

# Supplemental Math Note for “Topography in the bursting dynamics of entorhinal neurons”

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Our model is a noisy leaky integrate-and-fire model, with two additional factors: each neuron has 1) an intrinsic after-spike current, and 2) an intrinsic after-spike threshold. We will first demonstrate a model reduction where these two factors can be combined into a single, effective after-spike threshold. We then demonstrate how this effective intrinsic threshold is measured through in vitro experiments, and then how this intrinsic threshold predicts firing rates and burst scores for in vivo recordings. We then compare predictions of the model with in vivo neural data collected as mice freely forage open arenas (Materials and Methods).

## I. MATHEMATICAL MODEL

Consider a neuron with some dimensionless synaptic activation  $s(t)$  and a certain input  $I(t)$ . The integrate and fire dynamics are:

$$\frac{ds}{dt} = I(t) - \omega s(t) + \eta(t) + I_{\text{Int}}(t - t_0), \quad (1)$$

where  $\omega$  is the “leaking rate” of the neuron,  $\eta(t)$  is white input noise, and  $I_{\text{Int}}(t - t_0)$  is the intrinsic post-spike signal, where  $t_0$  is the time since the last spike.

**Spiking Rule:** To add spiking dynamics to Eq. 1, we add a rule that whenever  $s(t)$  rises above the intrinsic after-spike threshold  $s_{\text{Thr}}(t - t_0)$ , the neuron fires. After firing, the synaptic activation goes down to zero. As a convention, we say that the  $s_{\text{Thr}}(\Delta t)$  becomes 1 as  $\Delta t \rightarrow \infty$ .

### A. Equivalence between recent-spike-dependent input and recent-spike-dependent threshold

Above, we have characterized the intrinsic spiking dynamics of a neuron with two separate functions: 1) The intrinsic after-spike current  $I_{\text{Int}}(t - t_0)$ , and 2) The intrinsic after-spike threshold  $s_{\text{Thr}}(t - t_0)$ . We can simplify the model by observing that the spiking dynamics, both in vivo and in vitro, depend only on a *combination* of these two functions; every modification of the intrinsic after-spike current is equivalent to some modification of the intrinsic after-spike threshold, and vice versa.

Solving Eq. 1, we can show that at any given time, the synaptic activation will be:

$$s(t) = \int_{t'=t_0}^t e^{-\omega(t-t')} (I(t') + \eta(t') + I_{\text{Int}}(t' - t_0)).$$

We can divide the synaptic activation into two components, one which depends on the refractory input, and one which depends only on the base signal plus noise:

$$s(t) = s_{\text{Int}}(t - t_0) + \int_{t'=t_0}^t e^{-\omega(t-t')} (I(t') + \eta(t')), \quad \text{where } s_{\text{Int}}(t - t_0) = \int_{t'=0}^{t-t_0} e^{-\omega(t-t_0-t')} I_{\text{Int}}(t').$$

The distance between the synaptic activation and the threshold is:

$$s_{\text{Thr}}(t - t_0) - s(t) = \underbrace{(s_{\text{Thr}}(t - t_0) - s_{\text{Int}}(t - t_0))}_{\text{Intrinsic threshold and current}} - \underbrace{\int_{t'=t_0}^t (e^{-\omega(t-t')} (I(t') + \eta(t')))}_{\text{Extrinsic input and noise}}.$$

Therefore, the firing rate and interspike interval distribution depend only on the *combination* of the input and threshold of the firing rule  $s_{\text{Thr}}(t - t_0) - s_{\text{Int}}(t - t_0)$  (Table I). Because of this, it is mathematically convenient to combine these two effecting into an *effective* threshold function  $s_{\text{Thr}}^{\text{Eff}}(t - t_0) = s_{\text{Thr}}(t - t_0) - s_{\text{Int}}(t - t_0)$ .

Convention	Synaptic Input	Spiking Rule
Synaptic Input	$\frac{ds}{dt} = -\omega s + I(t) + \eta(t) + I_{\text{Int}}(t - t_0)$	Spike if $s(t) > s_{\text{Thr}}(t - t_0)$
Spike Threshold	$\frac{ds}{dt} = -\omega s + I(t) + \eta(t)$	Spike if $s(t) > s_{\text{Thr}}^{\text{Eff}}(t - t_0)$

TABLE I: Two sets of dynamics. In the first set of dynamics, there are both intrinsic after-spike dynamics as well as an intrinsic after-spike current. In the second, the intrinsic after-spike dynamics are rolled into an effective intrinsic after-spike threshold. When  $s_{\text{Thr}}^{\text{Eff}}(t - t_0) = s_{\text{Thr}}(t - t_0) - s_{\text{Int}}(t - t_0)$  these two sets of dynamics are equivalent.

## II. MODEL APPLIED TO IN VITRO EXPERIMENTS

The in vitro experiments consist of two inputs 1) A pulse of magnitude  $I_0$  at  $t = 0$ , where  $I_0$  is chosen to be the minimum size to elicit a response. 2) A second, scaled pulse of magnitude  $\gamma I_0$  after a delay of  $\Delta t$ . Other than that, there is no input signal or noise. Choosing a convention where the threshold activation at  $t = 0$  is 1, Eq. 1 reduces to:

$$\frac{ds}{dt} = \underbrace{\delta(t - 0) + \gamma \delta(t - \Delta t)}_{I(t)} - \omega s(t). \quad (2)$$

Where  $\delta(t)$  is the delta function (impulse function), which is infinitely thin, infinitely tall, and has an area under the curve of 1 at  $t = 0$ . We can solve for Eq. 2 to show that a second spike is elicited when  $\gamma > s_{\text{Thr}}^{\text{Eff}}(\Delta t)$ . Therefore, the experimentally normalized second spike threshold, i.e. the *minimum*  $\gamma$  required to elicit a second spike at  $\Delta t$ , is *exactly* the the model's threshold function  $s_{\text{Thr}}^{\text{Eff}}(\Delta t)$  (Fig. 1).

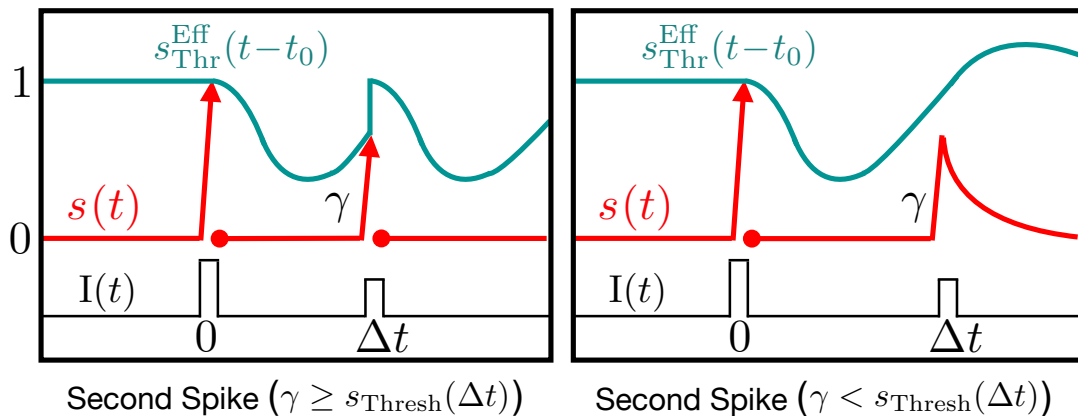


FIG. 1: Schematic of mathematical model applied to in vitro experiments (Eq. 2). The current profile  $I(t)$  consists of two pulses separated by time  $\Delta t$  and with ratio  $\gamma$ , which effects the synaptic activation  $s(t)$  (red). The first pulse is set to barely make the neuron spike, and then  $\gamma$  and  $\Delta t$  are varied. When the second impulse pushes the synaptic activation above the spiking threshold  $s_{\text{Thr}}^{\text{Eff}}(t - t_0)$  (green), a second spike is elicited; otherwise, it is not. Therefore, the experimentally measured threshold function  $\gamma_{\text{Thr}}(\Delta t)$  is the *exactly* the model's effective intrinsic post-spike threshold  $s_{\text{Thr}}^{\text{Eff}}(\Delta t)$ .

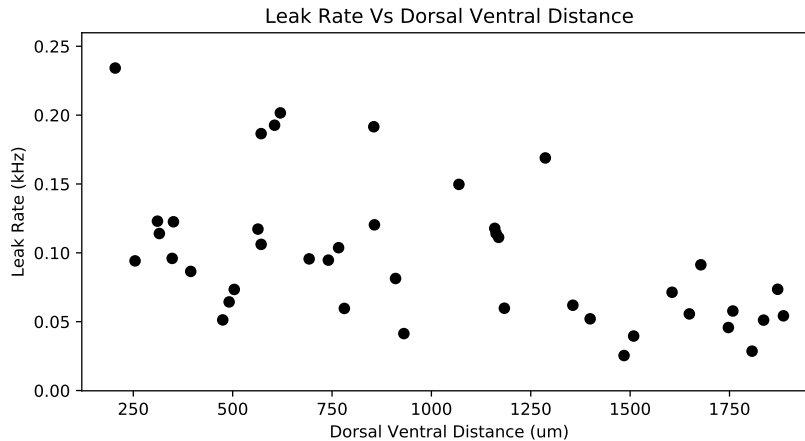


FIG. 2: Leak rate vs. dorsoventral distance for in vitro recorded cells.

### III. MODEL APPLIED TO IN VITRO EXPERIMENTS

Because the timescale at which an animal changes its speed, direction, or position is much slower than that of the average interspike interval, we can consider the average input signal  $I(t)$  to be *independent* of time. The dynamics of Eq. 1 reduce to

$$\frac{ds}{dt} = I - \omega s(t) + \eta(t) \quad (3)$$

with the same spiking threshold of  $s_{\text{Thr}}^{\text{Eff}}(t - t_0)$ . We simulated Eq. 3 combined with experimentally measured values of  $\omega$  (Fig. 2, measured using the decay of sub-threshold responses), along with the spiking rule using experimentally measured  $s_{\text{Thr}}^{\text{Eff}}(\Delta t)$  to yield both a mean firing rate as well as a burst score as a function of the mean input signal  $I$ ; because we do not know the input signal for in vivo recordings, we instead plot the burst score as a function of mean firing rate.

Fitting the magnitude of noise to be  $\sigma_{\eta}^2 = .02/\text{ms}$  [1], we simulate Eq. 3 for each individual neuron recorded in vivo, and find a qualitative agreement between experiment and simulations about the relative burst scores of dorsal and ventral neurons. We have not considered the effects of variable noise and input statistics or any higher-order network effects in our model; while these will change the burst-firing curve, the dominant effect can be explained by NSST and leak rate alone.

Performing a Pearson test of burst slope vs. dorsal ventral distance, we find that dorsal cells are predicted by the model to have a significantly higher bursting score than ventral cells ( $R = -.35$ ,  $P = .026$ ).

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[1] Magnitude of white noise has units of synaptic activation squared/time, but here we have chosen a dimensionless synaptic activation.

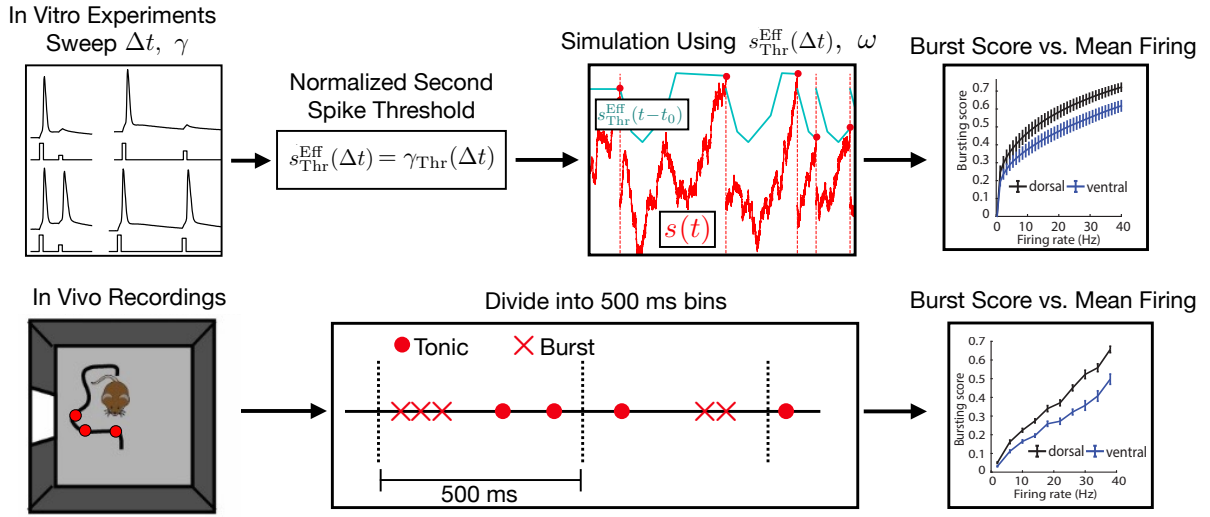


FIG. 3: **Top**) We performed delayed pulse experiments along the dorsal ventral axis. We then used experimentally recorded normalized second spike thresholds (NSSTs) and decay rates to calculate the intrinsic post spike thresholds  $s_{\text{Thr}}^{\text{Eff}}(\Delta t)$ . These values were then used to model the effect of NSST and decay rate on bursting by simulating Eq. 1 with the spiking rule for a range of mean inputs (simulated  $s(t)$  in red, spikes visualized as red dots, spiking threshold  $s_{\text{Thr}}^{\text{Eff}}(t-t_0)$  in green). Using these simulation results we plot burst score vs. mean firing rate. **Bottom**) To compare our simulations to cells recorded in vivo, we took the firing rate data and divide it into 500ms bins; for each bin we calculated the mean firing rate and burst score, and then grouped these bins to plot burst score vs. mean firing rate.