# Supplemental Online Material for the article "Nonparametric tests for transition probabilities in nonhomogeneous Markov processes"

**Abstract**

This Supplemental Online Material for the article "Nonparametric tests for transition probabilities in nonhomogeneous Markov processes" includes additional simulation resutls for the proposed tests with weight function $\hat{W}_{12}(t) = I[\prod_{l=1}^{2} \bar{Y}_l^{(1)}(t)\bar{Y}_l^{(2)}(t) > 0]$. This weight function assings a weight equal to 1 for observation times where the sizes of all the involved risk sets are non-zero.

Table 1: Simulation results about empirical type I error rates for the linear test (Linear), the $L^2$-norm-based test ($L^2$), and the Kolmogorov–Smirnov-type test (KS) for testing $H_0 : P^{(1)}_{0,12}(0,\cdot) = P^{(2)}_{0,12}(0,\cdot)$, under simulation scenarios 1 and 2. The weight function was $\hat{W}_{12}(t) = I[\prod_{l=1}^{2} \bar{Y}^{(1)}_l(t)\bar{Y}^{(2)}_l(t) > 0]$.

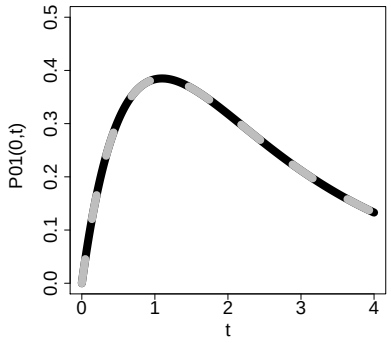
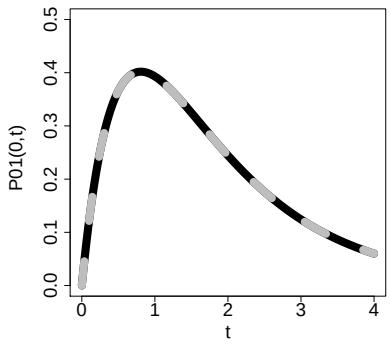
| | | | $\alpha = 0.01$ | | | $\alpha = 0.05$ | | |
|---|---|---|---|---|---|---|---|---|
| $H_0$ scenario | $n_1$ | $n_2$ | Linear | $L^2$ | KS | Linear | $L^2$ | KS |
|  | 50 | 50 | 0.007 | 0.009 | 0.014 | 0.043 | 0.048 | 0.065 |
| | 100 | 50 | 0.016 | 0.016 | 0.018 | 0.054 | 0.056 | 0.068 |
| | 100 | 100 | 0.015 | 0.016 | 0.016 | 0.049 | 0.054 | 0.062 |
| | 200 | 100 | 0.010 | 0.010 | 0.011 | 0.043 | 0.052 | 0.055 |
| | 200 | 200 | 0.011 | 0.013 | 0.019 | 0.053 | 0.052 | 0.057 |
|  | 50 | 50 | 0.009 | 0.009 | 0.011 | 0.043 | 0.049 | 0.062 |
| | 100 | 50 | 0.009 | 0.011 | 0.018 | 0.056 | 0.049 | 0.059 |
| | 100 | 100 | 0.013 | 0.012 | 0.012 | 0.045 | 0.048 | 0.056 |
| | 200 | 100 | 0.005 | 0.008 | 0.010 | 0.047 | 0.049 | 0.054 |
| | 200 | 200 | 0.010 | 0.014 | 0.016 | 0.060 | 0.056 | 0.060 |

Table 2: Simulation results about empirical type I error rates for the linear test (Linear), the $L^2$-norm-based test ($L^2$), and the Kolmogorov–Smirnov-type test (KS) for testing $H_0 : P^{(1)}_{0,12}(0,\cdot) = P^{(2)}_{0,12}(0,\cdot)$, under simulation scenarios 3 and 4. The weight function was $\hat{W}_{12}(t) = I[\prod_{l=1}^{2} \bar{Y}^{(1)}_l(t)\bar{Y}^{(2)}_l(t) > 0]$.

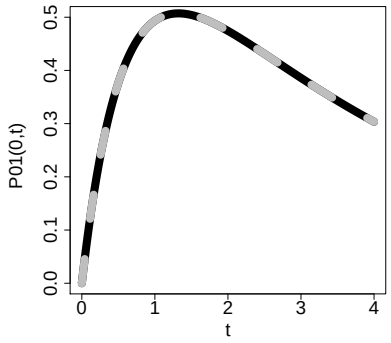
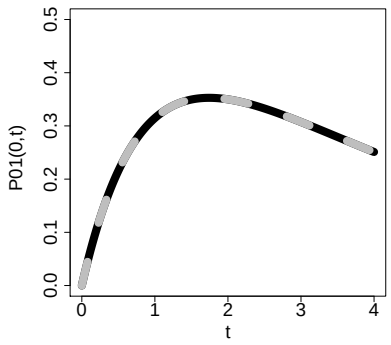
| | | | $\alpha = 0.01$ | | | $\alpha = 0.05$ | | |
|---|---|---|---|---|---|---|---|---|
| $H_0$ scenario | $n_1$ | $n_2$ | Linear | $L^2$ | KS | Linear | $L^2$ | KS |
|  | 50 | 50 | 0.010 | 0.012 | 0.014 | 0.056 | 0.055 | 0.064 |
| | 100 | 50 | 0.019 | 0.019 | 0.022 | 0.060 | 0.058 | 0.076 |
| | 100 | 100 | 0.012 | 0.014 | 0.020 | 0.047 | 0.056 | 0.075 |
| | 200 | 100 | 0.010 | 0.013 | 0.013 | 0.047 | 0.047 | 0.067 |
| | 200 | 200 | 0.010 | 0.012 | 0.012 | 0.057 | 0.056 | 0.062 |
|  | 50 | 50 | 0.007 | 0.010 | 0.013 | 0.045 | 0.046 | 0.071 |
| | 100 | 50 | 0.011 | 0.015 | 0.019 | 0.049 | 0.050 | 0.073 |
| | 100 | 100 | 0.012 | 0.013 | 0.014 | 0.048 | 0.049 | 0.061 |
| | 200 | 100 | 0.008 | 0.009 | 0.016 | 0.040 | 0.041 | 0.060 |
| | 200 | 200 | 0.010 | 0.013 | 0.013 | 0.056 | 0.056 | 0.061 |

Table 3: Simulation results about empirical power levels for the linear test (Linear), the $L^2$-norm-based test ($L^2$), and the Kolmogorov–Smirnov-type test (KS) for testing $H_0 : P^{(1)}_{0,12}(0,\cdot) = P^{(2)}_{0,12}(0,\cdot)$, under simulation scenarios 5 and 6. The weight function was $\hat{W}_{12}(t) = I[\prod_{l=1}^{2} \bar{Y}_l^{(1)}(t)\bar{Y}_l^{(2)}(t) > 0]$.

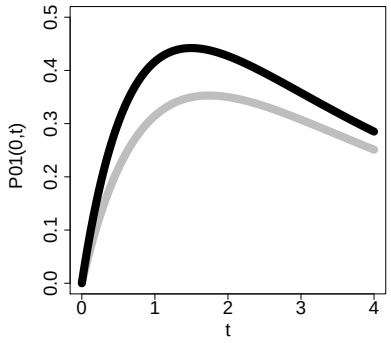
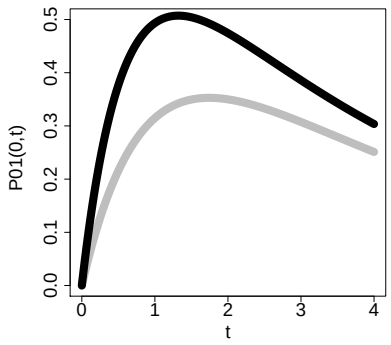
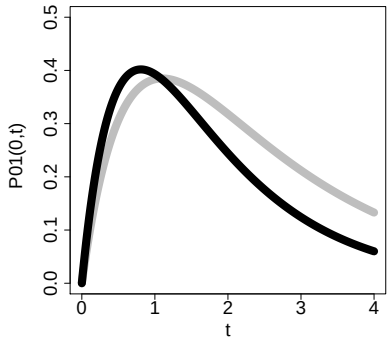
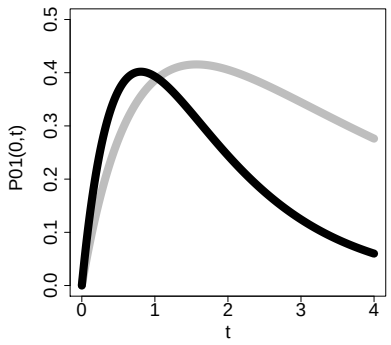
| | | | $\alpha = 0.01$ | | | $\alpha = 0.05$ | | |
|---|---|---|---|---|---|---|---|---|
| $H_1$ scenario | $n_1$ | $n_2$ | Linear | $L^2$ | KS | Linear | $L^2$ | KS |
|  | 50 | 50 | 0.032 | 0.032 | 0.049 | 0.116 | 0.112 | 0.153 |
| | 100 | 50 | 0.053 | 0.050 | 0.067 | 0.145 | 0.139 | 0.187 |
| | 100 | 100 | 0.049 | 0.050 | 0.072 | 0.142 | 0.147 | 0.214 |
| | 200 | 100 | 0.069 | 0.068 | 0.112 | 0.189 | 0.194 | 0.271 |
| | 200 | 200 | 0.075 | 0.073 | 0.154 | 0.215 | 0.240 | 0.352 |
|  | 50 | 50 | 0.092 | 0.093 | 0.149 | 0.231 | 0.239 | 0.329 |
| | 100 | 50 | 0.133 | 0.130 | 0.214 | 0.295 | 0.308 | 0.451 |
| | 100 | 100 | 0.143 | 0.154 | 0.293 | 0.319 | 0.375 | 0.576 |
| | 200 | 100 | 0.213 | 0.239 | 0.447 | 0.389 | 0.473 | 0.711 |
| | 200 | 200 | 0.248 | 0.324 | 0.640 | 0.456 | 0.609 | 0.854 |

Table 4: Simulation results about empirical power levels for the linear test (Linear), the $L^2$-norm-based test ($L^2$), and the Kolmogorov–Smirnov-type test (KS) for testing $H_0 : P^{(1)}_{0,12}(0,\cdot) = P^{(2)}_{0,12}(0,\cdot)$, under simulation scenarios 7 and 8. The weight function was $\hat{W}_{12}(t) = I[\prod_{l=1}^{2} \bar{Y}_l^{(1)}(t)\bar{Y}_l^{(2)}(t) > 0]$.

| | | | $\alpha = 0.01$ | | | $\alpha = 0.05$ | | |
|---|---|---|---|---|---|---|---|---|
| $H_1$ scenario | $n_1$ | $n_2$ | Linear | $L^2$ | KS | Linear | $L^2$ | KS |
|  | 50 | 50 | 0.015 | 0.028 | 0.038 | 0.060 | 0.112 | 0.142 |
| | 100 | 50 | 0.027 | 0.054 | 0.056 | 0.094 | 0.170 | 0.166 |
| | 100 | 100 | 0.036 | 0.079 | 0.075 | 0.125 | 0.237 | 0.212 |
| | 200 | 100 | 0.054 | 0.125 | 0.105 | 0.172 | 0.320 | 0.280 |
| | 200 | 200 | 0.099 | 0.243 | 0.164 | 0.264 | 0.503 | 0.380 |
|  | 50 | 50 | 0.073 | 0.202 | 0.241 | 0.207 | 0.464 | 0.493 |
| | 100 | 50 | 0.186 | 0.393 | 0.384 | 0.389 | 0.662 | 0.635 |
| | 100 | 100 | 0.352 | 0.679 | 0.622 | 0.589 | 0.878 | 0.844 |
| | 200 | 100 | 0.572 | 0.873 | 0.789 | 0.791 | 0.970 | 0.936 |
| | 200 | 200 | 0.846 | 0.982 | 0.952 | 0.948 | 0.997 | 0.990 |