**Supporting Information.** Luimstra, V. M., J. M. H. Verspagen, T. Xu, J. M. Schuurmans, and J. Huisman. 2020. Changes in water color shift competition between phytoplankton species with contrasting light-harvesting strategies. *Ecology.*

**Appendix S1**

## Section 1: Stability analysis

### The model

We consider two species competing for two colors of light, blue (*b*) and red (*r*). In this case, the competition model of Eq. 6 can be written as:

$$\frac{dC_1}{dt} = \left(f_{1b}\left(I_{out,b}\right) + f_{1r}\left(I_{out,r}\right) - m_1\right)C_1$$
$$\frac{dC_2}{dt} = \left(f_{2b}\left(I_{out,b}\right) + f_{2r}\left(I_{out,r}\right) - m_2\right)C_2 \tag{S1}$$

where the growth rates of species *i* on blue and red light are given by

$$f_{ib}(I_{out,b}) = \phi_{ib} k_{ib} \left(\frac{I_{in,b} - I_{out,b}}{\ln\left(I_{in,b}\right) - \ln(I_{out,b})}\right)$$

and

$$f_{ir}(I_{out,r}) = \phi_{ir} k_{ir} \left(\frac{I_{in,r} - I_{out,r}}{\ln\left(I_{in,r}\right) - \ln(I_{out,r})}\right) \tag{S2}$$

### Stability analysis

The local stability of the coexistence equilibrium is investigated by analyzing the Jacobian matrix of the system (e.g., Edelstein-Keshet 1988, Otto and Day 2007). The Jacobian matrix is given by

$$\boldsymbol{J} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial (dC_1/dt)^*}{\partial C_1} & \frac{\partial (dC_1/dt)^*}{\partial C_2} \\ \frac{\partial (dC_2/dt)^*}{\partial C_1} & \frac{\partial (dC_2/dt)^*}{\partial C_2} \end{pmatrix} \tag{S3}$$

where the superscript * indicates that the matrix is to be evaluated at the coexistence equilibrium. The coexistence equilibrium is stable if and only if:

$$trace(\boldsymbol{J}) = A_{11} + A_{22} < 0 \tag{S4}$$

$$det(\boldsymbol{J}) = A_{11}A_{22} - A_{12}A_{21} > 0$$

We note that, at the coexistence equilibrium, $f_{ib}(I_{out,b}) + f_{ir}(I_{out,r}) - m_i = 0$ for both species. Hence, the elements of the Jacobian matrix can be written as

$$A_{ij} = \frac{\partial f_{ib}}{\partial I_{out,b}} \frac{\partial I_{out,b}}{\partial C_j} + \frac{\partial f_{ir}}{\partial I_{out,r}} \frac{\partial I_{out,r}}{\partial C_j} \quad \text{(S5)}$$

It is straightforward to derive that both $\partial f_{ib}/\partial I_{out,b} > 0$ and $\partial f_{ir}/\partial I_{out,r} > 0$, whereas both $\partial I_{out,b}/\partial C_j < 0$ and $\partial I_{out,r}/\partial C_j < 0$. It follows that all $A_{ij} < 0$ and therefore $trace(\boldsymbol{J}) < 0$.

After some algebra, the determinant of the Jacobian matrix can be written as

$$det(\boldsymbol{J}) = \left(\frac{\partial f_{1b}}{\partial I_{out,b}} \frac{\partial f_{2r}}{\partial I_{out,r}} - \frac{\partial f_{2b}}{\partial I_{out,b}} \frac{\partial f_{1r}}{\partial I_{out,r}}\right) \left(\frac{\partial I_{out,b}}{\partial C_1} \frac{\partial I_{out,r}}{\partial C_2} - \frac{\partial I_{out,b}}{\partial C_2} \frac{\partial I_{out,r}}{\partial C_1}\right) \quad \text{(S6)}$$

From Lambert-Beer's law, we note that

$$\frac{\partial I_{out,b}}{\partial C_i} = -k_{ib} I_{out,b} \ \text{ and } \ \frac{\partial I_{out,r}}{\partial C_i} = -k_{ir} I_{out,r} \quad \text{(S7)}$$

Furthermore,

$$\frac{\partial f_{ib}}{\partial I_{out,b}} = \phi_{ib} k_{ib} \frac{\partial I_{avg,b}}{\partial I_{out,b}} \ \text{ and } \ \frac{\partial f_{ir}}{\partial I_{out,r}} = \phi_{ir} k_{ir} \frac{\partial I_{avg,r}}{\partial I_{out,r}} \quad \text{(S8)}$$

Hence, we obtain

$$det(\boldsymbol{J}) = (\phi_{1b}\phi_{2r}k_{1b}k_{2r} - \phi_{2b}\phi_{1r}k_{2b}k_{1r})\,(k_{1b}k_{2r} - k_{2b}k_{1r}) \frac{\partial I_{avg,b}}{\partial I_{out,b}} \frac{\partial I_{avg,r}}{\partial I_{out,r}} \, I_{out,b} I_{out,r} \quad \text{(S9)}$$

Since $\partial I_{avg,b}/\partial I_{out,b} > 0$ and $\partial I_{avg,r}/\partial I_{out,r} > 0$, the signs of the two bracketed terms in this equation determine whether the coexistence equilibrium is stable or unstable.

*Case 1: Photosynthetic efficiency independent of light color*

Suppose that photosynthetic efficiency is independent of light color, i.e., a species utilizes all its absorbed photons with the same efficiency irrespective of wavelength, as assumed by Stomp et al. (2004, 2007). Hence, $\phi_{1b} = \phi_{1r} = \phi_1$ and $\phi_{2b} = \phi_{2r} = \phi_2$, and the determinant simplifies to

$$det(\boldsymbol{J}) = \phi_1\phi_2(k_{1b}k_{2r} - k_{2b}k_{1r})^2 \frac{\partial I_{avg,1}}{\partial I_{out,1}} \frac{\partial I_{avg,2}}{\partial I_{out,2}} \, I_{out,1} I_{out,2} \quad \text{(S10)}$$

In this case, it follows that $det(\boldsymbol{J})>0$ and therefore the coexistence equilibrium is locally stable whenever it exists.

*Case 2: Photosynthetic efficiency depends on light color*

More generally, photosynthetic efficiency varies with light color. Let us arbitrarily assume that species 1 is a better competitor for blue light and species 2 a better competitor for red light. Graphically, this implies that species 1 has a steeper zero isocline than species 2 (as in Fig. 2B, where the green alga would be species 1 and the cyanobacterium species 2). According to Eqs. 8a,b, this difference in slope of the zero isoclines implies

$$\frac{\phi_{1b}k_{1b}}{\phi_{1r}k_{1r}} > \frac{\phi_{2b}k_{2b}}{\phi_{2r}k_{2r}} \tag{S11}$$

Hence, the first bracketed term in Eq. S9 is positive, and therefore the sign of *det*(***J***) depends only on the second bracketed term. This implies that *det*(***J***)>0 and, hence, the coexistence equilibrium is locally stable if

$$\frac{k_{2r}}{k_{2b}} > \frac{k_{1r}}{k_{1b}} \tag{S12}$$

whereas it is locally unstable if this inequality is reversed. In other words, if species 2 (the better competitor for red light) absorbs relatively more red than blue light in comparison to species 1, then coexistence of the two species is stable. Conversely, if species 2 absorbs relatively more blue than red light in comparison to species 1, then the coexistence equilibrium is unstable and the winner will depend on the initial abundances of the species.

**Literature Cited**


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**Table S1.** Model parameters estimated from the monoculture experiments

| Symbol | Definition | Value | | Units |
|---|---|---|---|---|
| **Variables:** | | | | |
| $C_i$ | Biomass of phytoplankton species *i* | - | | $mm^3 L^{-1}$ |
| $I_{out,blue}$ | Blue light transmitted through the chemostats | - | | μmol photons $m^{-2} s^{-1}$ |
| $I_{out,red}$ | Red light transmitted through the chemostats | - | | μmol photons $m^{-2} s^{-1}$ |
| **System parameters:** | | | | |
| $I_{in,j}$ | Incident blue or red light intensity | 45 | | μmol photons $m^{-2} s^{-1}$ |
| $K_{bg,blue}$ | Background turbidity of blue light | 7.5 | | $m^{-1}$ |
| $K_{bg,red}$ | Background turbidity of red light | 9 | | $m^{-1}$ |
| $K_{bg,green}$ | Background turbidity of green light | 8 | | $m^{-1}$ |
| $z_{max}$ | Maximum depth of water column | 0.05 | | m |
| *D* | Dilution rate* | 0.015 | | $h^{-1}$ |
| **Species parameters[1]:** | | *Chlorella* | *Synechocystis* | |
| $\phi_{i,blue}$ | Photosynthetic efficiency in blue light | $2.30\times10^{-3}$ | $0.69\times10^{-3}$ | $mm^3$ $\mu mol^{-1}$ |
| $\phi_{i,red}$ | Photosynthetic efficiency in red light | $3.00\times10^{-3}$ | $3.80\times10^{-3}$ | $mm^3$ $\mu mol^{-1}$ |
| $k_{i,blue}$ | Specific light absorption coefficient in blue light | $2.60\times10^{-4}$ | $2.10\times10^{-4}$ | $m^2$ $mm^{-3}$ |
| $k_{i,red}$ | Specific light absorption coefficient in red light | $1.33\times10^{-4}$ | $1.16\times10^{-4}$ | $m^2$ $mm^{-3}$ |
| | | *Prochlorococcus* | *Synechococcus* | |
| $\phi_{i,blue}$ | Photosynthetic efficiency in blue light | $2.30\times10^{-3}$ | $0.69\times10^{-3}$ | $mm^3$ $\mu mol^{-1}$ |
| $\phi_{i,green}$ | Photosynthetic efficiency in green light | $3.30\times10^{-3}$ | $3.80\times10^{-3}$ | $mm^3$ $\mu mol^{-1}$ |
| $k_{i,blue}$ | Specific light absorption coefficient in blue light | $2.60\times10^{-4}$ | $2.10\times10^{-4}$ | $m^2$ $mm^{-3}$ |
| $k_{i,green}$ | Specific light absorption coefficient in green light | $5.20\times10^{-5}$ | $2.02\times10^{-4}$ | $m^2$ $mm^{-3}$ |

*We assume that specific loss rates of the species are dominated by the dilution rate of the chemostat (i.e., $m_i$=*D*)

**Table S2.** Steady-state characteristics of monoculture experiments with the cyanobacterium *Synechocystis* and green alga *Chlorella* in blue and red light.

| | *Synechocystis* | | *Chlorella* | |
|---|---|---|---|---|
| | blue light | red light | blue light | red light |
| Population density (million cells $mL^{-1}$) | 7.5 ± 0.6 | 123.4 ± 12.6 | 25.4 ± 1.9 | 31.5 ± 5.5 |
| Total biovolume ($mm^3$ $L^{-1}$) | 57 ± 4 | 696 ± 66 | 427 ± 39 | 640 ± 118 |
| Cell volume (fL $cell^{-1}$) | 7.7 ± 0.3 | 5.6 ± 0.2 | 16.8± 0.7 | 19.2 ± 1.9 |
| Light transmission $I_{out}$ (µmol photons $m^{-2}$ $s^{-1}$) | 19.3 ± 0.5 | < 0.5 | < 0.5 | < 0.5 |
| Critical light intensity $I^*_{avg,ij}$ (µmol photons $m^{-2}$ $s^{-1}$) | 28.8 | 9.5 | 7.0 | 10.4 |

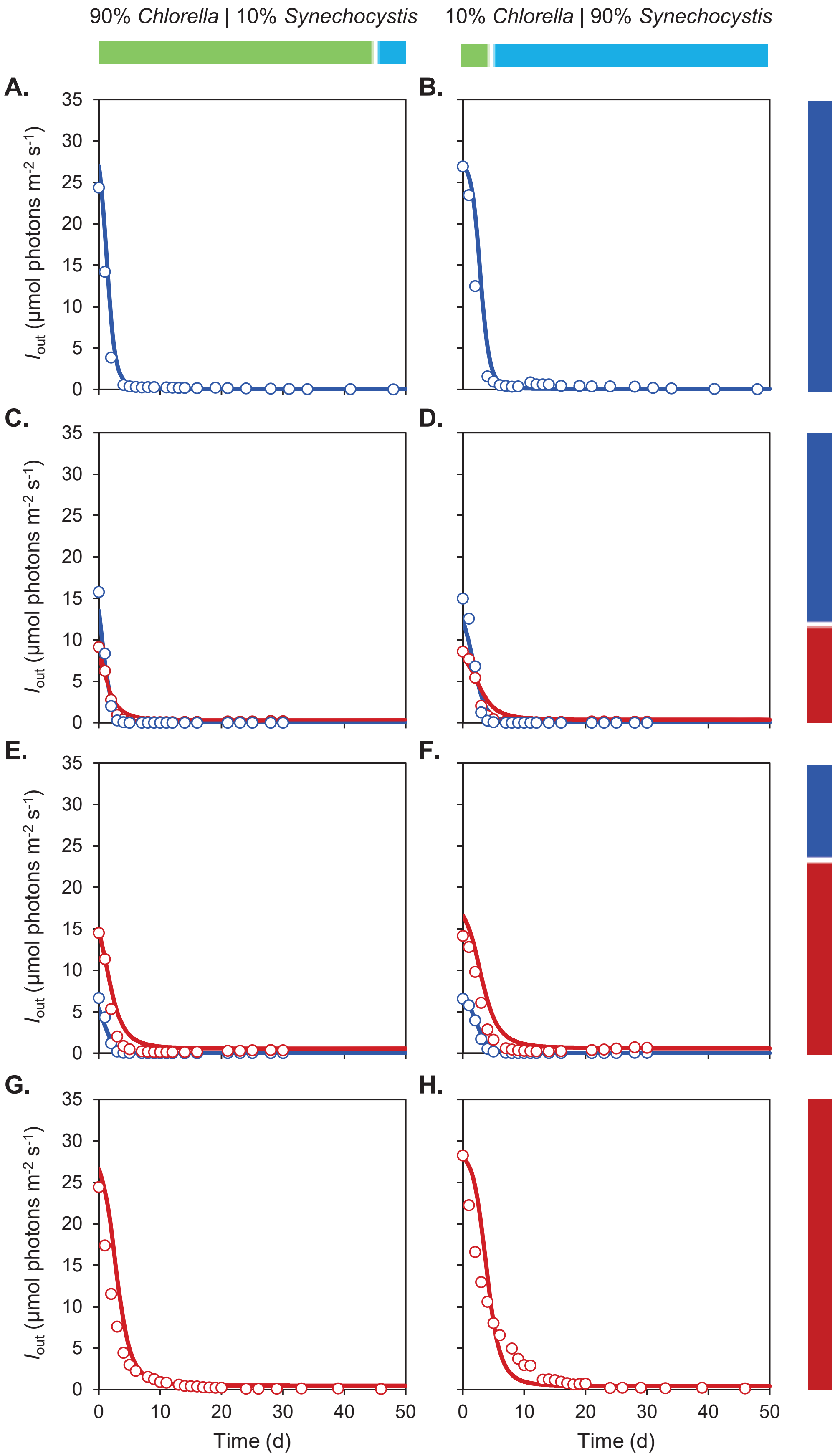


**Figure S1.** Light transmission ($I_{out}$) through the competition experiments between the cyanobacterium *Synechocystis* and the green alga *Chlorella*. Blue circles represent blue light, red circles represent red light, and solid lines represent the model predictions. The graph has the same layout as Figure 4 in the main text.