Supplementary Material

Movie S1 (Separate file). Transient behavior of rolling ring in the presence of illumination. Results are shown for $\theta_I = 0.2$ and $\theta_I = 0.3$ at various intensities of illumination.

Movie S2 (Separate file). Transient behavior of flapping beam in the presence of illumination. Results are shown for $\theta_I = \pi/4$ and various intensities of illumination. The top row is the case when $\Lambda > 0$ and the bottom row is when $\Lambda < 0$

A Computational Model

The numerical method is motivated by the discrete elastic rod model[1]. We partition the beam into $N-1$ segments $\mathcal{S}^i = (S_i, S_{i+1}),\ i = 1, \ldots, N-1$ which are all equal in arc-length by introducing N nodes: the ith node is at arc-length $S_i = (i-1)L/(N-1)$. We introduce the angle $\theta^i,\ i = 1, \ldots, N-1$ to be the angle that the segment $\mathcal{S}^i$ makes to the horizontal as our main kinematic variable. We can then obtain the current position of the nth node by exploiting the inextensibility condition as follows:

$$\mathbf{x}_n = \mathbf{x}_1 + \sum_{i=2}^{n} (S_i - S_{i-1}) \left(\cos\theta^{i-1}\mathbf{e}_1 + \sin\theta^{i-1}\mathbf{e}_2 \right).$$

The curvature is carried at the nodes and defined as $\kappa_i = \theta^i - \theta^{i-1}$ so that the total bending energy of the beam (discrete equivalent to (15)) is given by

$$E_B[\theta] = \sum_{i=2}^{N-1} \frac{1}{2} J_i (\theta^i - \theta^{i-1} - \kappa_i^0)^2 \tag{33}$$

where J_i is a bending modulus and κ_i^0 is the discrete natural curvature at the ith node.

We obtain the equilibrium equation (discrete equivalent to (12)) by taking the variation of E_B with respect to θ^j:

$$\frac{\partial E_B}{\partial \theta^j} = J_j(\theta^j - \theta^{j-1} - \kappa_j^0) - J_{j+1}(\theta^{j+1} - \theta^j - \kappa_{j+1}^0) = 0. \tag{34}$$

Given the spontaneous curvatures $\{\kappa_j^0\}$, we solve these equations for $\{\theta^j\}$ subject to appropriate boundary conditions. In order to improve the stability and convergence, it is convenient to have the Hessian,

$$\frac{\partial^2 E_B}{\partial \theta^j \partial \theta^k} = -J_j \delta_k^j + (J_j + J_{j+1})\delta_k^j - J_{j+1}\delta_k^{j+1}.$$

It remains to specify the spontaneous curvature. This evolves according to (13) whose discrete version is given by the set of ordinary differential equations:

$$\frac{\mathrm{d}\kappa_i^0}{\mathrm{d}t} + \kappa_i^0 = \Lambda f(\theta_i - \theta_I), \tag{35}$$

[1]Miklós Bergou, Max Wardetzky, Stephen Robinson, Basile Audoly, and Eitan Grinspun. Discrete elastic rods. *ACM Transactions on Graphics*, 27(3):63:1 – 63:12, August 2008.

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where $f(\theta_i - \theta_I)$ is as defined in Equation 13 and $\theta_i = (ave)(\theta^i, \theta^{i-1})$ is defined as the angle of the tangent of the ith node.

Equation (35) is discretized in time using an explicit Newton time stepping algorithm. Time dependent solutions are obtained by alternating the elastic relaxation in equation (34) and evolving of natural curvatures κ_i^0 based on equation (35) over a time step.

A.1 Elastic Ring

In Section 2, we analyzed rolling rings. They can be simulated by adapting the general numerical procedure outlined above as follows. The closure of the ring is imposed by the following constraints:

$$\theta^1 = 0 \qquad \mathbf{x}_1 = \mathbf{x}_{N-1} \qquad \mathbf{x}_2 = \mathbf{x}_N$$

The first of these can be implemented explicitly by freezing that degree of freedom and represents that the point of contact is tangent to the surface.

The last two enforce the closure constraint. The system is initialized by assuming a constant curvature which makes the last two nodes coincident with the first two. Then the system is relaxed by minimizing the energy while imposing the constraints. In order to stabilize the point of contact when the system is circular, a small amount of gravity is initially added and removed once the natural curvature deviates from its initial state.

The algorithm for calculating the translation and rotation of the system is as follows. Initially, the point of contact is defined to be the first and second nodes (second to last and last due to constraints). Then, given a natural curvature, κ_j^0, the energy is minimized to find the new configuration. The natural curvature is then updated using the explicit forward Euler scheme according to (35). Then, using a small window near the first and second nodes (which wraps around to nodes on the far end of the beam), the closest node to the calculated center of mass is found. Then, the nodes on either side of that node are tested to find the closest to the center of mass. This then forms an ordered pair of nodes $(\mathbf{x}_i, \mathbf{x}_{i+1})$ which defines the segment closest to the center of mass. Then, by shifting the minimized curvature $\theta^i \to \theta^1$, $\theta^{i+1} \to \theta^2$, etc in a cyclic manner (so the quantities at end points get wrapped around the beam). Similar transformations are done to the natural curvature ($\kappa_i^0 \to \kappa_{N-1}^0$, $\kappa_{i+1}^0 \to \kappa_2^0$). Note that these transformations are done in such a way that the ordering of the nodes is preserved and wrapped. At this point, the updated points of contact are now the 1st and 2nd nodes and the algorithm can be repeated to integrate the system in time. This solves for the rotation of the system while the translation can be found by using the rolling contact condition. Using the convention before, we had set $\mathbf{x}_1 = \mathbf{0}$. We can set this to be the relative position where the true position of node i is defined as $\tilde{\mathbf{x}}_i = \mathbf{x}_{S_c} + \mathbf{x}_i$ where $\mathbf{x}_{S_c}$ is the position of the point of contact. $\mathbf{x}_{S_c}$ is found using the rolling condition. Let $\mathbf{x}_{S_c}^k$ be the position of the point of contact at time step k and i be the shift necessary to establish that the point of contact is vertically aligned with the center of mass. Then,

$$\mathbf{x}_{S_c}^{k+1} = \begin{cases} \mathbf{x}_{S_c}^k + (S_i - S_1)\mathbf{E}_1 & \text{if } i \in [1, Ns] \\ \mathbf{x}_{S_c}^k + (S_i - S_{N-1})\mathbf{E}_1 & \text{if } i \in [N - Ns, N-1] \end{cases},$$

where N_s is a small window (usually set to $N/20$). If i is not in the range of values defined above, then the time discretization is made finer in order to ensure that the rotations induced in each time step correlate with a small translation. The results for various angles of incidence of light and intensities are given in Movie S1 in the supplementary material. The "velocity" of the system is

then found by finding the distance the point of contact travels over a small time window. Steady state velocities are found by iterating the time stepping procedure until the velocity reaches a steady value.

A.2 Doubly Clamped Beam

The doubly clamped system can be solved by setting up the following constraints

$$\theta^1 = 0 \qquad \theta^{N-1} = 0 \qquad \mathbf{x}_N = l_f \mathbf{e}_1$$

where $l_f < L$ is the distance between the two endpoints. As before, the first of these two constraints can be implemented explicitly by freezing those degrees of freedom and requires no special treatment, while the latter two constraints need to be implemented in the optimization engine. The initial solution is obtained numerically by decreasing l_f from 1 to its actual value in small steps. The system is integrated in time by alternating between relaxing the elastic energy and updating the natural curvature using an explicit Newton time stepping method. The results for various angles of incidence of light and intensities are given in Movie S2 in the supplementary material.

B Equilibrium and Stability Analysis

Investigation of the snapping instabilities from Section 3 requires obtaining the second variation of the energy $\mathcal{E}(\theta)$ in the presence of m constraints $c_i(\theta) = 0$, $i = 1, 2, ..., m$, where $\theta \in \mathbb{R}^n$ is the set of degrees of freedom. Denote the feasible set $\mathcal{C} = \{\theta \in \mathbb{R}^n \text{ s.t. } c_i(\theta) = 0\}$. We are interested in solutions $\bar{\theta} \in \mathcal{C} \subset \mathbb{R}^n$ such that

$$\mathcal{E}\left(\bar{\theta} + \varepsilon u + \frac{1}{2}\varepsilon^2 w\right) \geq \mathcal{E}(\bar{\theta}), \quad \forall u, w \in \mathbb{R}^n$$

satisfying $\bar{\theta} + \varepsilon u + \frac{1}{2}\varepsilon^2 w \in \mathcal{C}$, with $\varepsilon \to 0$. Expanding each of these out to first order and simplifying gives,

$$\nabla \mathcal{E}(\bar{\theta}) \cdot u = 0\,,$$
$$\nabla c_i(\bar{\theta}) \cdot u = 0\,.$$

where ∇ denotes the gradient operator relative to the degrees of freedom of the function ($(\nabla E)_i = \frac{\partial E}{\partial \theta_i}$). This gives the equilibrium condition,

$$\nabla \mathcal{E}(\bar{\theta}) + \sum_{i=1}^{m} \lambda_i \nabla c_i(\bar{\theta}) = 0,$$

where the parameters λ_i are Lagrange multipliers.

For stability, we require that any perturbation which satisfies the constraints will increase the energy. To do this, we expand our system to second order in ε and simplify:

$$u \cdot \nabla^2 \mathcal{E}(\bar{\theta}) u + \nabla \mathcal{E}(\bar{\theta}) \cdot w \geq 0\,,$$
$$u \cdot \nabla^2 c_i(\bar{\theta}) u + \nabla c_i(\bar{\theta}) \cdot w = 0\,,$$

where ∇^2 is the Hessian operator which returns the symmetric matrix of second derivatives. Using the equilibrium condition, we have

$$\nabla \mathcal{E}(\bar{\theta}) \cdot w = -\sum_{i=1}^{m} \lambda_i \nabla c_i(\bar{\theta}) \cdot w = u \cdot \sum_{i=1}^{m} \lambda_i \nabla^2 c_i(\bar{\theta}) u \,.$$

Plugging this into the above inequality, we have the stability condition that

$$u \cdot \left(\nabla^2 \mathcal{E}(\bar{\theta}) + \sum_{i=1}^{m} \lambda_i \nabla^2 c_i(\bar{\theta}) \right) u \geq 0 \,,$$

for all u such that

$$\nabla c_i(\bar{\theta}) \cdot u = 0 \,.$$

To determine whether a configuration satisfies this condition, we want to project $\mathbb{R}^n$ onto the space tangent to the constraints. This is done by a Gram-Schmidt process where

$$v_1 = \frac{\nabla c_1(\bar{\theta})}{\|\nabla c_1(\bar{\theta})\|} \,,$$

$$v_k = \frac{\nabla c_k(\bar{\theta}) - \sum_{i=1}^{k-1} (\nabla c_k(\bar{\theta}) \cdot v_i) v_i}{\|\nabla c_k(\bar{\theta}) - \sum_{i=1}^{k-1} (\nabla c_k(\bar{\theta}) \cdot v_i) v_i\|} \,,$$

$$P = I - \sum_{i=1}^{m} v_i \otimes v_i \,.$$

The stability analysis then boils down to calculating the eigenvalues of $P \left(\nabla^2 \mathcal{E}(\bar{\theta}) + \sum_{i=1}^{m} \lambda_i \nabla^2 c_i(\bar{\theta}) \right) P$. Due to the projection, there will be m zero eigenvalues and stability is implied when all other eigenvalues are greater than zero. This analysis determines if there exists feasible paths which locally lowers the energy; therefore, the existence of a non-positive eigenvalue implies a loss of stability of the configuration.

C Experimental Methods

Materials The monomers 1,4-Bis[4-(6-acryloyloxyhexyloxy)benzoyloxy]-2-methylbenzene (RM82) and 4,4'-Bis(6-acryloyloxyhexyloxy)azobenzene (Azo-6) were purchased from Synthon Chemicals and the photoinitiator phenylbis(2,4,6-trimethylbenzoyl)phosphine oxide) (Irgacure 819) was purchased from Sigma Aldrich. All chemicals were used as received. The polyimide alignment layer Elvamide was donated by Dupont.

Synthesis of LCN Beams Planar nematic LCN films were prepared following the procedure of Gelebart et al. (1) with modification. To synthesize films with the a penetration depth at 365 nm of 1.5 m, a formulation of 9.2 : 90.8 by weight of Azo-6 : RM82 was used, with 2.5 wt% of photoinitiator with respect to the total monomer weight. In a typical sample preparation, 4.6 mg Azo-6, 45.4 mg RM82, and 1.25 mg Irgacure 819 were melted together in a vial and vortexed repeatedly to ensure mixing. The molten monomer mixture was then infiltrated via capillary action into alignment cells on a hot plate at 100°C. The alignment cells were prepared by spin-coating

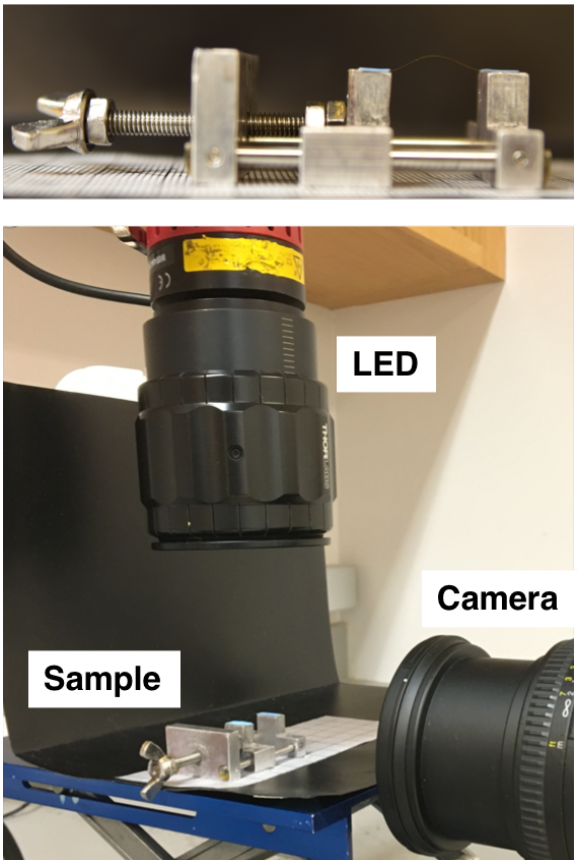

Figure S1: (Top) The sample is fixed at the ends in a home-made compression device. (Bottom) Experimental set-up is composed of a beam illuminated overhead by a UV LED and imaged from the side by a camera.

Elvamide onto clean glass slides, rubbing the slides with a velvet cloth, and gluing the two Elvamide sides facing each other with epoxy mixed with 15 m glass beads. The filled cells were subsequently cooled to 80°C, held isothermal for 5 minutes to induce alignment of the liquid crystalline mesogens, and photopolymerized for 30 minutes with 405 nm light. Following photopolymerization, samples were post-cured at 120°C for 10 minutes and the 15m thick LCNs were harvested by cracking open the alignment cells with a razor blade. Finally, beams of 1 mm in width were cut from the film with the nematic director along the long axis of the strip.

Photoactuation Experiments Buckled beams with dimensions 1 mm x 15 mm x 50 m were prepared by clamping the ends of the film in a home-made film clamp device and compressed to an end-to-end distance of Lfinal/Linitial = 0.95. The buckled film was subsequently illuminated from above with a 365 nm LED (ThorLabs) equipped with a Guassian profile focused onto the sample via an adjustable focusing lens. Each experiment is recorded using a camera (Nikon 5500) fitted with a macrolens operating at a recording speed of 60 frames per second.