## **Description of Additional Supplementary Files**

Supplementary Movie 1:

**Hopfion-assisted switching.** The video illustrates the evolution of the polarization configuration upon the system passing the polarization switching displayed on the hysteresis loop for the strongly densified nanoparticles with h=1 nm shown in Fig. 4e of the main text and going through the sequence of topological phase transitions.