	0.875	0.875		
0.03	0.8	0.5	10	5	3	0.847	0.881	0.857	0.899	0.898	0.847	0.847	0.778	0.778	0.822	0.822	0.822	0.822	0.822		
0.10	0.2	0.3	21	11	8	0.819	0.841	0.820	0.850	0.849	0.821	0.822	0.792	0.792	0.826	0.826	0.826	0.826	0.826		
0.10	0.2	0.3	24	11	7	0.835	0.854	0.837	0.859	0.858	0.838	0.838	0.814	0.814	0.841	0.841	0.841	0.841	0.841		
0.10	0.2	0.4	15	16	6	0.820	0.850	0.811	0.860	0.859	0.823	0.822	0.774	0.774	0.828	0.827	0.828	0.827	0.827		
0.10	0.2	0.4	18	8	7	0.853	0.879	0.856	0.887	0.887	0.861	0.861	0.827	0.827	0.864	0.864	0.864	0.864	0.864		
0.10	0.2	0.5	16	7	5	0.815	0.841	0.817	0.856	0.857	0.821	0.821	0.777	0.777	0.823	0.824	0.823	0.824	0.824		
0.10	0.8	0.2	20	18	5	0.808	0.830	0.806	0.840	0.839	0.808	0.808	0.776	0.776	0.808	0.808	0.808	0.808	0.808		
0.10	0.8	0.3	15	9	4	0.826	0.849	0.826	0.856	0.856	0.820	0.820	0.777	0.778	0.817	0.817	0.817	0.817	0.817		
0.10	0.8	0.4	10	20	3	0.843	0.889	0.849	0.900	0.901	0.849	0.850	0.779	0.781	0.823	0.824	0.823	0.824	0.824		
0.10	0.8	0.4	12	5	5	0.823	0.854	0.833	0.870	0.871	0.824	0.826	0.764	0.767	0.824	0.826	0.824	0.826	0.826		
0.10	0.8	0.5	9	7	4	0.835	0.884	0.852	0.897	0.898	0.833	0.834	0.743	0.744	0.825	0.825	0.825	0.825	0.825		

^a Pred: Predicted power.

^b MB: Model-based variance.

^c BC0: Uncorrected sandwich variance of Liang and Zeger (1986).

^d BC1: Bias-corrected sandwich variance of Kauermann and Carroll (2001).

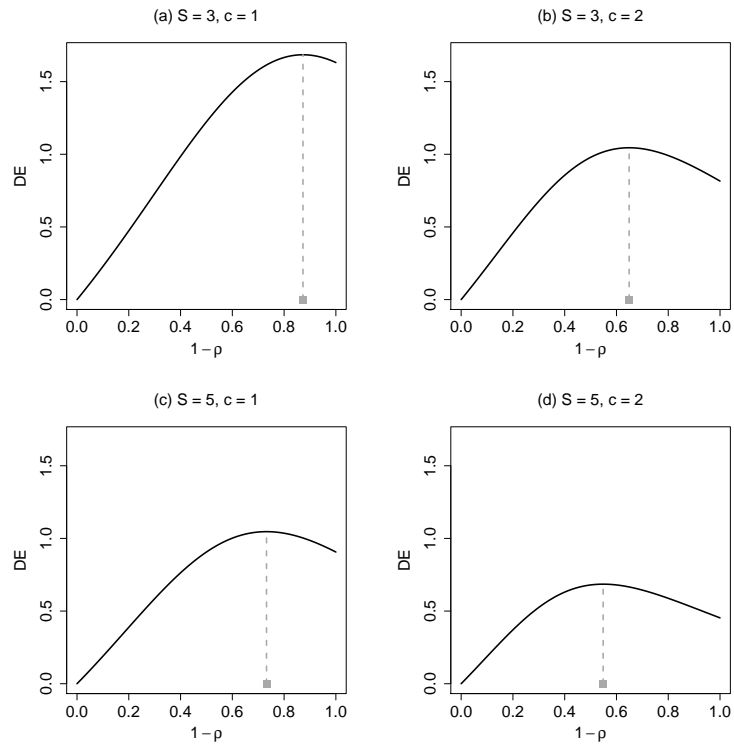
^e BC2: Bias-corrected sandwich variance of Mancl and DeRouen (2001).

^f BC3: Bias-corrected sandwich variance of Fay and Graubard (2001).

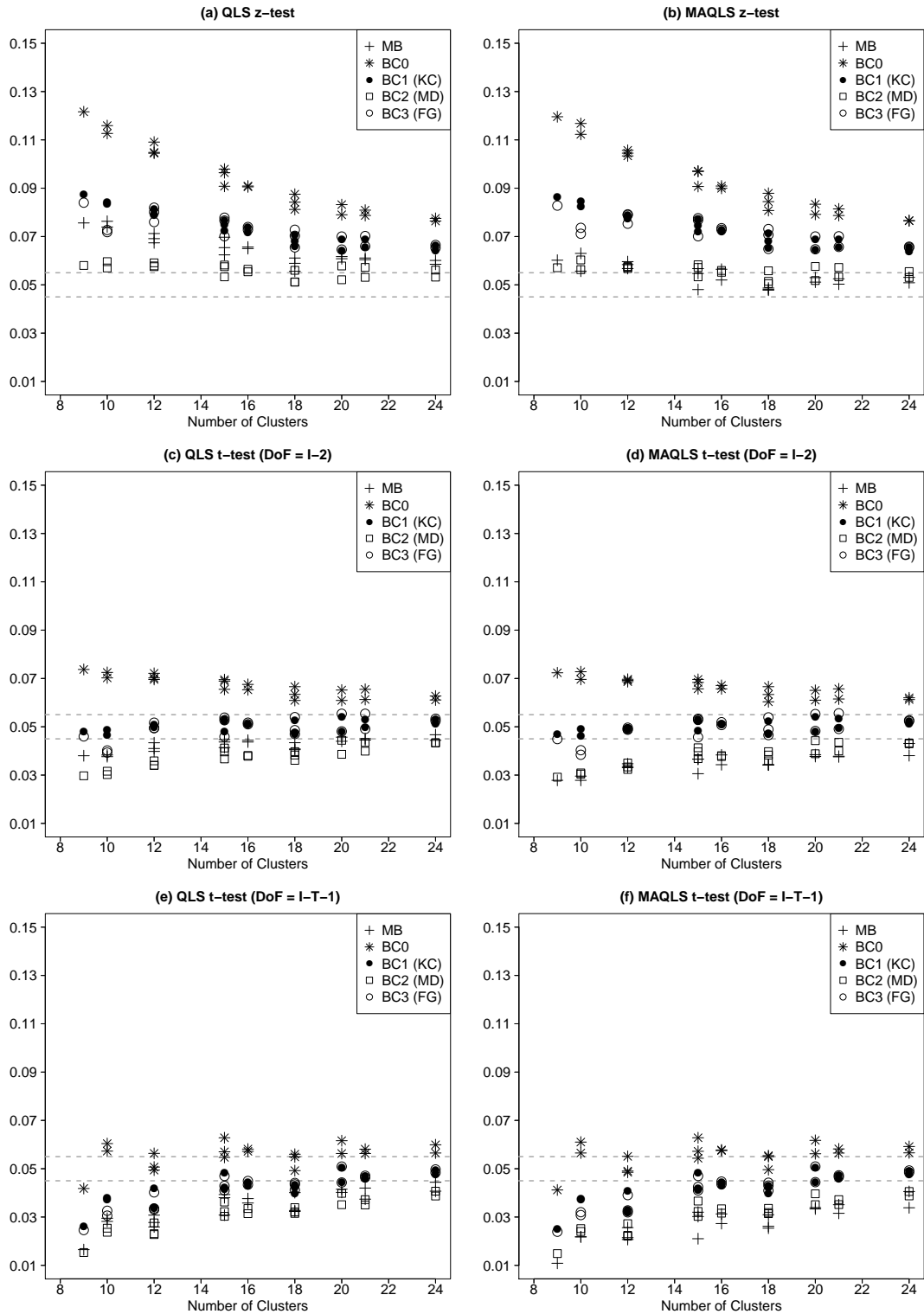
WEB APPENDIX E: WEB FIGURES

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
			$s = 1$	$s = 2$		$s = 3$
cluster 1	0	0	1	1	1	1
cluster 2	0	0	1	1	1	1
cluster 3	0	0	0	1	1	1
cluster 4	0	0	0	1	1	1
cluster 5	0	0	0	0	0	1
cluster 6	0	0	0	0	0	1
	$b = 2$		$c_1 = 1$	$c_2 = 2$		$c_3 = 1$

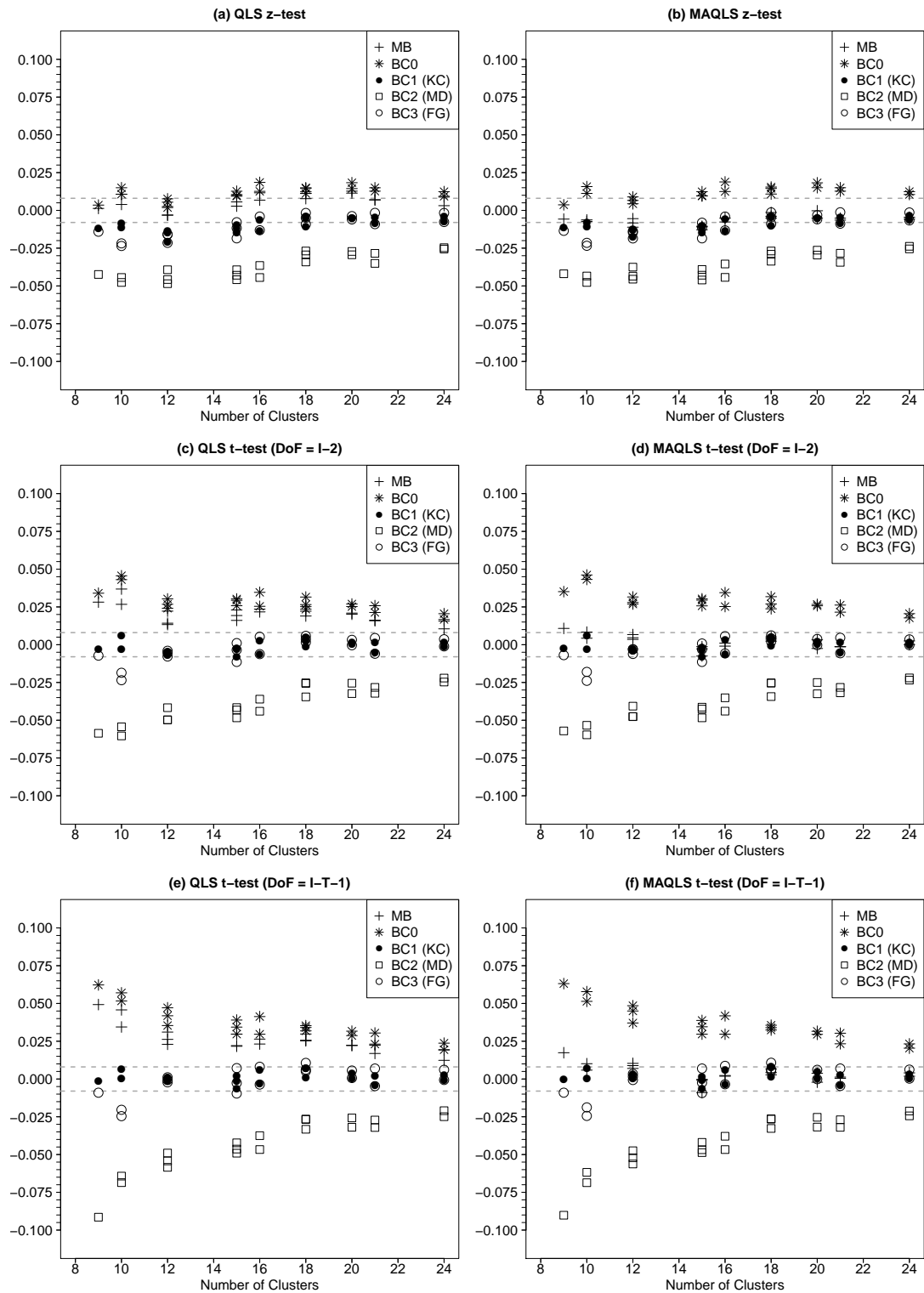
Web Figure 1 A schematic illustration of a stepped wedge design with $I = 6$ clusters and $T = 6$ periods, with $b = 2$ baseline periods, $c_1 = c_3 = 1$ and $c_2 = 2$. Each cell with a zero entry indicates a control cluster-period and each cell with a one entry indicates an intervention cluster-period.



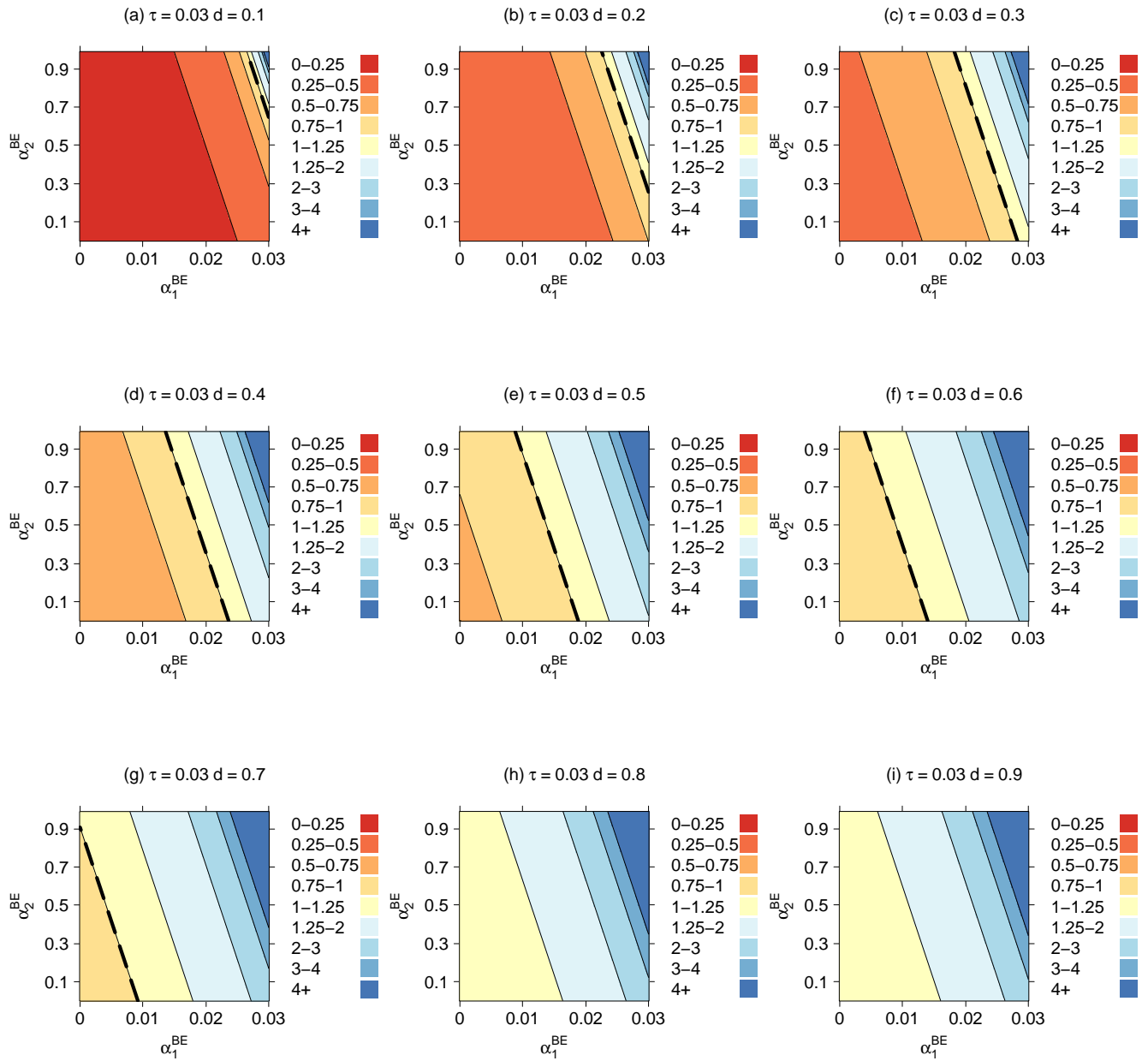
Web Figure 2 The design effect as a function of the degree of correlation decay, defined as $1 - \rho$. The x-axis is $1 - \rho$ so that the degree of decay increases from left to right. For illustration, the cohort size and the within-period correlation are fixed at $N = 20$ and $\tau = 0.1$. Corresponding to the analytical result in Section 4.2, the maximum design effect is attained when $\rho = r$ (gray square dot), a value between 0 and 1 and determined by the number of steps S and the number of measurements between steps c .



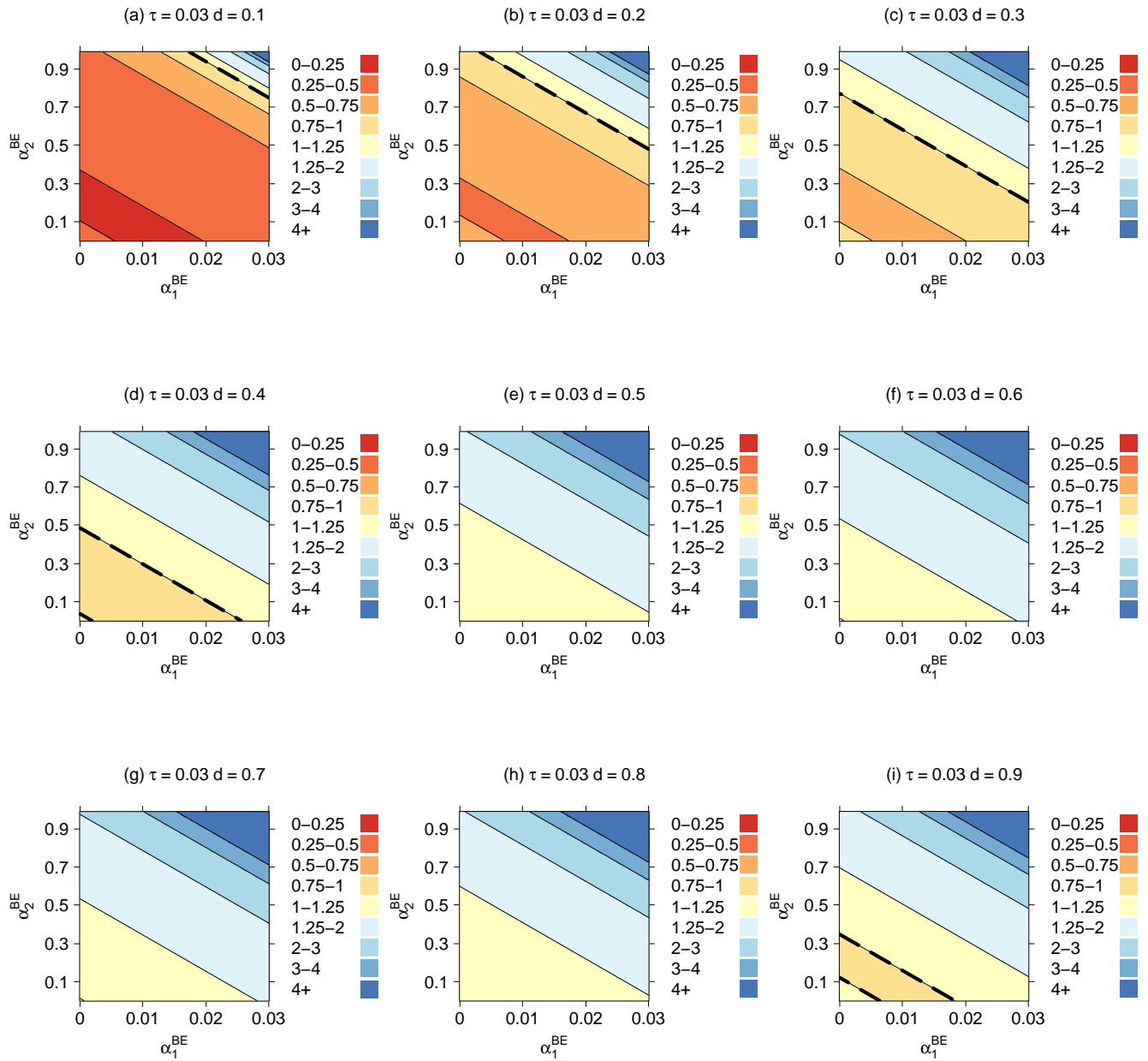
Web Figure 3 Empirical type I error rates for (a) QLS z-tests, (b) MAQLS z-tests, (c) QLS t-tests with $\text{DoF} = I - 2$, (d) MAQLS t-tests with $\text{DoF} = I - 2$, (e) QLS t-tests with $\text{DoF} = I - (T + 1)$ and (f) MAQLS t-tests with $\text{DoF} = I - (T + 1)$. MB: model-based variance; BC0: uncorrected sandwich variance; BC1: KC-corrected sandwich variance; BC2: MD-corrected sandwich variance; BC3: FG-corrected sandwich variance. The acceptable bounds are shown with the dashed horizontal lines. For each value of I , there may be multiple points with the same symbol indicating results with different combinations of design resources and correlation parameters.



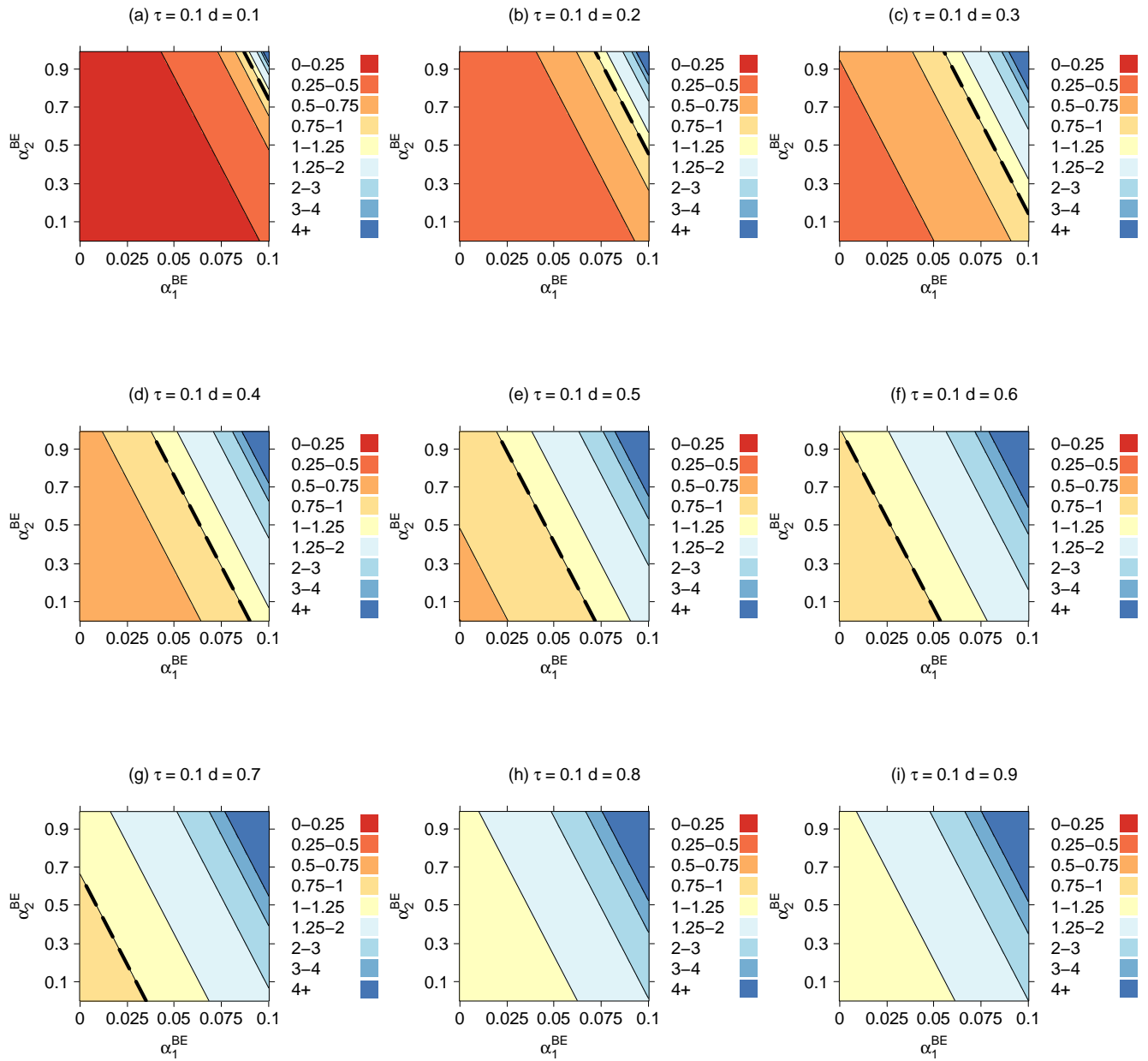
Web Figure 4 Differences between the empirical power and the predicted power of (a) QLS z-tests, (b) MAQLS z-tests, (c) QLS t-tests with $\text{DoF} = I - 2$, (d) MAQLS t-tests with $\text{DoF} = I - 2$, (e) QLS t-tests with $\text{DoF} = I - (T + 1)$ and (f) MAQLS t-tests with $\text{DoF} = I - (T + 1)$. MB: model-based variance; BC0: uncorrected sandwich variance; BC1: KC-corrected sandwich variance; BC2: MD-corrected sandwich variance; BC3: FG-corrected sandwich variance. The acceptable bounds are shown with the dashed horizontal lines. For each value of I , there may be multiple points with the same symbol indicating results with different combinations of design resources and correlation parameters.



Web Figure 5 Contour plots for the relative variance obtained under the proportional decay correlation model and the block exchangeable correlation model, for varying values of the proportional decay model decay parameter d and the block exchangeable model correlation parameters α_1^{BE} , α_2^{BE} . In all panels, the within-period correlation $\tau = 0.03$, the number of periods $T = 4$ and the cohort size $N = 100$. The dashed thick line indicates the equality of variances.



Web Figure 6 Contour plots for the relative variance obtained under the proportional decay correlation model and the block exchangeable correlation model, for varying values of the proportional decay model decay parameter d and the block exchangeable correlation parameters α_1^{BE} , α_2^{BE} . In all panels, the within-period correlation $\tau = 0.03$, the number of periods $T = 8$ and the cohort size $N = 20$. The dashed thick line indicates the equality of variances.



Web Figure 7 Contour plots for the relative variance obtained under the proportional decay correlation model and the block exchangeable correlation model, for varying values of the proportional decay model decay parameter d and the block exchangeable correlation parameters α_1^{BE} , α_2^{BE} . In all panels, the within-period correlation $\tau = 0.1$, the number of periods $T = 4$ and the cohort size $N = 20$. The dashed thick line indicates the equality of variances.

References

1. Shults J., Morrow A. L.. Use of quasi-least squares to adjust for two levels of correlation. *Biometrics*. 2002;58(3):521–530.
2. Preisser J. S., Lu B., Qaqish B. F.. Finite sample adjustments in estimating equations and covariance estimators for intracluster correlations. *Statistics in Medicine*. 2008;27(8):5764–5785.
3. Li F., Turner E. L., Preisser J. S.. Sample size determination for GEE analyses of stepped wedge cluster randomized trials. *Biometrics*. 2018;74(4):1450–1458.
4. Rochon J.. Application of GEE procedures for sample size calculations in repeated measures experiments. *Statistics in Medicine*. 1998;17(14):1643–1658.

