S1 Appendix - Interpretation of the test Δ^m_{DD-DA}

Let us re-parametrize model $\mathbf{M_3}$ so that

$$\beta^m_{ijk} = \alpha_i + \gamma_j + \delta_k + (\alpha\gamma)_{ij} + (\alpha\delta)_{ik} + (\gamma\delta)_{jk} + (\alpha\gamma\delta)_{ijk}$$

where:

- α_i : effect of genotypic allele i with $i \in \{0,1\}$

- γ_j : effect of allele ancestry j with $j \in \{D,F\}$

- δ_k : effect of genetic background k with $k \in \{D,A,F\}$

- $(\alpha\gamma)_{ij}$: effect of the "genotypic allele - allele ancestry" interaction

- $(\alpha\delta)_{ik}$: effect of the "genotypic allele - genetic background" interaction

- $(\gamma\delta)_{jk}$: effect of the "allele ancestry - genetic background" interaction

- $(\alpha\gamma\delta)_{ijk}$: effect of the "genotypic allele - allele ancestry - genetic background" interaction

Let us write the test Δ^m_{DD-DA} according to the new parametrization:

$$
\begin{aligned}
\Delta^m_{DD-DA} =\ & (\beta^m_{1DD} - \beta^m_{0DD}) - (\beta^m_{1DA} - \beta^m_{0DA}) \\
\Delta^m_{DD-DA} =\ & (\alpha_1 + \gamma_D + \delta_D + (\alpha\gamma)_{1D} + (\alpha\delta)_{1D} + (\gamma\delta)_{DD} + (\alpha\gamma\delta)_{1DD} \\
& -\alpha_0 - \gamma_D - \delta_D - (\alpha\gamma)_{0D} - (\alpha\delta)_{0D} - (\gamma\delta)_{DD} - (\alpha\gamma\delta)_{0DD}) \\
& - (\alpha_1 + \gamma_D + \delta_A + (\alpha\gamma)_{1D} + (\alpha\delta)_{1A} + (\gamma\delta)_{DA} + (\alpha\gamma\delta)_{1DD} \\
& -\alpha_0 - \gamma_D - \delta_A - (\alpha\gamma)_{0D} - (\alpha\delta)_{0A} - (\gamma\delta)_{DA} - (\alpha\gamma\delta)_{0DA}) \\
\Delta^m_{DD-DA} =\ & ((\alpha\delta)_{1D} - (\alpha\delta)_{0D}) - ((\alpha\delta)_{1A} - (\alpha\delta)_{0A}) \\
& + ((\alpha\gamma\delta)_{1DD} - (\alpha\gamma\delta)_{0DD}) - ((\alpha\gamma\delta)_{1DA} - (\alpha\gamma\delta)_{0DA})
\end{aligned}
$$

In this last expression, all terms involve an interaction between the genotypic allele and the genotypic background.