Fast Local Phase Velocity Based Imaging (LPVI): Shear Wave Particle Velocity and Displacement Motion Study

Supplementary Material

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This document serves as the Supplementary Material for the main manuscript titled *Fast Local Phase Velocity Based Imaging (LPVI): Shear Wave Particle Velocity and Displacement Motion Study* by Piotr Kijanka and Matthew W. Urban.

Fig. S1 presents boxplots of the phase velocity within inclusions, for the four models investigated, calculated for the clean data and with an SNR of 15 and 5 dB. The bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. White circles represent mean values of the phase velocity whereas, a solid line within each box corresponds to a median value. Outliers were also plotted if values are greater than $q_3 + w(q_3 - q_1)$ or less than $q_1 - w(q_3 - q_1)$, where w is the maximum whisker length, and q_1 and q_3 are the 25th and 75th percentiles of the sample data, respectively. w was selected to be 1.5. Results for three different frequencies of 500, 750 and 1000 Hz are presented in Figs. S1a, S1b and S1c, respectively.

Tables S1 - S10 present the 25th and 75th percentiles of the sample data for the reconstructed shear wave phase velocity, using the new implementation of LPVI, for the clean data and with an SNR of 20, 15, 10, and 5 dB. Results for six different frequencies of 500, 600, 700, 800, 900, and 1000 Hz are summarized. Models, A, B, C, and D are investigated, respectively. A graphical representation of these data, for selected frequencies and noise levels, is shown in Fig. S1 within this document, and in Fig. 9, in the main manuscript.
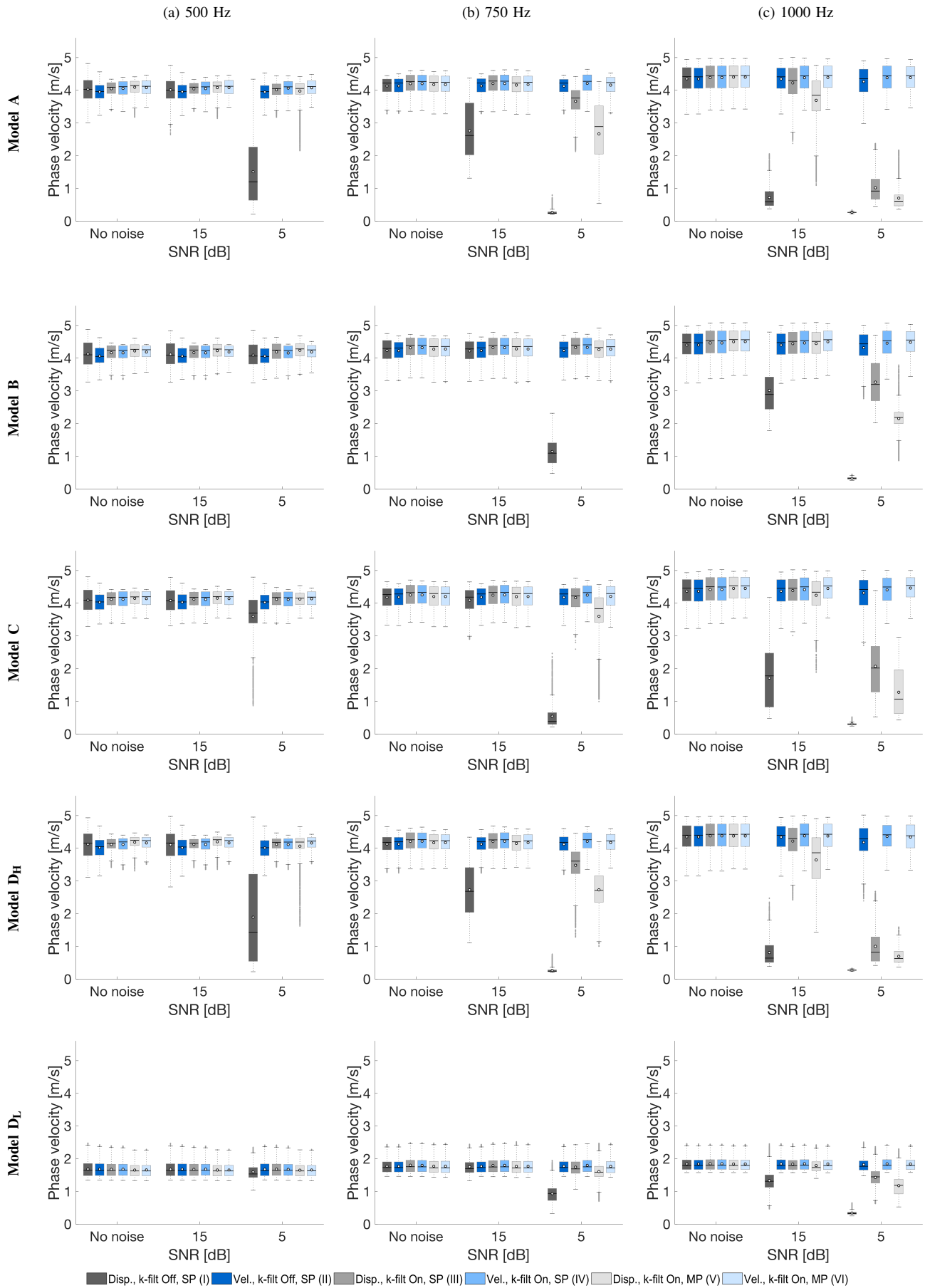
Fig. S1: Boxplots of phase velocity, of the inclusion for various numerical LISA models data and noise levels. White circles represent mean values whereas, a solid line within the box corresponds to a median value. Results were calculated for various LPVI processing scenarios tabulated in Table I in the main manuscript.

TABLE S1: The 25th percentile phase velocity of the inclusion for Model A and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	3.75	3.93	3.83	4.00	3.97	4.06
	II	3.76	3.88	3.91	3.96	3.99	4.06
	III	3.91	3.93	3.95	4.03	4.07	4.08
	IV	3.88	3.92	3.97	4.02	4.06	4.07
	V	3.94	3.90	3.91	4.01	4.08	4.09
	VI	3.90	3.91	3.93	4.01	4.07	4.09
20	I	3.74	3.92	3.83	3.97	3.51	2.50
	II	3.75	3.88	3.91	3.96	3.98	4.06
	III	3.91	3.94	3.94	4.03	4.04	4.07
	IV	3.88	3.92	3.97	4.02	4.06	4.07
	V	3.95	3.91	3.91	3.99	4.02	4.07
	VI	3.90	3.91	3.93	4.01	4.07	4.09
15	I	3.75	3.88	3.08	1.71	0.83	0.48
	II	3.76	3.88	3.91	3.95	3.99	4.06
	III	3.90	3.92	3.93	3.99	3.99	3.89
	IV	3.88	3.92	3.96	4.02	4.05	4.07
	V	3.94	3.93	3.92	3.91	3.88	3.37
	VI	3.90	3.91	3.93	4.01	4.07	4.08
10	I	3.50	1.64	0.52	0.32	0.29	0.30
	II	3.76	3.89	3.91	3.95	3.98	4.07
	III	3.90	3.92	3.92	3.91	3.72	2.11
	IV	3.88	3.92	3.97	4.01	4.06	4.07
	V	3.94	3.89	3.87	3.42	2.32	0.89
	VI	3.90	3.91	3.94	4.01	4.08	4.09
5	I	0.64	0.26	0.23	0.23	0.25	0.26
	II	3.75	3.87	3.91	3.95	3.98	3.96
	III	3.87	3.93	3.72	2.82	1.49	0.67
	IV	3.88	3.92	3.96	4.01	4.06	4.07
	V	3.88	3.86	2.84	1.09	0.57	0.47
	VI	3.90	3.91	3.93	4.00	4.05	4.09

TABLE S3: The 25th percentile phase velocity of the inclusion for Model B and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	3.81	4.01	3.94	4.04	4.07	4.12
	II	3.87	3.95	3.99	4.05	4.06	4.12
	III	4.01	4.07	4.09	4.13	4.15	4.16
	IV	4.01	4.06	4.10	4.13	4.15	4.16
	V	4.05	4.04	4.02	4.10	4.18	4.22
	VI	4.04	4.03	4.03	4.11	4.18	4.22
20	I	3.82	4.00	3.94	4.04	4.07	4.00
	II	3.86	3.95	3.99	4.04	4.06	4.12
	III	4.01	4.07	4.09	4.13	4.15	4.16
	IV	4.01	4.05	4.10	4.13	4.15	4.16
	V	4.05	4.04	4.01	4.10	4.17	4.21
	VI	4.04	4.03	4.03	4.11	4.19	4.21
15	I	3.82	4.00	3.93	4.03	3.61	2.44
	II	3.86	3.95	3.99	4.05	4.06	4.12
	III	4.02	4.07	4.07	4.12	4.14	4.14
	IV	4.01	4.05	4.10	4.13	4.15	4.17
	V	4.06	4.03	4.01	4.09	4.15	4.15
	VI	4.05	4.03	4.03	4.11	4.19	4.22
10	I	3.83	3.97	3.64	2.75	1.27	0.55
	II	3.86	3.96	3.99	4.04	4.06	4.12
	III	4.01	4.08	4.07	4.10	4.11	4.05
	IV	4.01	4.06	4.10	4.13	4.15	4.16
	V	4.07	4.05	4.00	4.11	4.15	3.53
	VI	4.05	4.03	4.03	4.11	4.19	4.22
5	I	3.82	3.49	1.28	0.50	0.35	0.30
	II	3.86	3.96	3.98	4.04	4.06	4.08
	III	3.99	4.07	4.05	4.09	3.95	2.70
	IV	4.01	4.05	4.09	4.13	4.14	4.15
	V	4.07	4.04	4.01	3.96	3.13	2.00
	VI	4.04	4.02	4.02	4.11	4.19	4.21

TABLE S2: The 75th percentile phase velocity of the inclusion for Model A and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	4.31	4.17	4.34	4.40	4.59	4.69
	II	4.15	4.14	4.27	4.45	4.57	4.68
	III	4.23	4.30	4.43	4.52	4.65	4.75
	IV	4.26	4.31	4.41	4.53	4.65	4.75
	V	4.28	4.30	4.41	4.49	4.64	4.75
	VI	4.29	4.32	4.39	4.50	4.64	4.75
20	I	4.30	4.16	4.37	4.38	4.39	3.32
	II	4.16	4.14	4.28	4.45	4.58	4.68
	III	4.24	4.30	4.43	4.53	4.65	4.74
	IV	4.26	4.31	4.41	4.53	4.65	4.75
	V	4.26	4.31	4.43	4.49	4.62	4.70
	VI	4.29	4.32	4.40	4.50	4.64	4.74
15	I	4.28	4.15	3.84	2.67	1.51	0.91
	II	4.16	4.14	4.28	4.45	4.58	4.68
	III	4.21	4.29	4.41	4.52	4.60	4.68
	IV	4.26	4.30	4.41	4.53	4.65	4.74
	V	4.26	4.31	4.39	4.43	4.47	4.29
	VI	4.30	4.31	4.39	4.50	4.64	4.74
10	I	4.19	3.43	1.17	0.57	0.39	0.34
	II	4.16	4.15	4.28	4.46	4.58	4.67
	III	4.23	4.27	4.40	4.36	4.38	3.85
	IV	4.26	4.31	4.41	4.53	4.65	4.75
	V	4.25	4.29	4.33	4.14	3.54	2.47
	VI	4.29	4.29	4.39	4.48	4.64	4.75
5	I	2.26	0.63	0.30	0.26	0.27	0.28
	II	4.14	4.13	4.26	4.46	4.57	4.64
	III	4.19	4.18	4.20	3.70	2.70	1.28
	IV	4.27	4.30	4.42	4.53	4.66	4.74
	V	4.20	4.22	3.79	2.00	1.08	0.80
	VI	4.29	4.33	4.40	4.50	4.64	4.72

TABLE S4: The 75th percentile phase velocity of the inclusion for Model B and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	4.47	4.25	4.52	4.56	4.67	4.74
	II	4.31	4.30	4.45	4.56	4.67	4.75
	III	4.38	4.45	4.58	4.65	4.74	4.83
	IV	4.38	4.46	4.58	4.65	4.74	4.83
	V	4.41	4.43	4.55	4.63	4.74	4.83
	VI	4.37	4.45	4.55	4.63	4.74	4.84
20	I	4.45	4.22	4.53	4.56	4.67	4.68
	II	4.31	4.30	4.45	4.56	4.67	4.75
	III	4.39	4.45	4.58	4.65	4.74	4.83
	IV	4.38	4.46	4.58	4.65	4.74	4.84
	V	4.44	4.44	4.54	4.64	4.73	4.83
	VI	4.37	4.45	4.55	4.64	4.74	4.83
15	I	4.44	4.23	4.52	4.54	4.49	3.42
	II	4.30	4.29	4.45	4.56	4.67	4.74
	III	4.38	4.45	4.58	4.67	4.75	4.80
	IV	4.38	4.45	4.58	4.65	4.74	4.83
	V	4.42	4.43	4.53	4.64	4.71	4.79
	VI	4.38	4.45	4.54	4.63	4.74	4.83
10	I	4.45	4.21	4.36	3.44	2.17	0.97
	II	4.29	4.29	4.44	4.55	4.67	4.74
	III	4.38	4.43	4.55	4.64	4.74	4.76
	IV	4.38	4.46	4.58	4.66	4.74	4.84
	V	4.43	4.41	4.55	4.60	4.65	4.54
	VI	4.38	4.44	4.55	4.64	4.74	4.83
5	I	4.41	4.06	2.01	0.76	0.45	0.35
	II	4.29	4.29	4.43	4.56	4.66	4.72
	III	4.43	4.41	4.54	4.61	4.55	3.84
	IV	4.38	4.45	4.58	4.66	4.73	4.83
	V	4.44	4.35	4.44	4.54	3.97	2.34
	VI	4.39	4.45	4.53	4.63	4.71	4.82

TABLE S5: The 25th percentile phase velocity of the inclusion for Model C and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	3.80	3.93	3.84	3.99	3.98	4.07
	II	3.81	3.89	3.91	3.97	3.98	4.06
	III	3.95	3.98	3.95	4.02	4.05	4.09
	IV	3.92	3.96	3.98	4.02	4.05	4.09
	V	3.98	3.93	3.89	3.99	4.09	4.16
	VI	3.96	3.92	3.90	3.99	4.09	4.16
20	I	3.78	3.93	3.83	3.99	3.87	3.33
	II	3.81	3.88	3.91	3.98	3.98	4.06
	III	3.95	3.98	3.95	4.02	4.05	4.09
	IV	3.92	3.96	3.97	4.02	4.05	4.09
	V	3.99	3.93	3.88	3.99	4.08	4.10
	VI	3.95	3.92	3.90	3.99	4.09	4.16
15	I	3.80	3.93	3.83	3.54	1.82	0.83
	II	3.82	3.89	3.91	3.97	3.98	4.06
	III	3.95	3.98	3.95	4.01	4.05	4.08
	IV	3.92	3.96	3.98	4.02	4.05	4.09
	V	3.99	3.93	3.91	3.97	3.97	3.94
	VI	3.96	3.92	3.90	3.99	4.08	4.15
10	I	3.79	3.80	2.37	0.62	0.39	0.34
	II	3.80	3.88	3.91	3.97	3.97	4.06
	III	3.94	3.98	3.94	4.00	3.93	3.57
	IV	3.92	3.97	3.97	4.01	4.04	4.09
	V	3.99	3.92	3.87	3.82	3.54	2.06
	VI	3.96	3.93	3.90	3.99	4.09	4.16
5	I	3.39	0.84	0.32	0.28	0.27	0.28
	II	3.82	3.89	3.91	3.98	3.97	3.95
	III	3.95	3.97	3.93	3.67	2.76	1.29
	IV	3.91	3.97	3.98	4.02	4.04	4.09
	V	3.96	3.92	3.83	1.92	0.77	0.63
	VI	3.95	3.92	3.90	3.98	4.08	4.19

TABLE S6: The 75th percentile phase velocity of the inclusion for Model C and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	4.40	4.21	4.46	4.50	4.64	4.72
	II	4.26	4.25	4.39	4.51	4.63	4.72
	III	4.33	4.34	4.52	4.59	4.68	4.79
	IV	4.35	4.40	4.50	4.59	4.68	4.78
	V	4.35	4.33	4.47	4.56	4.68	4.80
	VI	4.36	4.39	4.45	4.56	4.68	4.80
20	I	4.40	4.19	4.47	4.49	4.58	4.26
	II	4.25	4.25	4.38	4.51	4.63	4.72
	III	4.32	4.35	4.51	4.59	4.67	4.76
	IV	4.35	4.40	4.50	4.59	4.69	4.79
	V	4.36	4.34	4.47	4.55	4.68	4.75
	VI	4.36	4.40	4.46	4.57	4.68	4.80
15	I	4.39	4.20	4.44	4.34	3.55	2.48
	II	4.26	4.24	4.39	4.52	4.63	4.72
	III	4.34	4.35	4.52	4.58	4.67	4.73
	IV	4.35	4.40	4.51	4.59	4.68	4.78
	V	4.35	4.35	4.43	4.52	4.62	4.66
	VI	4.36	4.39	4.46	4.56	4.68	4.79
10	I	4.38	4.15	3.53	2.42	1.20	0.77
	II	4.26	4.25	4.38	4.52	4.62	4.72
	III	4.34	4.34	4.51	4.58	4.66	4.42
	IV	4.35	4.40	4.51	4.59	4.69	4.79
	V	4.32	4.35	4.52	4.47	4.42	3.59
	VI	4.36	4.39	4.45	4.57	4.69	4.79
5	I	4.10	3.05	1.09	0.48	0.36	0.32
	II	4.25	4.23	4.38	4.51	4.61	4.71
	III	4.33	4.35	4.55	4.41	4.06	2.68
	IV	4.35	4.38	4.51	4.58	4.69	4.77
	V	4.31	4.27	4.39	4.03	2.44	1.97
	VI	4.35	4.39	4.45	4.55	4.68	4.78

TABLE S7: The 25th percentile phase velocity of the inclusion for Model D_H and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	3.78	3.85	3.85	4.02	3.92	4.06
	II	3.79	3.81	3.92	3.98	3.93	4.06
	III	4.01	3.95	3.96	4.05	4.04	4.06
	IV	3.97	3.96	3.98	4.04	4.04	4.07
	V	4.08	3.97	3.93	4.00	4.03	4.05
	VI	4.03	3.97	3.95	4.00	4.02	4.06
20	I	3.78	3.84	3.81	4.01	2.75	2.43
	II	3.79	3.81	3.92	3.98	3.93	4.06
	III	4.01	3.95	3.96	4.05	4.05	4.03
	IV	3.97	3.96	3.98	4.04	4.04	4.07
	V	4.07	3.96	3.93	4.01	4.00	3.98
	VI	4.03	3.97	3.95	4.00	4.02	4.05
15	I	3.77	3.84	2.82	1.55	0.71	0.51
	II	3.79	3.81	3.92	3.98	3.93	4.05
	III	4.01	3.95	3.97	4.01	4.00	3.92
	IV	3.97	3.96	3.98	4.04	4.04	4.07
	V	4.10	3.94	3.92	3.95	3.82	3.07
	VI	4.04	3.97	3.95	4.00	4.02	4.06
10	I	3.55	1.48	0.44	0.33	0.29	0.29
	II	3.79	3.81	3.91	3.98	3.93	4.04
	III	4.00	3.95	3.90	3.93	3.43	2.20
	IV	3.97	3.96	3.99	4.04	4.03	4.07
	V	4.06	3.95	3.85	3.59	2.21	1.04
	VI	4.03	3.97	3.95	3.99	4.02	4.06
5	I	0.55	0.27	0.21	0.23	0.24	0.27
	II	3.78	3.80	3.92	3.98	3.90	3.93
	III	4.01	3.92	3.61	2.69	1.05	0.55
	IV	3.98	3.96	3.98	4.04	4.04	4.06
	V	3.99	3.74	3.35	1.43	0.49	0.51
	VI	4.04	3.97	3.94	3.99	4.01	4.02

TABLE S8: The 75th percentile phase velocity of the inclusion for Model D_H and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	4.44	4.25	4.35	4.35	4.59	4.69
	II	4.25	4.21	4.28	4.39	4.58	4.68
	III	4.27	4.29	4.44	4.50	4.61	4.74
	IV	4.29	4.27	4.42	4.51	4.61	4.74
	V	4.34	4.33	4.40	4.43	4.59	4.74
	VI	4.33	4.34	4.39	4.43	4.59	4.74
20	I	4.43	4.23	4.33	4.34	4.04	3.24
	II	4.25	4.21	4.28	4.39	4.57	4.68
	III	4.27	4.30	4.43	4.50	4.58	4.73
	IV	4.29	4.27	4.42	4.51	4.61	4.74
	V	4.34	4.32	4.40	4.41	4.55	4.68
	VI	4.33	4.34	4.39	4.43	4.60	4.74
15	I	4.44	4.23	3.75	2.81	1.47	1.03
	II	4.25	4.20	4.28	4.39	4.56	4.67
	III	4.27	4.26	4.42	4.46	4.55	4.62
	IV	4.29	4.27	4.43	4.52	4.61	4.75
	V	4.35	4.33	4.40	4.41	4.52	4.32
	VI	4.33	4.33	4.39	4.43	4.58	4.74
10	I	4.31	3.63	0.99	0.54	0.38	0.34
	II	4.26	4.19	4.28	4.39	4.58	4.67
	III	4.27	4.30	4.46	4.44	4.33	3.05
	IV	4.29	4.26	4.42	4.51	4.60	4.74
	V	4.32	4.29	4.27	4.16	3.12	2.36
	VI	4.33	4.34	4.37	4.42	4.58	4.76
5	I	3.21	0.70	0.31	0.28	0.26	0.28
	II	4.24	4.21	4.27	4.37	4.52	4.61
	III	4.29	4.29	4.25	3.69	1.97	1.29
	IV	4.29	4.26	4.43	4.51	4.60	4.70
	V	4.31	4.31	3.98	2.66	1.03	0.85
	VI	4.33	4.34	4.38	4.44	4.59	4.71

TABLE S9: The 25th percentile phase velocity of the inclusion for Model D_L and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	1.49	1.53	1.57	1.61	1.63	1.67
	II	1.49	1.53	1.57	1.61	1.63	1.67
	III	1.49	1.53	1.57	1.59	1.63	1.68
	IV	1.50	1.53	1.57	1.59	1.63	1.68
	V	1.48	1.53	1.56	1.59	1.63	1.67
	VI	1.48	1.53	1.56	1.59	1.63	1.67
20	I	1.48	1.53	1.57	1.59	1.60	1.61
	II	1.49	1.53	1.57	1.61	1.63	1.67
	III	1.49	1.53	1.57	1.58	1.63	1.68
	IV	1.50	1.53	1.57	1.59	1.63	1.68
	V	1.48	1.54	1.56	1.59	1.63	1.66
	VI	1.48	1.53	1.56	1.59	1.63	1.67
15	I	1.48	1.53	1.56	1.53	1.31	1.13
	II	1.49	1.53	1.57	1.61	1.63	1.67
	III	1.49	1.53	1.57	1.59	1.63	1.67
	IV	1.50	1.53	1.57	1.59	1.63	1.68
	V	1.48	1.53	1.57	1.59	1.63	1.63
	VI	1.48	1.53	1.56	1.59	1.63	1.67
10	I	1.48	1.52	1.44	1.10	0.79	0.56
	II	1.49	1.53	1.57	1.61	1.63	1.67
	III	1.49	1.53	1.57	1.57	1.61	1.58
	IV	1.49	1.52	1.57	1.59	1.63	1.68
	V	1.48	1.54	1.56	1.53	1.58	1.48
	VI	1.48	1.53	1.56	1.59	1.63	1.67
5	I	1.43	1.27	0.80	0.51	0.35	0.31
	II	1.49	1.53	1.56	1.61	1.62	1.66
	III	1.49	1.53	1.56	1.49	1.39	1.26
	IV	1.49	1.53	1.57	1.59	1.63	1.67
	V	1.48	1.54	1.51	1.30	1.23	0.93
	VI	1.48	1.53	1.56	1.58	1.63	1.67

TABLE S10: The 75th percentile phase velocity of the inclusion for Model D_L and various LPVI processing scenarios. Values are presented in the unit of m/s.

Noise level [dB]	Processing	Frequency [Hz]					
		500	600	700	800	900	1000
No noise	I	1.86	1.86	1.89	1.92	1.93	1.96
	II	1.85	1.86	1.89	1.92	1.93	1.96
	III	1.83	1.88	1.92	1.91	1.92	1.97
	IV	1.83	1.88	1.92	1.91	1.92	1.97
	V	1.80	1.88	1.92	1.91	1.93	1.96
	VI	1.80	1.88	1.92	1.91	1.93	1.96
20	I	1.85	1.86	1.89	1.90	1.88	1.86
	II	1.84	1.86	1.89	1.91	1.93	1.96
	III	1.83	1.88	1.93	1.91	1.92	1.97
	IV	1.83	1.88	1.92	1.91	1.92	1.97
	V	1.80	1.88	1.92	1.91	1.93	1.93
	VI	1.80	1.88	1.92	1.91	1.93	1.96
15	I	1.85	1.86	1.89	1.83	1.67	1.51
	II	1.84	1.86	1.89	1.92	1.93	1.97
	III	1.83	1.88	1.93	1.91	1.92	1.95
	IV	1.83	1.88	1.92	1.91	1.92	1.97
	V	1.80	1.88	1.92	1.91	1.93	1.91
	VI	1.80	1.88	1.91	1.92	1.93	1.96
10	I	1.82	1.85	1.78	1.50	1.21	0.89
	II	1.85	1.86	1.89	1.91	1.93	1.96
	III	1.83	1.88	1.93	1.89	1.88	1.84
	IV	1.83	1.87	1.92	1.91	1.92	1.97
	V	1.80	1.88	1.91	1.84	1.81	1.74
	VI	1.80	1.87	1.91	1.91	1.93	1.96
5	I	1.73	1.72	1.28	0.91	0.51	0.36
	II	1.84	1.86	1.89	1.92	1.92	1.93
	III	1.84	1.88	1.88	1.77	1.65	1.61
	IV	1.83	1.88	1.93	1.90	1.92	1.97
	V	1.79	1.88	1.88	1.57	1.56	1.37
	VI	1.80	1.88	1.91	1.90	1.93	1.96