

Supplemental material: “Numerical modeling of the effect of intermittent upwelling events on plankton blooms”

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I. HYDRODYNAMIC MODEL

The analytically defined model describes the velocity field for an incompressible viscid fluid with a Reynolds number at which the solution of the Navier-Stokes equation is time periodic. As described in the main text, the period of the flow is T . During this time, two vortices are created in the wake, with a phase shift of $T/2$, and move away from the island. Although this flow field can be obtained as a solution of the Navier-Stokes equation, we use an analytically generated field [1] that captures the main characteristics of this solution, but with smaller numerical efforts. Please note that the assumption of a two dimensional velocity field relies on the fact that the vertical velocities in the ocean are significantly smaller compared to the horizontal ones. Additional dynamical properties of the flow relevant to this work are reviewed in Sec. 3.1.

The parameters used for the flow field are shown in Tab. S1 and mimic the situation in the region of the Canary islands off northwest Africa, for details please check [2] and the references therein. Considering the unperturbed flow velocity u_0 in this geographical region the shedding time for the vortices $T/2$ is 16 days, which sets the period of the flow T to 32 days. The rotation of a vortex, on the other hand, is much shorter and is of approximately 3 days, according to observations [3]. Also following [2], and in agreement to the observation of currents in the region of Canary Islands, we superimpose the Ekman flow u_E in the y direction, perpendicular to the main flow, for $x > 1$ (length is given in units of $r = 25$ km), see Fig.1 (a) of the main text. The values for the Ekman drift are derived from the wind speed magnitudes of the region, for details see [2].

II. BIOLOGICAL MODEL

The biological model is based on some traditional NPZ models, such as of Steele & Henderson [4] and Edwards & Brindley [5]. All the parameters used in the model are listed in Tab. S2. The parameters are taken from ranges established in [6]. These ranges are widely used for modeling ecosystems in the open ocean and do not correspond to any particular geographical location. Additionally we would like to emphasize that there is no direct influence of the toxic species on the mortality of zooplankton in our model. Therefore this model is not restricted to HAB formation, but can be also used to describe the emergence of any phytoplankton bloom, in which the two different competing species are involved. The notation of toxic and non-toxic species simplifies the extension of our findings. While we discuss the net-growth rate of the phytoplankton in the main text, here we illustrate the difference in the top-down control in Fig. S1 (a, b) by showing the net growth of zooplankton for the two systems. While for system (II) the net growth rate of zooplankton is always positive Fig. S1 (b), this does not hold for system (I). For very low abundances of phytoplankton, there is not enough food for zooplankton to survive. Therefore, for very low nutrient supply and subsequently very low phytoplankton abundance, zooplankton would go extinct.

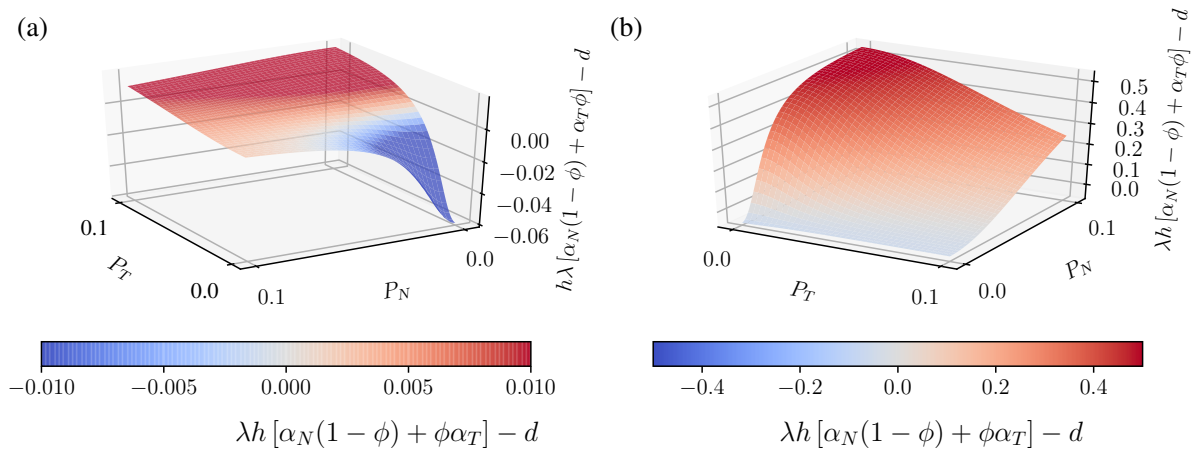
Next we shortly derive the range of values used for k in this work. The input rate through diffusion is given by $D_v \frac{\partial^2 N}{\partial z^2} \sim D_v \frac{(N_0 - N)}{h^2}$, where N_0 is the concentration of nutrients below the mixed layer and h is an average extension of the gradient. By using the definition $k_d = \frac{D_v}{h^2}$ we can rewrite the relation as $k_d(N_0 - N)$. For the ocean we find in the literature values of $D_v \sim 0.1 - 2.6 \text{ m}^2 \text{ day}^{-1}$ [7, 8]. We use

Parameter	Symbol	Used value
Island radius	r	25 km
Horizontal main flow velocity	u_0	0.18 m s^{-1}
Velocity of the Ekman flow	u_E	0.018 m s^{-1}
Vortex strength	ω	$55 \cdot 10^3 \text{ m}^2 \text{ s}^{-1}$

TABLE S1: Parameters used in the hydrodynamic flow model (for details of parameters see [2]).

Parameter	Symb.	Sys.(I)	Sys.(II)	Units
a/b maximum daily nutrient uptake	a	0.2	0.2	$\text{m}^{-1}\text{day}^{-1}$
Light attenuation by water	b	0.1	0.1	m^{-1}
Phytoplankton self-shading coefficient	c	0.4	0.4	$\text{m}^2 \text{gC}^{-1}$
Mortality rate of Zooplankton	d	0.065	0.065	day^{-1}
Half-saturation rate for N uptake of P_N	e_N	0.02	0.02	gCm^{-3}
Half-saturation rate for N uptake of P_T	e_T	0.1	0.1	gCm^{-3}
Respiration rate of P_N	r_N	0.1	0.1	day^{-1}
Respiration rate of P_T	r_T	0.05	0.1	day^{-1}
Conversion rate of nutrients into P_N	θ_N	0.4	0.4	
Conversion rate of nutrients into P_T	θ_T	0.8	0.4	
Phytoplankton sinking rate	s	0.08	0.08	day^{-1}
Growth efficiency of Z due to P_N	α_N	0.25	0.5	
Growth efficiency of Z due to P_T	α_T	0.2	0.2	
Z excretion fraction	β	0.33	0.33	
Excretion factor of Z	γ	0.5	0.5	
Maximum grazing rate of Z	λ	0.65	1.3	day^{-1}
Grazing of Z half saturation constant	μ	0.02	0.02	gC m^{-3}
Intensity of grazing on P_T	ϕ	0.5	0.05	

TABLE S2: The values used are taken from ranges given in [6]

FIG. S1: (a, b) Net-growth rate in day^{-1} of the zooplankton feeding as a function of concentrations (in gC m^{-3}) of toxic and non-toxic species of phytoplankton for: (a) system (I) and (b) system (II).

the known extension of the thermocline for the tropical region to estimate h and therefore adopt values from 10 to 25 m [7]. With these parameters we can evaluate k_d in the range of $10^{-2} - 10^{-4} \text{ day}^{-1}$. The nutrient transport due to upwelling, on the other hand, is defined as $u_z \frac{\partial N}{\partial z} \sim u_z \frac{\Delta N}{\Delta z}$. Therefore for the situation with upwelling we can define the thermocline exchange rate as $k_{\text{up}}(N_0 - N)$ with $k_{\text{up}} = \frac{u_z}{h} + k_d$. It is known that the vertical velocity u_z may reach values as large as $\sim 40 \text{ m day}^{-1}$ [8], however specifically for the region of the Canarian Archipelago we find estimations close to 10 m day^{-1} [9]. This gives us k_{up} of 1 day^{-1} . Therefore in this work we restrict ourselves to value for k_{up} of the order of unity.

III. COUPLED MODEL

The full system of equations is solved by the using a semi-Lagrangian algorithm [10], the code can be found at <https://github.com/kсениаguseva/Upwelling>. We use a grid of $[500 \times 300]$ points, the integration is carried out for step size $dt = 0.01T$. In particular, the diffusion is carried out in intermediate smaller steps $dt_D = \frac{dt}{10}$ using an Eulerian scheme. Small steps t_D are required for the numerical stability of the diffusion process, which can be guaranteed by $Dt_D/(dx dy) < 0.5$. Please note that the current model does not capture the small scale heterogeneities which can arise in the plankton populations and approximates them to average values, dismissing any spatial patterns within a single grid cell. This spatial averaging in turn may influence the biological dynamics since the trophic interactions occur at the individual level [11, 12].

Everywhere in the observation area except for the prescribed upwelling region located above the island, the cross thermocline exchange rate is set to k_d . In the upwelling region the value is exchanged between k_d and k_{up} in time, corresponding to intermittent upwelling events. The upwelling region, if not stated otherwise, spans the region: $x \in [-1, 1]$ and $y \in [2, 2.5]$, see Fig.1 of the main text.

All the modeled species enter the system from the left at $x = -2$ with the same concentration at all y values and are advected across the observation area. We assume that they arrive from the open ocean, an environment poor in nutrients and plankton. Therefore we use as the influx 20% of the steady state concentration value reached by each given species for a cross thermocline exchange rate k_d . Please note that all species of phytoplankton are present in the system in the influx. As we show in Sec.2.2 of the main text the non-toxic species dominates in the influx conditions, for both scenarios that we analyse.

IV. BIOLOGICAL MODEL WITH HYDRODYNAMICS, IN THE ABSENCE OF UPWELLING

Here we present the dynamics of the coupled biological-hydrodynamic model without upwelling to allow a comparison with the results in the main text. First, we can mention that system (I) has very similar properties as the population dynamics analysed in [13]. But in contrast to [13] we are here more interested in the dynamics of plankton in the whole area and not only in the development of a plankton bloom related to specific regions such as vortices. When the upwelling is not present the non-toxic species is found in higher concentrations in a region around the island and inside every vortex formed in the wake. This spatial distribution of the non-toxic species reflects the nutrient accumulation in those regions, which results from the vertical exchange of nutrients across the thermocline while the horizontal advection is slow. Subsequently, this high nutrient concentration is captured by the vortices behind the island, where it creates good conditions for the growth of the non-toxic species. However, the nutrient accumulation cannot be directly observed, since the nutrients are promptly consumed by the phytoplankton, see spatial plots in Fig. S2 (a). Note that nutrient amount never reaches the threshold to trigger a HAB in these conditions. Notably, system (I) is characterized by a periodic timeseries for the spatial average of the non-toxic species following the vortex formation and advection: $\langle P_N \rangle$ has a period $T/2$. For system (I) the averages correspond to: $\langle P_N \rangle^* = 0.042 \text{ gC m}^{-3}$, $\langle P_T \rangle^* = 0.019 \text{ gC m}^{-3}$ and $\langle Z \rangle^* = 0.005 \text{ gC m}^{-3}$, $\langle N \rangle^* = 0.025 \text{ gC m}^{-3}$.

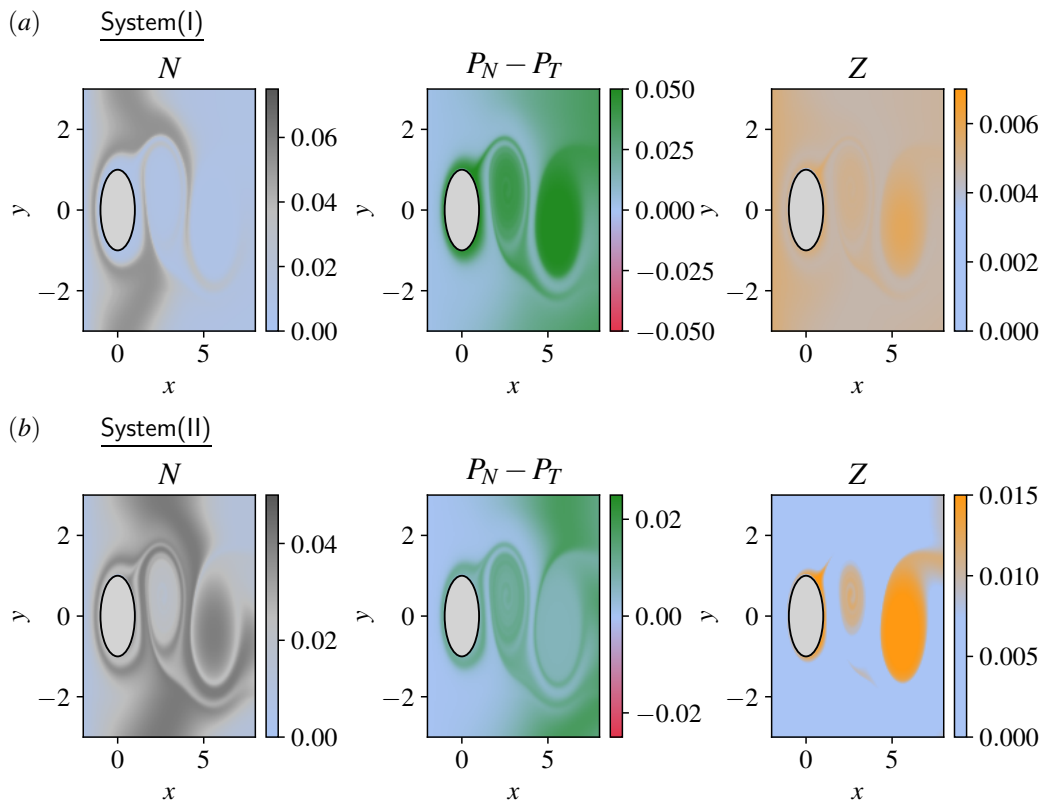


FIG. S2: Spatial distribution of dominance patterns with concentrations given in gC m^{-3} , at $t = 1.25 T$, for the system in the absence of upwelling (a) system (I) and (b) system (II).

We continue with the characterization of the spatio-temporal patterns formed in system (II). For the scenario without upwelling the non-toxic species, as in the previous case, develops around the island and inside the vortices. As in system (I), the growth of phytoplankton also follows the large concentration of nutrients. The large presence of non-toxic species creates now good conditions for the development of zooplankton, that also grows in the same location but at a slower rate. As a result, this system displays more complex spatio-temporal patterns, where as the vortices are advected by the flow field

the concentration of zooplankton increases as well. Note that here as well, due to the growth of the non-toxic species, the vortices become depleted in nutrients. However as the zooplankton grows and feeds increasingly on the non-toxic species this nutrient concentration is partially recovered, although not sufficiently to create an advantage to the development of the toxic species (see Fig. S2 (b)). In summary, in system (II) the non-toxic species is the dominant species everywhere in space with the average concentration $\langle P_N \rangle^* = 0.0084 \text{ gC m}^{-3}$, which is in agreement with low nutrient concentration in the system $\langle N \rangle^* = 0.0286 \text{ gC m}^{-3}$. The average concentration of the other plankton species are: $\langle P_T \rangle^* = 0.0010 \text{ gC m}^{-3}$ and $\langle Z \rangle^* = 0.0054 \text{ gC m}^{-3}$. Furthermore, the average concentration of phytoplankton species is about one order of magnitude smaller than for system (I). This is a direct consequence of the used values of the thermocline exchange rate.

V. INTERMITTENCY

To model the time series of intermittent upwelling events we use the absolute value of the x_1 variable from the system of the following differential equations:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -x_1^3 - 2x_1x_3 + x_1x_5 - \mu x_2, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= -x_3^3 - \nu_1 x_1^2 + x_3x_3 - \nu_2 x_4, \\ \dot{x}_5 &= -\nu_3 x_5 - \nu_4 x_1^2 - \nu_5 (x_3^2 - 1), \end{aligned} \tag{1}$$

where $\mu = 1.815$, $\nu_1 = 1.00$, $\nu_2 = 1.815$, $\nu_3 = 0.44$, $\nu_4 = 2.86$, $\nu_5 = 2.86$. For these parameters the system displays on-off intermittency, for details see [14].

To lead to upwelling events of the adequate duration, corresponding to few days as shown in [7], we rescale the time in Eqs.(1) using $t' = t/T$, where T is the period of the flow field. The “off” states of the time series from Eqs.(1) are characterized by $|x_1| \sim 0$, and the “on” state by $\max(|x_1|) \sim 1$. Therefore, we transform this time series to:

$$k(t') = (k_{\text{up}} - k_d)|x_1(t')| + k_d. \tag{2}$$

With this transformation the new time series has the “off” state $\sim k_d$ and an “on” state which can be at most k_{up} . The time series $k(t')$ is used to define the cross thermocline exchange rate at the upwelling region.

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