A. Technical algorithmic details

A.1. Identifying conditionally independent parameters

The following algorithm (called `find_conditionally_independent`) is used by MultiBUGS to identify sets of conditionally-independent parameters $W_1, \ldots, W_l \subseteq U$:

Input: $G = (E, V)$, a DAG; U, a set of nodes (with identical topological depth)

$l \leftarrow 1$
$M \leftarrow \varnothing$
while $|U| > 0$ **do**
  for u in U **do**
    if $\mathrm{ch}_G(u) \cap M = \varnothing$ **then**
      $W_l \leftarrow W_l \cup \{u\}$
      $U \leftarrow U \setminus \{u\}$
      $M \leftarrow M \cup \mathrm{ch}_G(u)$
    end if
  end for
  $M \leftarrow \varnothing$
  $l \leftarrow l + 1$
end while

Output: $\{W_1, \ldots, W_l\}$

A.2. Identifying parallelisable likelihoods

Nodes for which the likelihood calculations should be partitioned across cores are identified using the following algorithm, called `find_partial_product_parallel`:

Input: $G = (E, V)$, a DAG; C, a number of cores; h, a topological depth; $h^\star$, the maximum topological depth in G; T, a computation schedule; r, the current schedule row

$U \leftarrow D_G^h$
$\overline{\mathrm{ch}} \leftarrow \mathrm{mean}_{v \in S_G} |\,\mathrm{ch}_G(v)|$
for u in U **do**
  if $|\,\mathrm{ch}_G(u)| > 2 \times \overline{\mathrm{ch}}$ **or** $h^\star = 1$ **then**
    $r \leftarrow r + 1$
    for c in 1 to C **do**
      $T_{rc} \leftarrow u$
    end for
    $U \leftarrow U \setminus \{u\}$
  end if
end for

Output: $\{T, U, r\}$

A.3. Building a computation schedule

The overall algorithm for allocating compution to cores is as follows:

Input: $G = (E, V)$, a DAG; C, a number of cores

Initialise T, a table with C columns
$r \leftarrow 0$
$h^\star \leftarrow \max_{v \in S_G} d_G(v)$
for h in $h^\star$ to 1 **do**
  $\{T, U, r\} \leftarrow$ `find_partial_product_parallel`$(G, C, h, h^\star, T, r)$
  $\{W_1, \ldots, W_l\} \leftarrow$ `find_conditionally_independent`(G, U)
  for i in 1 to l **do**
    $c \leftarrow 0$
    for j in $\max_{w \in W_i} |\,\mathrm{ch}_G(w)|$ to 1 **do**
      for x in $\{w \in W_i : |\,\mathrm{ch}_G(w)| = j\}$ **do**
        if $c \bmod C = 0$ **then**
          $r \leftarrow r + 1$

$c \leftarrow 0$
end if
$T_{r(c+1)} \leftarrow x$
$c \leftarrow c + 1$
end for
end for
end for
end for
Output: T