1 Relating the overall reproduction number to the reproduction numbers of index and secondary cases

Let $R_{overall}$ denote the number of new infections caused by each case, averaged over all cases. Let μ be the average size of transmission chain (i.e. all cases that are linked to an index case). Then on average, μ cases cause $\mu - 1$ infections (because the index case is caused by any case in the chain). Therefore [Harris, 2002, Lange, 2010],

$$R_{overall} = \frac{\mu - 1}{\mu} \tag{1}$$

$$\mu = \frac{1}{1 - R_{overall}} \tag{2}$$

Meanwhile the index case is expected to generate R_i cases in the first generation of secondary cases. These cases are expected to generate $R_i \cdot R_s$ cases in the second generation. These in turn produce $R_i \cdot R_s^2$ cases in the third generation and so forth. Therefore,

$$\mu = 1 + R_i + R_i \cdot R_s + R_i \cdot R_s^2 + \cdots \tag{3}$$

$$= 1 + \frac{R_i}{1 - R_s} \tag{4}$$

Equating the two formulas for μ and rearranging gives,

$$\mu = \frac{1}{1 - R_{overall}} = 1 + \frac{R_i}{1 - R_s} \tag{5}$$

$$R_{overall} = 1 - \frac{1 - R_s}{1 - R_s + R_i} \tag{6}$$

$$R_{overall} = \frac{R_i}{1 - R_s + R_i} \tag{7}$$

2 Adjusting observed R_i and $R_{overall}$ for the proportion of asymptomatic index cases that are unobserved

Here we consider adjustments to the observed value of R_i and $R_{overall}$ for the possibility that asymptomatic, unobserved index cases were not incorporated into the initial data analysis. We assume that the proportion of symptomatic, observed index cases is $\rho_{i,c}$ and that the proportion of unobserved index cases is $1-\rho_{i,c}$. Let R_i^o and $R_{overall}^o$ and μ^o represent the values of R_i,R_s and the average chain size based solely on observed data. Let R_i^a and $R_{overall}^a$ and μ^a represents the corresponding values, once adjusted for unobserved asymptomatic index cases.

Recognizing that R_i^a has a contribution from both symptomatic and asymptomatic cases,

$$R_i^a = \rho_{i,c} \cdot R_i^o + (1-\rho_{i,c}) \cdot 0 \quad (8)$$

$$R_i^a = \rho_{i,c} \cdot R_i^o \quad (9)$$

Likewise, given that asymptomatic index cases result in a transmission chain of size one,

$$\mu_i^a = \rho_{i,c} \cdot \mu_i^o + (1-\rho_{i,c}) \cdot 1 \quad (10)$$

Coupling this to equation 1,

$$\frac{1}{1-R_{overall}^a} = \frac{\rho_{i,c}}{1-R_{overall}^o} + (1-\rho_{i,c}) \cdot 1 \quad (11)$$

$$R_{overall}^a = 1 - \frac{1}{\frac{\rho_{i,c}}{1-R_{overall}^o} + (1-\rho_{i,c})} \quad (12)$$

References

[Harris, 2002] Harris, Theodore E. 2002. *The Theory of Branching Processes*. Toronto: Dover.

[Lange, 2010] Lange, Kenneth. 2010. *Applied Probability*. Second edn. New York: Springer.