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Supplemental Information

Model Plasma Membrane Exhibits a Microemulsion in Both Leaves Providing a Foundation for "Rafts"

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Supplementary Material for

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We write the fluctuation part of the free energy, given by Eqs (39) and (40) of the paper in the form

$$\frac{\delta F}{k_BT} = \frac{A^2}{(2\pi)^2 a^2} \int d^2p\ \phi_i^T(p) \cdot M_{i,j} \cdot \phi_j(p), \tag{1}$$

where $M_{i,j} = M_{j,i}$ and $p \equiv a^{1/2}k$.

We recall the expressions for the cholesterol-dependent spontaneous curvature of the outer leaf, Eq.(15),

$$\frac{\kappa_m^O H_0^O}{k_BT} = \left[\frac{y_{SM}}{y_{SM}+y_{PCo}}(a_{SM}-b_{SM}y_{Co}) + \frac{y_{PCo}}{y_{SM}+y_{PCo}}(a_{PC}-b_{PC}y_{Co})\right], \qquad 0.25 \le y_{Ci} \le 0.6 \tag{2}$$

and for the inner leaf, Eq.(16),

$$\begin{aligned}\frac{\kappa_m^I H_0^I}{k_BT} &= -\left[\frac{y_{PE}}{y_{PE}+y_{PS}+y_{PCi}}(a_{PE}-b_{PE}y_{Ci}) + \frac{y_{PS}}{y_{PE}+y_{PS}+y_{PCi}}(a_{PS}-b_{PS}y_{Ci}) +\right.\\ &+ \left.\frac{y_{PCi}}{y_{PE}+y_{PS}+y_{PCi}}(a_{PC}-b_{PC}y_{Ci})\right], \qquad 0.25 \le y_{Ci} \le 0.6\end{aligned} \tag{3}$$

To evaluate the derivatives of the H_0^O and H_0^I with respect to the various mol fractions y, we define the quantities

$$z_o = (1-\bar{y}_{Co}),$$

where the overbar denotes the equlibrium value, and

$$C^O = f_{SM}a_{SM} + f_{PCo}a_{PC} - \bar{y}_{Co}[f_{SM}b_{SM} + f_{PCo}b_{PC}] \tag{4}$$

Here f_i is the fraction of phospholipids in the monolayer of type i, that is

$$f_{SM} = \frac{\bar{y}_{SM}}{\bar{y}_{SM} + \bar{y}_{PCo}} \qquad f_{PCo} = \frac{\bar{y}_{PCo}}{\bar{y}_{SM} + \bar{y}_{PCo}}$$

$$\begin{aligned}
C_{SM} &= (a_{SM} - a_{PC}) - (\bar{y}_{Co})[b_{SM} - b_{PC}] \\
C_{Co} &= f_{SM}Q_{SM} - z_o b_{PC} \qquad \text{where} \\
Q_{SM} &= (a_{SM} - b_{SM}) - (a_{PC} - b_{PC}) \\
C_{SM-C} &= Q_{SM} \\
C_{Co-Co} &= (f_{SM})Q_{SM}.
\end{aligned}$$

$$z_i = (1 - \bar{y}_{Ci})$$

$$C^I = f_{PE}a_{PE} + f_{PS}a_{PS} + f_{PCi}a_{PC} - \bar{y}_{Ci}[f_{PE}b_{PE} + f_{PS}b_{PS} + f_{PCi}b_{PC}]$$

$$\begin{aligned}
C_{PE} &= (a_{PE} - a_{PC}) - (\bar{y}_{Ci})[b_{PE} - b_{PC}] \\
C_{PS} &= (a_{PS} - a_{PC}) - (\bar{y}_{Ci})[b_{PS} - b_{PC}] \\
C_{Ci} &= f_{PE}Q_{PE} + f_{PS}Q_{PS} - z_i b_{PC} \\
Q_{PE} &= (a_{PE} - b_{PE}) - (a_{PC} - b_{PC}) \\
Q_{PS} &= (a_{PS} - b_{PS}) - (a_{PC} - b_{PC}) \\
C_{PE-C} &= Q_{PE} \\
C_{PS-C} &= Q_{PE}
\end{aligned}$$

$$C_{Ci-Ci} = (f_{PE})Q_{PE} + f_{PS}Q_{PS}$$

With the above definitions, and Eqs. (5) and (6) of the text for $\rho(y_C)$, the matrix elements are as follows:

$$\begin{aligned} M_{11} \equiv M_{SM,SM} &= a\rho(\bar{y}_{Co})\left[\frac{1}{2\bar{y}_{PCo}} + \frac{1}{2\bar{y}_{SM}} - 6\frac{\epsilon_{SM,PC}}{k_BT}\right] \\ &+ \frac{b}{k_BT}p^2 + \left(\frac{k_BT}{\kappa_b}\right)a\frac{C_{SM}^2}{z_o^2} \\ &- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\left(\frac{C_{SM}}{z_0}\right)^2 \end{aligned} \quad (5)$$

$$\begin{aligned} M_{12} \equiv M_{SM,Co} &= a\rho(\bar{y}_{Co})\left[\frac{1}{2\bar{y}_{PCo}} + 3\frac{\epsilon_{SM,C}}{k_BT} - 3\frac{\epsilon_{SM,PC}}{k_BT} - 3\frac{\epsilon_{PC,C}}{k_BT}\right] \\ &- a^2(1-r_a)\rho^2(\bar{y}_{Co})\left[(3\frac{\epsilon_{SM,PC}}{k_BT}\bar{y}_{SM} + 3\frac{\epsilon_{PC,C}}{k_BT}\bar{y}_{Co}) + \frac{1}{2}(\ln\bar{y}_{PCo}+1)\right] \\ &+ a^2(1-r_a)\rho^2(\bar{y}_{Co})\left[(3\frac{\epsilon_{SM,PC}}{k_BT}\bar{y}_{PCo} + 3\frac{\epsilon_{SM,C}}{k_BT}\bar{y}_{Co}) + \frac{1}{2}(\ln\bar{y}_{SM}+1)\right] \\ &+ \frac{b}{2k_BT}p^2 + \left(\frac{k_BT}{\kappa_b}\right)a\frac{C^OC_{SM-C} + C_{SM}C_{Co}}{z_o^2} \\ &- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{SM}C_{Co}}{z_0^2} \end{aligned} \quad (6)$$

$$M_{13} \equiv M_{SM,PE} = +\left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{SM}C_{PE}}{z_oz_i}. \quad (7)$$

$$M_{14} \equiv M_{SM,PS} = +\left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{SM}C_{PS}}{z_oz_i}. \quad (8)$$

$$M_{15} \equiv M_{SM,Ci} = +\left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{SM}C_{Ci}}{z_oz_i}. \quad (9)$$

$$
\begin{aligned}
M_{22} \equiv M_{Co,Co} &= a\rho(\bar{y}_{Co})\left[\frac{1}{2\bar{y}_{PCo}} + \frac{1}{2\bar{y}_{Co}} - 6\frac{\epsilon_{PC,C}}{k_BT}\right] \\
&- a^2(1-r_a)\rho^2(\bar{y}_{Co})[(6\frac{\epsilon_{SM,PC}}{k_BT}\bar{y}_{SM} + 6\frac{\epsilon_{PC,C}}{k_BT}\bar{y}_{Co}) + (\ln\bar{y}_{PCo}+1)] \\
&+ a^2(1-r_a)\rho^2(\bar{y}_{Co})[(6\frac{\epsilon_{SM,C}}{k_BT}\bar{y}_{SM} + 6\frac{\epsilon_{PC,C}}{k_BT}\bar{y}_{PCo}) + (\ln\bar{y}_{Co}+1)] \\
&+ a^3(1-r_a)^2\rho^3(\bar{y}_{Co})\left[\frac{f^O_{int}(\{\bar{y}_i\})}{k_BT} + \frac{f^O_{ent}(\{\bar{y}_i\})}{k_BT}\right] \\
&+ \frac{b}{k_BT}p^2 + \left(\frac{k_BT}{\kappa_b}\right)a\frac{2C^OC_{Co-Co} + (C_{Co})^2}{z_o^2} \\
&- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\left(\frac{C_{co}}{z_0}\right)^2. \qquad (10)
\end{aligned}
$$

$$
M_{23} \equiv M_{Co,PE} = +\left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{Co}C_{PE}}{z_oz_i}. \qquad (11)
$$

$$
M_{24} \equiv M_{Co,PS} = +\left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{Co}C_{PS}}{z_oz_i}. \qquad (12)
$$

$$
M_{25} \equiv M_{Co,Ci} = +\left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{Co}C_{Ci}}{z_oz_i}. \qquad (13)
$$

$$
\begin{aligned}
M_{33} \equiv M_{PE,PE} &= a\rho(\bar{y}_{Ci})\left[\frac{1}{2\bar{y}_{PE}} + \frac{1}{2\bar{y}_{PCi}} - 6\frac{\epsilon_{PE,PC}}{k_BT}\right] \\
&+ \frac{b}{k_BT}p^2 + \left(\frac{k_BT}{\kappa_b}\right)a\frac{(C_{PE})^2}{z_i^2} \\
&- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\left(\frac{C_{PE}}{z_i}\right)^2. \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
M_{34} \equiv M_{PE,PS} &= a\rho(\bar{y}_{Ci})\left[3\frac{\epsilon_{PS,PE}}{k_BT} - 3\frac{\epsilon_{PE,PC}}{k_BT} - 3\frac{\epsilon_{PS,PC}}{k_BT} + \frac{1}{2\bar{y}_{PCi}}\right] \\
&+ \frac{b}{2k_BT}p^2 + \left(\frac{k_BT}{\kappa_b}\right)a\frac{C_{PE}C_{PS}}{z_i^2}
\end{aligned}
$$

$$- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{PE}C_{PS}}{z_i^2}. \qquad (15)$$

$$\begin{aligned}
M_{35} \equiv M_{PE,Ci} &= a^2(1-r_a)\rho^2(\bar{y}_{Ci})\left[(3\frac{\epsilon_{PE,PC}}{k_BT}\bar{y}_{PCi}+3\frac{\epsilon_{PE,PS}}{k_BT}\bar{y}_{PS}+3\frac{\epsilon_{PE,C}}{k_BT}\bar{y}_{Ci})+\frac{1}{2}(\ln\bar{y}_{PE}+1)\right] \\
&- a^2(1-r_a)\rho^2(\bar{y}_{Ci})\left[(3\frac{\epsilon_{PE,PC}}{k_BT}\bar{y}_{PE}+3\frac{\epsilon_{PS,PC}}{k_BT}\bar{y}_{PS}+3\frac{\epsilon_{PC,C}}{k_BT}\bar{y}_{Ci})+\frac{1}{2}(\ln\bar{y}_{PCi}+1)\right] \\
&+ a\rho(\bar{y}_{Ci})\left[3\frac{\epsilon_{PE,C}}{k_BT}-3\frac{\epsilon_{PC,C}}{k_BT}-3\frac{\epsilon_{PE,PC}}{k_BT}+\frac{1}{2\bar{y}_{PCi}}\right] \\
&+ \frac{b}{2k_BT}p^2+\left(\frac{k_BT}{\kappa_b}\right)a\frac{[C^IC_{PE-C}+C_{PE}C_{Ci}]}{z_i^2} \\
&- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{PE}C_{Ci}}{z_i^2}. \qquad (16)
\end{aligned}$$

$$\begin{aligned}
M_{44} \equiv M_{PS,PS} &= a\rho(\bar{y}_{Ci})\left[\frac{1}{2\bar{y}_{PS}}+\frac{1}{2\bar{y}_{PCi}}-6\frac{\epsilon_{PS,PC}}{k_BT}\right] \\
&+ \frac{b}{k_BT}p^2+\left(\frac{k_BT}{\kappa_b}\right)a\frac{(C_{PS})^2}{z_i^2} \\
&- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\left(\frac{C_{PS}}{z_i}\right)^2. \qquad (17)
\end{aligned}$$

$$\begin{aligned}
M_{45} \equiv M_{PS,Ci} &= a^2(1-r_a)\rho^2(\bar{y}_{Ci})\left[(3\frac{\epsilon_{PS,PC}}{k_BT}\bar{y}_{PCi}+3\frac{\epsilon_{PE,PS}}{k_BT}\bar{y}_{PE}+3\frac{\epsilon_{PS,C}}{k_BT}\bar{y}_{Ci})+\frac{1}{2}(\ln\bar{y}_{PS}+1)\right] \\
&- a^2(1-r_a)\rho^2(\bar{y}_{Ci})\left[(3\frac{\epsilon_{PS,PC}}{k_BT}\bar{y}_{PS}+3\frac{\epsilon_{PE,PC}}{k_BT}\bar{y}_{PE}+3\frac{\epsilon_{PC,C}}{k_BT}\bar{y}_{Ci})+\frac{1}{2}(\ln\bar{y}_{PCi}+1)\right] \\
&+ a\rho(\bar{y}_{Ci})\left[3\frac{\epsilon_{PS,C}}{k_BT}-3\frac{\epsilon_{PC,C}}{k_BT}-3\frac{\epsilon_{PS,PC}}{k_BT}+\frac{1}{2\bar{y}_{PCi}}\right] \\
&+ \frac{b}{2k_BT}p^2+\left(\frac{k_BT}{\kappa_b}\right)a\frac{[C^IC_{PS-C}+C_{PS}C_{Ci}]}{z_i^2} \\
&- \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\frac{C_{PS}C_{Ci}}{z_i^2}. \qquad (18)
\end{aligned}$$

$$M_{55} \equiv M_{Ci,Ci} = a^3(1-r_a)^2\rho^3(\bar{y}_{Ci})\left[\frac{f^I_{int}(\{\bar{y}\}_i)}{k_BT}+\frac{f^I_{ent}(\{\bar{y}\}_i)}{k_BT}\right]$$

$$
\begin{aligned}
+ \quad & a^2(1-r_a)\rho^2(\bar{y}_{Ci})\left[(6\frac{\epsilon_{PE,C}}{k_BT}\bar{y}_{PE} + 6\frac{\epsilon_{PS,C}}{k_BT}\bar{y}_{PS} + 6|frac\epsilon_{PC,C}k_BT\bar{y}_{PCi}) + (\ln\bar{y}_{Ci}+1)\right] \\
- \quad & a^2(1-r_a)\rho^2(\bar{y}_{Ci})\left[(6\frac{\epsilon_{PE,PC}}{k_BT}\bar{y}_{PE} + 6\frac{\epsilon_{PS,PC}}{k_BT}\bar{y}_{PS} + 6\frac{\epsilon_{PC,C}}{k_BT}\bar{y}_{Ci}) + (\ln\bar{y}_{PCi}+1)\right] \\
+ \quad & a\rho(\bar{y}_{Ci})\left[\frac{1}{2\bar{y}_{Ci}} + \frac{1}{2\bar{y}_{PCi}} - 3\frac{\epsilon_{PC,C}}{k_BT}\right] \\
+ \quad & \frac{b}{k_BT}p^2 + \left(\frac{k_BT}{\kappa_b}\right)a\frac{[2C^I C_{Ci-Ci} + (C_{Ci})^2]}{z_i^2} \\
- \quad & \left(\frac{k_BT}{\kappa_b}\right)\frac{p^2}{2((\sigma a/\kappa_b)+p^2)}a\left(\frac{C_{Ci}}{z_i}\right)^2 \qquad (19)
\end{aligned}
$$