Additional file 3: Derivation of an alternative computation of $\mathbf{A}^{gn}\left(\mathbf{A}^{nn}\right)^{-1}\mathbf{A}^{ng}$

Let the inverse of the pedigree relationship matrix of all animals in the pedigree, $\mathbf{A}^{-1}$, be partitioned between the genotyped animals g, and the non-genotyped animals n:
$\mathbf{A}^{-1} = \begin{bmatrix} \mathbf{A}^{nn} & \mathbf{A}^{ng} \\ \mathbf{A}^{gn} & \mathbf{A}^{gg} \end{bmatrix}$.

Let the inverse of the pedigree relationship matrix of all genotyped animals and their ancestors, $\mathbf{A}_{anc}^{-1}$, be partitioned between the genotyped animals g, and the non-genotyped ancestors of the genotyped animals:
$\mathbf{A}_{anc}^{-1} = \begin{bmatrix} \mathbf{A}_{anc,nn} & \mathbf{A}_{anc,ng} \\ \mathbf{A}_{anc,gn} & \mathbf{A}_{anc,gg} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}_{anc}^{nn} & \mathbf{A}_{anc}^{ng} \\ \mathbf{A}_{anc}^{gn} & \mathbf{A}_{anc}^{gg} \end{bmatrix}$.
It is worth noting that a non-genotyped ancestor can be an offspring of a genotyped animal.

The inverse of the pedigree relationship matrix among genotyped animals g is then equal to:
$\mathbf{A}_{gg}^{-1} = \mathbf{A}^{gg} - \mathbf{A}^{gn}\left(\mathbf{A}^{nn}\right)^{-1}\mathbf{A}^{ng} = \mathbf{A}_{anc}^{gg} - \mathbf{A}_{anc}^{gn}\left(\mathbf{A}_{anc}^{nn}\right)^{-1}\mathbf{A}_{anc}^{ng}$

Therefore, it follows that:

$$\begin{aligned} \mathbf{Q} &= \mathbf{A}^{gn}\left(\mathbf{A}^{nn}\right)^{-1}\mathbf{A}^{ng} \\ &= \mathbf{A}_{anc}^{gn}\left(\mathbf{A}_{anc}^{nn}\right)^{-1}\mathbf{A}_{anc}^{ng} + \mathbf{A}^{gg} - \mathbf{A}_{anc}^{gg} \\ &= \mathbf{A}_{anc}^{gn}\left(\mathbf{A}_{anc}^{nn}\right)^{-1}\mathbf{A}_{anc}^{ng} + \mathbf{\Delta} \end{aligned}$$

From Henderson's rules to construct directly the inverse of a pedigree relationship matrix $\mathbf{A}^{-1}$ [22], adding the contribution of an animal to $\mathbf{A}^{-1}$ will modify at most for 9 entries of $\mathbf{A}^{-1}$: three diagonal elements of $\mathbf{A}^{-1}$ corresponding to the entries of the animal, its sire, and its dam, four off-diagonal elements of $\mathbf{A}^{-1}$ corresponding to the entries between the animal and its sire or its dam, and two off-diagonal elements of $\mathbf{A}^{-1}$ corresponding to the entries between its sire and its dam.

Therefore, only the genotyped animals and their non-genotyped offspring that are also ancestors of genotyped animals, contribute to $\mathbf{A}_{anc}^{gg}$, and only the genotyped animals and all their non-genotyped offspring contribute to $\mathbf{A}^{gg}$. It follows that the difference between $\mathbf{A}^{gg}$ and $\mathbf{A}_{anc}^{gg}$, $\mathbf{\Delta}$, is only due to the contributions to $\mathbf{A}^{-1}$ of the non-genotyped offspring of the genotyped animals that are not ancestors of genotyped animals. The matrix $\mathbf{\Delta}$ can be then easily and directly constructed using Henderson's rules by reading the pedigree only once.