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Supplementary information to :

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Carbon and phosphorus allocation in annual plants: an optimal functioning approach

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1 Supplementary Tables

Supplementary Table 1: Calibrated values for the maize field trial. Parameter k_{CR} is given for each of the 12 P experiments (3 P levels x 4 blocks) and all others kept the same. Parameter description can be found in Tables 1 and 2.

| Parameter | Initial guess | Calibrated values \pm std. error |
|----------------|---------------|------------------------------------|
| k_{CL} | 1.5 | 1.7137 ± 0.1106 |
| $k_{CR,1}$ | 1.0 | 0.9101 ± 0.1017 |
| $k_{CR,2}$ | 1.0 | 0.8302 ± 0.0922 |
| $k_{CR,3}$ | 1.0 | 0.8545 ± 0.0925 |
| $k_{CR,4}$ | 1.0 | 0.8376 ± 0.0969 |
| $k_{CR,5}$ | 1.0 | 1.4577 ± 0.1673 |
| $k_{CR,6}$ | 1.0 | 1.4526 ± 0.1529 |
| $k_{CR,7}$ | 1.0 | 1.1963 ± 0.1111 |
| $k_{CR,8}$ | 1.0 | 1.4287 ± 0.1573 |
| $k_{CR,9}$ | 1.0 | 1.7878 ± 0.2064 |
| $k_{CR,10}$ | 1.0 | 1.8376 ± 0.1784 |
| $k_{CR,11}$ | 1.0 | 1.7906 ± 0.1966 |
| $k_{CR,12}$ | 1.0 | 1.8377 ± 0.1859 |
| $C_{L,max}$ | 1.5 | 0.9042 ± 0.0862 |
| $\rho_{L,max}$ | 1.0 | 1.0927 ± 0.0904 |
| λ_{CR} | 0.30 | 0.1362 ± 0.0291 |
| λ_{CL} | 0.10 | 0.1608 ± 0.0141 |
| λ_{CS} | 0.03 | 0.0117 ± 0.0020 |
| λ_{CG} | 0.01 | 0.0153 ± 0.0048 |
| f_{CS} | 0.5 | 0.4036 ± 0.0495 |
| f_{CG} | 1.0 | 2.3523 ± 0.2097 |
| f_{PS} | 0.8 | 1.5956 ± 0.0660 |
| f_{PR} | 0.1 | 0.0399 ± 0.0850 |
| k_{mPL} | 0.05 | 0.0584 ± 0.0081 |
| k_{mPR} | 0.05 | 0.2541 ± 0.1193 |
| k_{mPS} | 0.05 | 0.0686 ± 0.0096 |
| $C_{L,0}$ | 0.1 | 0.0064 ± 0.0250 |
| $P_{L,0}$ | 0.1 | 0.0298 ± 0.0180 |

Supplementary Table 2: Calibrated parameter values for the hydroponic studies. Parameter k_{CR} is given for each of the five P experiments and all others kept the same for each species (Asher and Loneragan, 1967; Fageria and Baligar, 1989). Parameter description can be found in Tables 1 and 2.

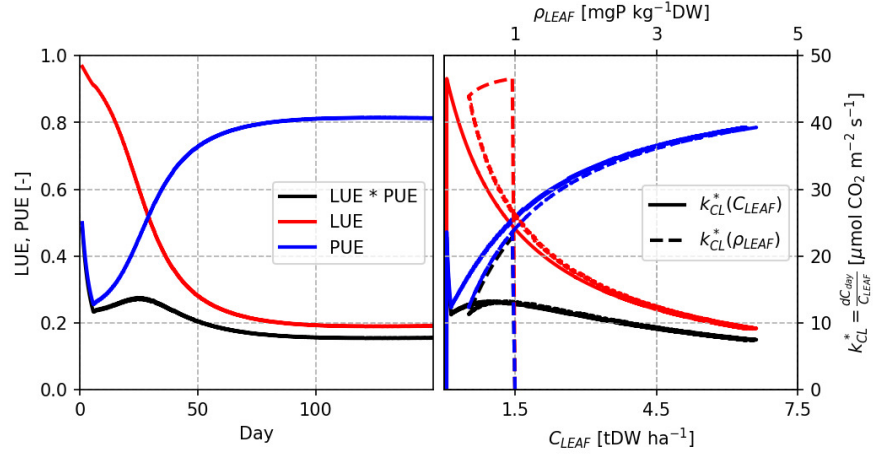
| Variable | k_{CL} | $C_{L,max}$ | $\rho_{L,max}$ | λ_{CR} | λ_{CL} | $C_{L,0}$ |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Initial guess | 1.5 | 1.0 | 1.5 | 0.3 | 0.1 | 0.1 |
| Barrel medic | 1.2217 ± 1.2479 | 1.4816 ± 0.9336 | 0.8967 ± 0.2541 | 0.3261 ± 0.4065 | 0.1195 ± 0.1727 | 0.0441 ± 0.2109 |
| Brome grass | 1.8221 ± 0.5271 | 0.4289 ± 0.0644 | 1.2571 ± 0.2453 | 0.4145 ± 0.1946 | 0.1187 ± 0.0651 | 0.1211 ± 0.4818 |
| Capeweed | 1.2570 ± 0.5783 | 1.6330 ± 1.1550 | 0.9148 ± 0.4691 | 0.2489 ± 1.2535 | 0.1224 ± 0.4597 | 0.0524 ± 0.8952 |
| Clover | 1.7286 ± 0.3377 | 0.4744 ± 0.1035 | 1.1299 ± 0.2715 | 0.4721 ± 0.1357 | 0.0847 ± 0.0482 | 0.1395 ± 0.1960 |
| Erodium | 2.0614 ± 2.4349 | 0.7016 ± 1.0327 | 1.1258 ± 1.3077 | 0.6225 ± 1.1166 | 0.0720 ± 0.3201 | 0.0956 ± 0.5452 |
| Flatweed | 2.0698 ± 0.6796 | 0.4879 ± 0.1347 | 2.0429 ± 0.4540 | 0.4136 ± 0.2458 | 0.1366 ± 0.1897 | 0.1208 ± 0.2399 |
| Silver grass | 1.7724 ± 0.7744 | 0.4667 ± 0.2487 | 1.4596 ± 1.0746 | 0.4734 ± 0.7083 | 0.1048 ± 0.2218 | 0.1309 ± 0.4042 |
| Alfalfa | 1.7676 ± 1.0372 | 0.4844 ± 0.1900 | 1.7205 ± 0.4770 | 0.4672 ± 0.5006 | 0.1132 ± 0.2824 | 0.1419 ± 0.4264 |
| Bean | 1.4107 ± 0.4049 | 0.3595 ± 0.0642 | 1.3110 ± 0.2805 | 0.5136 ± 0.3943 | 0.1099 ± 0.1339 | 0.1300 ± 0.3123 |
| Red clover | 1.5970 ± 1.4740 | 0.3258 ± 0.2304 | 1.9161 ± 0.7067 | 0.3124 ± 0.3857 | 0.1165 ± 0.1431 | 0.0873 ± 0.5166 |
| Rice | 1.4473 ± 0.9212 | 0.1676 ± 0.0903 | 3.7253 ± 1.1436 | 0.3411 ± 1.1679 | 0.1157 ± 0.6166 | 0.0858 ± 0.8162 |
| Wheat | 1.7524 ± 0.7460 | 0.6345 ± 0.2700 | 1.2003 ± 0.4162 | 0.3365 ± 0.3818 | 0.1091 ± 0.2223 | 0.0447 ± 0.5497 |

| Variable | $P_{L,0}$ | $k_{CR,1}$ | $k_{CR,2}$ | $k_{CR,3}$ | $k_{CR,4}$ | $k_{CR,5}$ |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Initial guess | 0.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| Barrel medic | 0.0214 ± 0.5136 | 0.1588 ± 3.3886 | 0.6731 ± 2.0856 | 0.8325 ± 1.6676 | 1.1213 ± 1.0650 | 1.2884 ± 0.7138 |
| Brome grass | 0.0723 ± 0.2866 | 0.0360 ± 0.1446 | 0.2408 ± 0.2552 | 1.1367 ± 0.3647 | 1.3340 ± 0.4073 | 1.8140 ± 0.3765 |
| Capeweed | 0.0609 ± 0.3320 | 0.2978 ± 0.4454 | 0.4815 ± 0.5419 | 0.5965 ± 0.6870 | 0.8538 ± 0.8076 | 1.1816 ± 0.9310 |
| Clover | 0.1203 ± 0.2558 | 0.0348 ± 0.0862 | 0.2690 ± 0.1442 | 1.1347 ± 0.3132 | 2.2824 ± 0.6717 | 2.0929 ± 0.5477 |
| Erodium | 0.0415 ± 1.6314 | 0.0215 ± 1.1265 | 0.3436 ± 2.0639 | 0.7719 ± 2.3206 | 1.6700 ± 2.8556 | 1.8104 ± 3.0153 |
| Flatweed | 0.0961 ± 0.4203 | 0.0782 ± 0.1879 | 0.3508 ± 0.1543 | 0.9953 ± 0.1750 | 1.6565 ± 0.4316 | 1.4294 ± 0.2642 |
| Silver grass | 0.1066 ± 1.3494 | 0.0809 ± 0.5959 | 0.3281 ± 0.6141 | 0.7678 ± 0.4249 | 1.8235 ± 0.6355 | 1.7826 ± 0.8745 |
| Alfalfa | 0.0713 ± 0.8038 | 0.0259 ± 0.4894 | 0.2313 ± 0.4821 | 0.9014 ± 0.3704 | 1.5848 ± 0.3303 | 2.1503 ± 0.6900 |
| Bean | 0.0464 ± 0.1495 | 0.0630 ± 0.1667 | 0.5332 ± 0.2332 | 0.8130 ± 0.2835 | 1.2538 ± 0.3362 | 1.4391 ± 0.3720 |
| Red clover | 0.0719 ± 0.7136 | 0.1330 ± 0.5427 | 0.4411 ± 0.5719 | 0.6859 ± 0.4559 | 0.8600 ± 0.5091 | 1.0676 ± 0.4154 |
| Rice | 0.0468 ± 0.4628 | 0.0548 ± 0.8097 | 0.4648 ± 0.5638 | 1.4896 ± 0.9318 | 1.2552 ± 0.7613 | 1.2643 ± 0.8733 |
| Wheat | 0.0440 ± 0.3336 | 0.1627 ± 0.4041 | 0.2031 ± 0.4443 | 0.5298 ± 0.6615 | 0.8987 ± 1.0080 | 1.0202 ± 0.9067 |

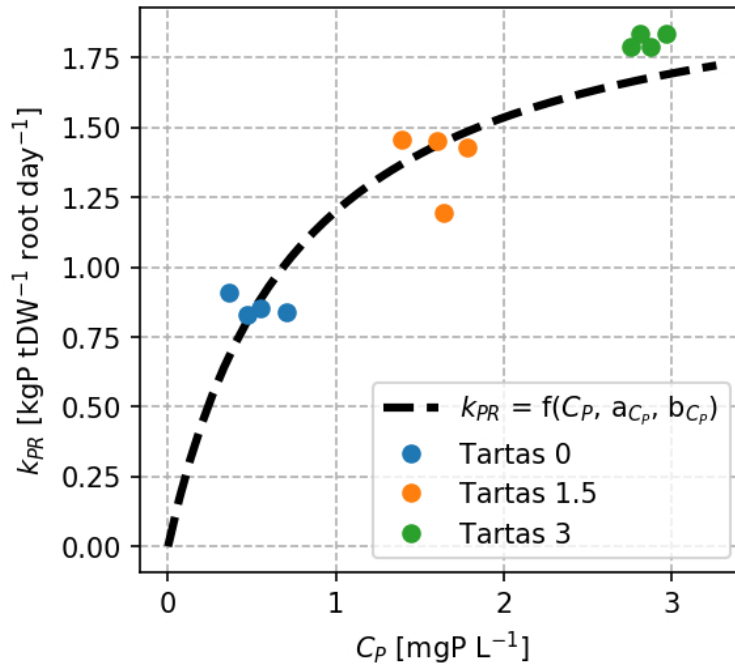
Supplementary Table 3: Calibrated parameter values for the Michaelis-Menten kinetic (Eq. 15). a_{CP} unit is $\text{kgP t}^{-1}\text{DW root ha}^{-1} \text{ day}^{-1}$ in the field maize trial, and $\text{mgP g}^{-1}\text{DW root pot}^{-1} \text{ day}^{-1}$ in the hydroponic studies. b_{CP} unit is mgP L^{-1} in the field maize trial, and $\mu\text{P L}^{-1}$ in the hydroponic studies.

| Scale | Species | a_{CP} | b_{CP} |
|------------|--------------|-----------------|--------------------|
| Field | Maize | 2.13 ± 0.12 | 0.78 ± 0.13 |
| Hydroponic | Barrel medic | 1.72 ± 0.06 | 0.74 ± 0.13 |
| | Brome grass | 2.39 ± 0.09 | 0.88 ± 0.18 |
| | Capeweed | 1.99 ± 0.04 | 1.31 ± 0.11 |
| | Clover | 1.63 ± 0.05 | 0.51 ± 0.12 |
| | Erodium | 2.03 ± 0.07 | 1.21 ± 0.20 |
| | Flatweed | 2.22 ± 0.04 | 1.68 ± 0.11 |
| | Silver grass | 1.49 ± 0.09 | 0.23 ± 0.07 |
| Hydroponic | Alfalfa | 1.46 ± 0.04 | 64.78 ± 5.77 |
| | Bean | 1.24 ± 0.17 | 67.24 ± 30.39 |
| | Red clover | 1.95 ± 0.06 | 126.51 ± 10.55 |
| | Rice | 1.25 ± 0.05 | 80.83 ± 9.96 |
| | Wheat | 1.72 ± 0.27 | 227.72 ± 73.89 |

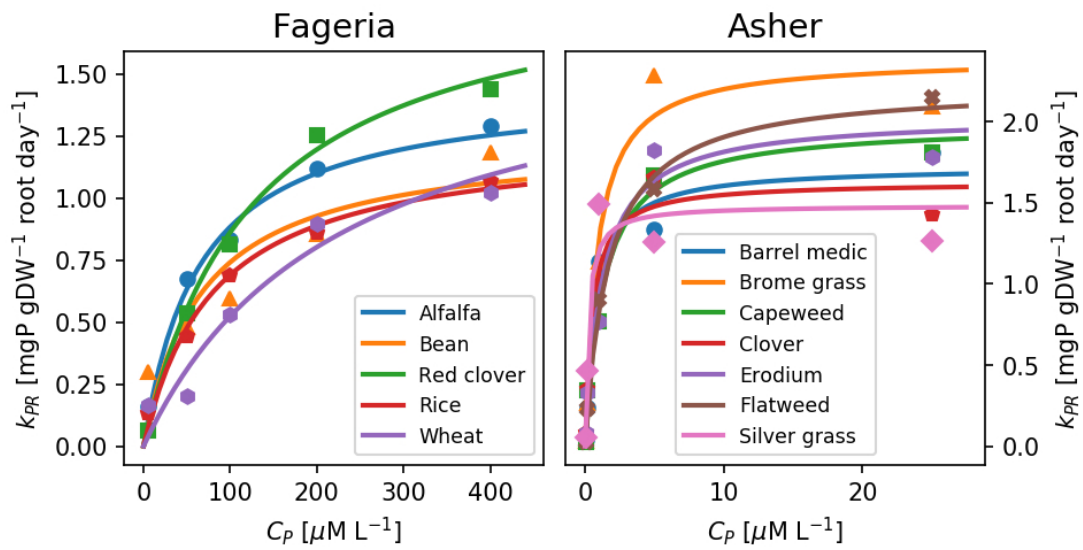
1 2 Supplementary Figures



Supplementary Figure 1: Time evolution of leaf LUE and PUE (left) and the effective assimilation rate (k_{CL}^* , right) as function of leaf biomass and concentration. Root uptake rate k_{CR} is set to $1.0 \text{ kgP tDW}^{-1} \text{ root ha}^{-1} \text{ day}^{-1}$.



Supplementary Figure 2: Predicted field trial root P uptake (k_{CR}) vs. measured soil solution P concentration. Different markers and colors depict different levels of P addition. Dark dashed line depicts a Michaelis-Menten kinetic (similar to Eq. 15) which best describes the relationship between the two.



Supplementary Figure 3: Predicted hydroponic studies root P uptake (k_{CR}) vs. measured solution P concentration. Different markers and colors depict different levels of P addition. Dark dashed line depicts a Michaelis-Menten kinetic (similar to Eq. 15) which best describes the relationship between the two.