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Supplementary Information for

Paternal provisioning results from ecological change

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Supplementary text
Figs. S1 to S5

Supporting Information Text

Model analysis

We derive analytical results for how the replicator dynamic (see equation (5) in the main text) is affected by changes in one or several of the model parameters, by way of analyzing how these changes impact the class of the evolutionary game. We use a slightly mode general model than the one in the main text, by assuming that the amount of food that a Dad brings to his mate and her offspring when his neighbor is a Dad and a Cad, respectively, are

$$y_{DD} = \delta \cdot \frac{(1-\mu) \cdot 2Y_D + \mu \cdot Y_D^2}{2}. \tag{1}$$

$$y_{DC} = \delta \cdot \frac{(1-\mu) \cdot (Y_D + Y_C) + \mu \cdot Y_D Y_C}{2}, \tag{2}$$

where the parameter $\delta \in [0,1]$ is the share of the collected food that a Dad brings back to his mate and her offspring.

In the section entitled **Male productive behaviors** below, we examine the effects of changes in the parameters pertaining to male productive behavior, (δ, Y_D, Y_C), holding the female openness to extra-pair matings (ϕ) and the complementarity parameters (μ, κ) fixed. Then, in the section entitled **Female sexual behavior**, we will confirm analytically the intuition that, for any parameter configuration $(\delta, Y_D, Y_C, \mu, \kappa)$, a decrease in female openness to extra-pair matings (ϕ) unambiguously facilitates the spread of Dads. These results are then used to analyze the effects of changes in the complementarity parameters (μ, κ), presented in the section entitled **Male-female and male-male complementarities**.

Prior to deriving these results, however, we begin by presenting some general observations that will be used to conduct the analysis. While these observations are necessary to understand the proofs of the results, readers who are not interested in the proofs can proceed directly to the sections with the results.

Preliminaries. Recall that, at the level of reproductive success, the replicator dynamic writes (equation (5) in the main text), with the average reproductive success of Dads, $\bar{\pi}_D(d_t)$, and that of Cads, $\bar{\pi}_C(d_t)$, being written explicitly in terms of the reproductive successes π_{ij}, $i,j \in \{C,D\}$, and d_t):

$$\dot{d}_t = d_t\,(1-d_t)\,[d_t \cdot \pi_{DD} + (1-d_t) \cdot \pi_{DC} - d_t \cdot \pi_{CD} - (1-d_t) \cdot \pi_{CC}]. \tag{3}$$

Using the expressions for the reproductive successes (1)-(4) in the main text, Eq. (3) can be written as follows:

$$\dot{d}_t = d_t\,(1-d_t)\,[d_t \cdot s_{DD} + (1-d_t) \cdot (1-\phi) \cdot s_{DC} - d_t \cdot (s_{CD} + \phi \cdot s_{DC}) - (1-d_t) \cdot s_{CC}]2N, \tag{4}$$

where

$$s_{CC} = s_{CD} = \frac{x}{A} \tag{5}$$

$$s_{DD} = \frac{(1-\kappa) \cdot (x + y_{DD}) + \kappa \cdot x \cdot (y_{DD} + 1)}{A}, \tag{6}$$

$$s_{DC} = \frac{(1-\kappa) \cdot (x + y_{DC}) + \kappa \cdot x \cdot (y_{DC} + 1)}{A}, \tag{7}$$

$$y_{DD} = \delta \cdot Y_{DD} = \delta \cdot \frac{2\,(1-\mu)\,Y_D + \mu \cdot Y_D^2}{2}, \tag{8}$$

and

$$y_{DC} = \delta \cdot Y_{DC} = \delta \cdot \frac{(1-\mu)\,(Y_D + Y_C) + \mu \cdot (Y_D Y_C)}{2}. \tag{9}$$

Below we will simplify the notation slightly by letting $s_{CC} = s_{CD} \equiv s_C$ (that the survival probability of the offspring of a Cad's pair-bonded mate does not depend on whether the neighbor is a Cad or a Dad comes from the assumption that a Cad brings back no food).

Defining Δ_1 and Δ_2 as follows:

$$\Delta_1 = s_{DD} - \phi \cdot s_{DC} - s_C \tag{10}$$

$$\Delta_2 = s_C - (1-\phi) \cdot s_{DC}, \tag{11}$$

the conditions for the four different evolutionary game classes to obtain can be written as follows:
Game class [Cads] (which leads to a population with only Cads) obtains if $\Delta_1 < 0$ and $\Delta_2 > 0$.
Game class [Dads] (which leads to a population with only Dads) obtains if $\Delta_1 > 0$ and $\Delta_2 < 0$.
Game class [Cads-and-Dads] (which leads to a stable polymorphism)obtains if $\Delta_1 < 0$ and $\Delta_2 < 0$.
Game class [Cads-or-Dads] (which leads to a population with either only Dads or only Cads) obtains if $\Delta_1 > 0$ and $\Delta_2 > 0$.

Figure S1 illustrates these conditions. In this figure, the horizontal axis measures the value of Δ_1 and the vertical axis measures the value of Δ_2. Each orthant corresponds to one game class.

Figure S1 shows that in order for the game class to transition from **[Cads]** to **[Cads-and-Dads]**, Δ_2 must decrease and Δ_1 cannot increase too much; likewise, in order for the game class to transition from **[Cads]** to **[Dads]**, there must be a large enough decrease in Δ_2 and a large enough increase in Δ_1; etc.

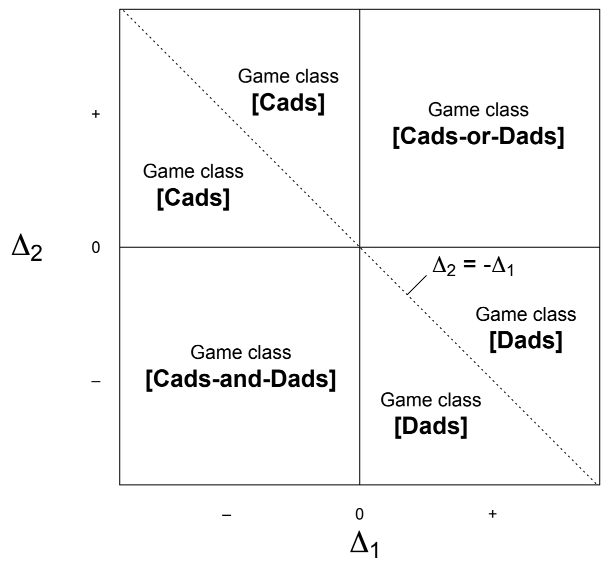

Fig. S1. The class of the evolutionary game depends on the signs of Δ_1 and Δ_2.

Importantly for the analysis below, Figure S1 shows that analysis of the sum $\Delta_1 + \Delta_2$ is informative. The downward-sloping dashed line through the origin in the figure shows values of Δ_1 and Δ_2 for which this sum is nil, $\Delta_1 + \Delta_2 = 0$. The figure thus shows that at points above the line $\Delta_1 + \Delta_2 = 0$ (i.e., if $\Delta_1 + \Delta_2 > 0$), the game class is either **[Cads]**, **[Cads-or-Dads]**, or **[Dads]**; likewise, at points below the line $\Delta_1 + \Delta_2 = 0$ (i.e., if $\Delta_1 + \Delta_2 < 0$), the game class is either **[Cads]**, **[Cads-and-Dads]**, or **[Dads]**. Noting that the sum $\Delta_1 + \Delta_2$ simplifies to:

$$\Delta_1 + \Delta_2 = s_{DD} - \phi \cdot s_{DC} - s_C + s_C - (1 - \phi) \cdot s_{DC} = s_{DD} - s_{DC}, \qquad [12]$$

the preceding arguments and observations together prove the following lemma:

Lemma 1 *If $s_{DD} - s_{DC} > 0$, the evolutionary game class is either **[Cads]**, **[Cads-or-Dads]**, or **[Dads]**. If $s_{DD} - s_{DC} = 0$, the evolutionary game class is either **[Cads]** or **[Dads]**. If $s_{DD} - s_{DC} < 0$, the evolutionary game class is either **[Cads]**, **[Cads-and-Dads]**, or **[Dads]**.*

Below we will thus come to study the sign of

$$s_{DD} - s_{DC} \quad = \quad \frac{(1-\kappa) \cdot (x + y_{DD}) + \kappa \cdot x \cdot (y_{DD} + 1)}{A} - \frac{(1-\kappa) \cdot (x + y_{DC}) + \kappa \cdot x \cdot (y_{DC} + 1)}{A} \qquad [13]$$

In the second part of this preliminary analysis, we derive some basic comparative statics results on how the survival probability (see equation (9) in the main text for the general expression) varies with the mother's input x, the mother's mate's input y, and the parameters δ and κ.

First, for any $\kappa \in [0, 1]$ and any (x, y), the survival probability is strictly increasing in x and in y:

$$\frac{\partial s(x, y)}{\partial x} = \frac{1}{A} > 0 \qquad [14]$$

$$\frac{\partial s(x, y)}{\partial y} = \frac{\kappa \cdot x}{A} > 0. \qquad [15]$$

Importantly, this in turn implies that the value of $s(x, y)$ would increase as a result of an increase in δ, since y is strictly increasing in δ.

Second, to study the marginal effect of κ on survival, we replace y_{DD} and y_{DC} in Eq. (6) and Eq. (7) by the expressions in Eq. (8) Eq. (9), and write the survival probabilities s_{DD} and s_{DC} as functions of δ, μ, and κ (a tilde has been added to differentiate these functions from the function s, which maps female and male contributions x and y to the survival probability):

$$\tilde{s}_{DD}(\delta, \mu, \kappa) = \left[(1-\kappa) \cdot \left(x + \delta \cdot \frac{2(1-\mu) Y_D + \mu \cdot Y_D^2}{2}\right) + \kappa \cdot x \cdot \left(\delta \cdot \frac{2(1-\mu) Y_D + \mu \cdot Y_D^2}{2} + 1\right)\right] / A, \qquad [16]$$

and

$$\tilde{s}_{DC}(\delta,\mu,\kappa) = \left[(1-\kappa)\cdot\left(x+\delta\cdot\frac{(1-\mu)\,(Y_D+Y_C)+\mu\cdot(Y_DY_C)}{2}\right)+\kappa\cdot x\cdot\left(\delta\cdot\frac{(1-\mu)\,(Y_D+Y_C)+\mu\cdot(Y_DY_C)}{2}+1\right)\right]/A. \quad [17]$$

Hence, for $i = D, C$:

$$\frac{\partial \tilde{s}_{Di}(\delta,\mu,\kappa)}{\partial\kappa} = \left[-\left(x+\delta\cdot\frac{(1-\mu)\,(Y_D+Y_i)+\mu\cdot(Y_DY_i)}{2}\right)+x\cdot\left(\delta\cdot\frac{(1-\mu)\,(Y_D+Y_i)+\mu\cdot(Y_DY_i)}{2}+1\right)\right]/A. \quad [18]$$

This is strictly positive if $x > 1$ and $Y_D > 0$, which is true.

We collect these observations in the following lemma:

Lemma 2 *The survival probability is strictly increasing in x, y, δ and κ.*

We now turn to the results.

Male productive behaviors. Since we assume that Cads do not bring any food back home, the productive behavior of Dads and Cads may differ in two dimensions: their productivities ($Y_C \geq 2$ and $Y_D \geq 2$) and the amount of food brought home by Dads ($\delta \in (0,1]$). Here we show how the values of these parameters affect the game class that obtains. We summarize the results in propositions.

We first show two results pertaining to the difference in effectiveness at producing food for Dads and Cads.

Proposition 1 *Consider any Dad type $(\delta, Y_D) \in (0,1] \times [2,+\infty)$. Then:*
*(i) If $Y_C = Y_D$, the evolutionary game is either in class **[Cads]** or in class **[Dads]**.*
*(ii) If $Y_C > Y_D$, the evolutionary game is either in class **[Cads]**, in class **[Cads-and-Dads]**, or in class **[Dads]**.*
*(iii) If $Y_C < Y_D$, the evolutionary game is either in class **[Cads]**, in class **[Cads-or-Dads]**, or in class **[Dads]**.*

Proof: Simplification of Eq. (13) reveals that the difference $s_{DD} - s_{DC}$ has the same sign as:

$$[(1-\kappa)\cdot\delta\cdot(1-\mu)+(1-\kappa)\cdot\mu\cdot Y_D+\kappa\cdot x\cdot\delta\cdot(1-\mu)+\kappa\cdot\mu\cdot Y_D]\cdot(Y_D-Y_C)\,, \quad [19]$$

which has the same sign as the difference $Y_D - Y_C$ (since the term inside the square brackets is strictly positive). This together with Lemma 1 implies claims *(i)-(iii)*. **Q.E.D.**

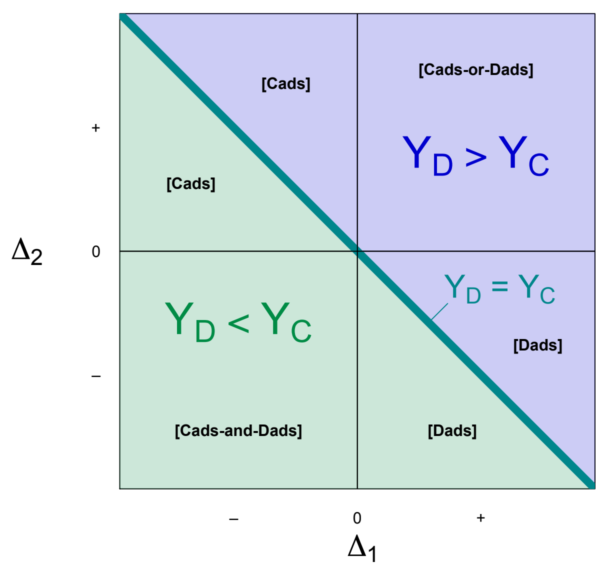

Fig. S2. The sign of the difference $Y_C - Y_D$ has an impact on the class of the evolutionary game (see Proposition 1).

This proposition is illustrated in Figure S2. The difference $Y_D - Y_C$ is nil at any point on the downward-sloping line through the origin, strictly positive at any point in the blue region, and strictly negative at any point in the green region. An important implication is that the difference $Y_D - Y_C$ must be strictly negative for the evolutionary game to be in class **[Cads-and-Dads]**, while it must be strictly positive for the evolutionary game to be in class **[Cads-or-Dads]**.

The next two propositions state results on how the evolutionary game class may change as a consequence of an increase in the difference $Y_C - Y_D$ when $Y_C > Y_D$ (Proposition 2), and of an increase in the difference $Y_D - Y_C$ when $Y_D > Y_C$ (Proposition 3).

Proposition 2 *Consider any given Dad type* $(\delta, Y_D) \in (0,1] \times [2, +\infty)$*, and suppose that the Cad's productivity increases from* $Y_C = Y_D + \varepsilon$ *to* $Y_C = Y_D + \varepsilon_A$ *for some* $\varepsilon_A > \varepsilon > 0$*. Then:*
*(2.i) If the evolutionary game is initially in class [**Cads**], it remains in this class if* ε_A *is sufficiently small, and it switches to class [**Cads-and-Dads**] if* ε_A *is sufficiently large.*
*(2.ii) If the evolutionary game is initially in class [**Cads-and-Dads**], it remains in this class for any* $\varepsilon_A > 0$*.*
*(2.iii) If the evolutionary game is initially in class [**Dads**], it remains in this class if* ε_A *is sufficiently small, and it switches to class [**Cads-and-Dads**] if* ε_A *is sufficiently large.*

Proof: Fix Y_D, let $Y_C - Y_D = \varepsilon$ for some $\varepsilon > 0$, and consider an increase in $Y_C - Y_D$ to some $\varepsilon_A > \varepsilon$. Inspection of Eq. (5)-Eq. (11) reveals that the values of Δ_1 and Δ_2 may vary with ε_A. By some abuse of notation, let us thus write them as functions of ε_A: $\Delta_1(\varepsilon_A)$ and $\Delta_2(\varepsilon_A)$. Specifically, replacing Y_C by $Y_D + \varepsilon_A$ in Eq. (9), we see that y_{DC} is increasing in ε_A. It follows that s_{DC} is increasing in ε_A for any $\delta > 0$. Importantly, holding Y_D, δ, κ, μ, and ϕ fixed, this is the only term in $\Delta_1(\varepsilon_A)$ and $\Delta_2(\varepsilon_A)$ that varies with ε_A (s_C is unaffected). This in turn implies that both $\Delta_1(\varepsilon_A)$ and $\Delta_2(\varepsilon_A)$ are decreasing in ε_A. Now, from Proposition 1 we know that $\Delta_1(\varepsilon_A) + \Delta_2(\varepsilon_A) < 0$ for any $Y_C - Y_D = \varepsilon_A > 0$. Three cases arise (see Figure S2 for visual support for the following arguments). Case *(i)*: $\Delta_1(\varepsilon) < 0$ and $\Delta_2(\varepsilon) > 0$, in which case the initial game class is [**Cads**]. Since $\Delta_1(\varepsilon_A)$ is decreasing in ε_A, $\Delta_1(\varepsilon_A)$ remains negative for any $\varepsilon_A > \varepsilon$; by contrast, for large enough values of ε_A, $\Delta_2(\varepsilon_A)$ becomes negative. In other words, the evolutionary game either remains in class [**Cads**], or it transitions to class [**Cads-and-Dads**]. Case *(ii)*: $\Delta_1(\varepsilon) < 0$ and $\Delta_2(\varepsilon) < 0$, in which case the initial game class is [**Cads-and-Dads**]. Since both $\Delta_1(\varepsilon_A)$ and $\Delta_2(\varepsilon_A)$ are decreasing in ε_A, they both remain negative for any $\varepsilon_A > \varepsilon$. Hence, the evolutionary game remains in class [**Cads-and-Dads**]. Case *(iii)*: $\Delta_1(\varepsilon) > 0$ and $\Delta_2(\varepsilon) < 0$, in which case the initial game class is [**Dads**]. Since $\Delta_2(\varepsilon_A)$ is decreasing in ε_A, $\Delta_2(\varepsilon_A)$ remains negative for any $\varepsilon_A > \varepsilon$; by contrast, for large enough values of ε_A, $\Delta_1(\varepsilon_A)$ becomes negative. In other words, the evolutionary game then either remains in class [**Dads**], or that it transitions to class [**Cads-and-Dads**]. **Q.E.D.**

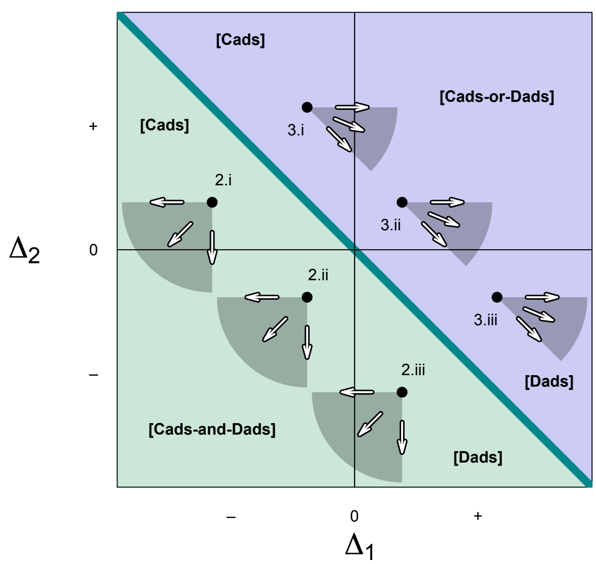

Fig. S3. The grey shaded areas in the green region show the effects of an increase in Y_C on the game class, when $Y_C > Y_D$ (see Proposition 2). The grey shaded areas in the blue region show the effects of an increase in Y_D on the game class, when $Y_D > Y_C$ (see Proposition 3).

The grey shaded areas in the green region in Figure S3 illustrate the proposition. The grey shaded areas associated with points *(2.i)*, *(2.ii)*, and *(2.iii)* show the directions in which the vector (Δ_1, Δ_2) can move as a result of an increase in Y_C (see Proposition 2). Thus, as can be seen in the figure, if the evolutionary game is initially in class [**Cads**] (for example at point *(2.i)*), it can either remain in this class or transition to class [**Cads-and-Dads**]; if it is initially in class [**Cads-and-Dads**] (for example at point *(2.ii)*), it must remain so; and finally, if it is initially in class [**Dads**] (for example at point *(2.iii)*), it either remains in this class or transitions to class [**Cads-and-Dads**].

Proposition 2 makes intuitive sense. By stealing paternity from Dads with whom they are matched, Cads inflict a cost on Dads. However, if Cads are better at producing food (i.e., if $Y_C > Y_D$), they also bestow a benefit on Dads. If this benefit is high enough, then a Dad may be better off when matched with a Cad than with a Dad, and a stable polymorphic population state then exists.

Proposition 3 *Consider a given Dad type with $\delta \in (0,1]$, and a given Cad type with $Y_C \geq 2$, and suppose that the Dad's productivity increases from $Y_D = Y_C + \varepsilon$ to $Y_D = Y_C + \varepsilon_B$ for some $\varepsilon_B > \varepsilon > 0$. Then:*
*(3.i) If the evolutionary game is initially in class **[Cads]**, it remains in this class if ε_B is sufficiently small, it switches to class **[Cads-or-Dads]** if ε_B is intermediate, while it switches to class **[Dads]** if ε_B is sufficiently large.*
*(3.ii) If the evolutionary game is initially in class **[Cads-or-Dads]**, it remains in this class if ε_B is sufficiently small, and it switches to class **[Dads]** if ε_B is sufficiently large.*
*(3.iii) If the evolutionary game is initially in class **[Dads]**, it remains in this class for any $\varepsilon_B > 0$.*

Proof: Unless otherwise specified, we use the same notation as in the proof to the preceding proposition. Fix Y_C, let $Y_D = Y_C + \varepsilon$ for some $\varepsilon > 0$, and consider an increase in Y_D to $Y_C + \varepsilon_B$ for some $\varepsilon_B > \varepsilon$. Replacing Y_D by $Y_C + \varepsilon_B$ in Eq. (8) and Eq. (9), we see that both y_{DD} and y_{DC} are increasing in ε_B, and that any given increase in ε_B induces a greater increase in y_{DD} than in y_{DC}. It follows from this observation that $\Delta_1(\varepsilon_B) + \Delta_2(\varepsilon_B)$ is increasing in ε_B. Moreover, it clearly also follows that $\Delta_1(\varepsilon_B)$ is increasing and $\Delta_2(\varepsilon_B)$ is decreasing in ε_B. Now, from Proposition 1 we know that $\Delta_1(\varepsilon_B) + \Delta_2(\varepsilon_B) > 0$ for any $Y_D - Y_C = \varepsilon_B > 0$. Three cases arise (see Figure S2 for visual support for the following arguments). Case *(i)*: $\Delta_1(\varepsilon) < 0$ and $\Delta_2(\varepsilon) > 0$, in which case the initial game class is **[Cads]**. For small enough values of ε_B, $\Delta_1(\varepsilon_B)$ remains negative and $\Delta_2(\varepsilon_B)$ remains positive. For intermediate values of ε_B, $\Delta_1(\varepsilon_B)$ becomes positive while $\Delta_2(\varepsilon_B)$ remains positive. For large enough values of ε_B, $\Delta_1(\varepsilon_B)$ becomes positive while $\Delta_2(\varepsilon_B)$ becomes negative. Hence, the evolutionary game remains in class **[Cads]** for small values of ε_B, it switches to class **[Cads-or-Dads]** for intermediate values of ε_B, and it switches to class **[Dads]** for large enough values of ε_B. Case *(ii)*: $\Delta_1(\varepsilon) > 0$ and $\Delta_2(\varepsilon) > 0$, in which case the initial game class is **[Cads-or-Dads]**. Since $\Delta_1(\varepsilon_B)$ is increasing, $\Delta_2(\varepsilon)$ is decreasing, while their sum is increasing in ε_B, the evolutionary game either remains in class **[Cads-or-Dads]**, or it transitions to class **[Dads]**. Case *(iii)*: $\Delta_1(\varepsilon) > 0$ and $\Delta_2(\varepsilon) < 0$, in which case the initial game class is **[Dads]**. Since $\Delta_1(\varepsilon_B)$ remains positive and $\Delta_2(\varepsilon_B)$ remains negative for any $\varepsilon_B > \varepsilon$, the evolutionary game remains in class **[Dads]**. **Q.E.D.**

The grey shaded areas in the blue region in Figure S3 illustrate this proposition. As was shown in the proof of Proposition 3, this is because for any $Y_D > Y_C$ (i.e., at any point in the blue region), an increase in Y_D brings about a change in the vector (Δ_1, Δ_2) towards the East-South-East, as indicated by the arrows in the shaded areas associated with points *(3.i)*, *(3.ii)*, and *(3.iii)*.

Proposition 3 also makes intuitive sense. When Dads are more productive than Cads, a Dad's reproductive success is always higher when his neighbor is a Dad rather than a Cad. This effect is amplified if Dads become even more productive, and this favors the propagation of Dads.

We next turn to the parameter δ. The proofs of Propositions 4 and 5 are collected in a single proof following Proposition 5.

Proposition 4 *Assume that $\kappa > 0$, and suppose that the Dad type becomes more "Daddish" (that is, the amount of food brought home by a Dad, δ, increases). If $Y_C > Y_D$, then:*
*(4.i) if the evolutionary game is initially in class **[Cads]**, it remains in this class for a small enough increase in δ, it switches to class **[Cads-and-Dads]** for an intermediate increase in δ, and it may switch to class **[Dads]** for a large enough increase in δ;*
*(4.ii) if the evolutionary game is initially in class **[Cads-or-Dads]**, it either remains in this class for any increase in δ, or it switches to class **[Dads]** for a large enough increase in δ;*
*(4.iii) if the evolutionary game is initially in class **[Dads]**, it remains so for any increase in δ for sufficiently small values of ϕ, while it switches to class **[Cads-and-Dads]** for a large enough increase in δ and a large enough value of ϕ.*

Proposition 5 *Assume that $\kappa > 0$, and suppose that the Dad type becomes more "Daddish" (that is, the amount of food brought home by a Dad, δ, increases. If $Y_D > Y_C$, then:*
*(5.i) if the evolutionary game is initially in class **[Cads]**, it remains in this class for a small enough increase in δ, it switches to class **[Cads-or-Dads]** for an intermediate increase in δ, and to class **[Dads]** for a large enough increase in δ;*
*(5.ii) if the evolutionary game is initially in class **[Cads-or-Dads]**, it remains in this class for a small enough increase in δ, and it switches to class **[Dads]** for a large enough increase in δ;*
*(5.iii) if the evolutionary game is initially in class **[Dads]**, it remains so for any increase in δ.*

Proof: Note, first, that since s_{DC} is increasing in δ (recall Lemma 2), $\Delta_2 = s_C - (1-\phi) \cdot s_{DC}$ is decreasing in δ. It remains to be determined how Δ_1 and $\Delta_1 + \Delta_2$ vary with δ. To this end, we analyze how $\Delta_1 + \Delta_2 = s_{DD} - s_{DC}$ varies with δ.

Recalling the definitions of $\tilde{s}_{DD}$ and $\tilde{s}_{DC}$ (see Eq. (16) and Eq. (17)), we obtain the following expressions for the partial derivatives:

$$\frac{\partial\left[\tilde{s}_{DD}(\delta,\mu,\kappa) - \tilde{s}_{DC}(\delta,\mu,\kappa)\right]}{\partial\delta} = \left[(1-\kappa)\cdot\left(\frac{2(1-\mu)Y_D + \mu\cdot Y_D^2}{2}\right) + \kappa\cdot x\cdot\left(\frac{2(1-\mu)Y_D + \mu\cdot Y_D^2}{2}\right)\right]/A - \left[(1-\kappa)\cdot\left(\frac{(1-\mu)(Y_D+Y_C) + \mu\cdot(Y_D Y_C)}{2}\right) + \kappa\cdot x\cdot\left(\frac{(1-\mu)(Y_D+Y_C) + \mu\cdot(Y_D Y_C)}{2}\right)\right]/A. \quad [20]$$

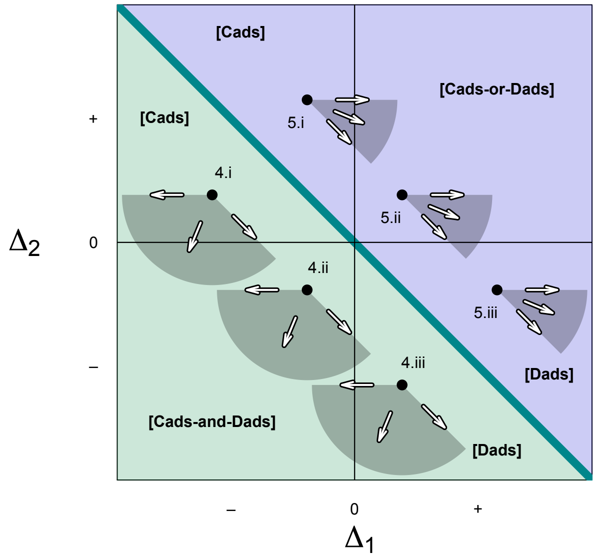

Fig. S4. The grey shaded areas in the green region show the effects of an increase δ on the game class, when $Y_C > Y_D$ (see Proposition 4). The grey shaded areas in the blue region show the effects of an increase in δ on the game class, when $Y_D > Y_C$ (see Proposition 5).

First, consider the case $Y_D > Y_C$ (Proposition 5). Inspection of Eq. (20) immediately reveals that this expression is strictly positive when $Y_D > Y_C$. Since it was observed above that Δ_2 is decreasing in δ, this in turn implies that Δ_1 must be strictly increasing in δ (since $\Delta_1 + \Delta_2 = \tilde{s}_{DD}(\delta, \mu, \kappa) - \tilde{s}_{DC}(\delta, \mu, \kappa)$). In sum, if $Y_D > Y_C$, then an increase in δ induces an increase in Δ_1, a decrease in Δ_2, and an increase in $\Delta_1 + \Delta_2$.

Second, consider the case $Y_C > Y_D$ (Proposition 4). Inspection of Eq. (20) immediately reveals that this expression is then strictly negative. In sum, if $Y_C > Y_D$, an increase in δ induces a decrease in both Δ_2 and $\Delta_1 + \Delta_2$. When it comes to Δ_1 (see Eq. (10)), we see that if $\phi = 1$, $\Delta_1 = s_{DD} - s_{DC} - s_C$, in which case its value would decrease as a result of an increase in δ, while if $\phi = 0$, $\Delta_1 = s_{DD} - s_C$, in which case its value would increase as a result of an increase in δ. Statement *(4.iii)* follows from the fact that Δ_1 varies continuously with ϕ. **Q.E.D.**

Proposition 5 says that when Dads cannot benefit from having a Cad as a neighbor (i.e., when $Y_D > Y_C$), an increase in the positive effect that a Dad has on the survival of his mate's children facilitates the propagation of Dads. While this is intuitive, it is by no means trivial, since an increase in δ also means that the value of stealing paternity increases. This latter effect appears in Proposition 4, which indicates that when Dads benefit from having a Cad as a neighbor, an increase in the positive effect that a Dad has on the survival of his mate's children may also facilitate the propagation of Cads: an increase in δ can then render the benefit of stealing paternity large enough (even though μ is constant) for Cads to see their share increase (part *(4.iii)* of the proposition). Figure S4 illustrates the two propositions.

Female sexual behavior. Here we focus on the effect that female openness to extra-pair copulations ϕ has on the game class.

Proposition 6 *Suppose that the female openness to extra-pair copulations, i.e., the value of ϕ, decreases. If this induces a transition from one game class to another, then: (i) if $Y_D > Y_C$, the transition is from **[Cads]** to **[Cads-or-Dads]** (for an intermediate decrease in ϕ) or to **[Dads]** (for a large decrease in ϕ), or from **[Cads-or-Dads]** to **[Dads]**; (ii) if $Y_D < Y_C$, the transition is from **[Cads]** to **[Cads-and-Dads]** (for an intermediate decrease in ϕ) or to **[Dads]** (for a large decrease in ϕ), or from **[Cads-and-Dads]** to **[Dads]**.*

Proof: Since both s_{DD} and s_{DC} are independent of ϕ, the value of $\Delta_1 + \Delta_2$ is unaffected by the value of ϕ. Moreover, Δ_2 is increasing in ϕ. Taken together, these observations imply that, in Figure S1, a decrease in ϕ entails a movement towards the South-East, on some line parallel to the one along which $\Delta_1 + \Delta_2 = 0$. Hence, the stated result immediately obtains. **Q.E.D.**

This result simply means that Dads cannot become less prevalent if females become more faithful. Figure S5 gives an indication of how sensitive the values of Δ_1 and Δ_2 are to changes in ϕ. This figure shows the vector (Δ_1, Δ_2), and the associated game class, for different values of ϕ along the line for which $\Delta_1 + \Delta_2 = 0$. For the parameter values used here, the game class switches from **[Cads]** to **[Dads]** for some value of ϕ slightly above 0.5.

Male-female and male-male complementarities. We finally turn to the heart of the argument developed in the main text, namely, the ecological shifters: κ (male-female complementarity) and μ (male-male complementarity). The proofs of Propositions 7 and 8 are collected in a single proof following Proposition 8.

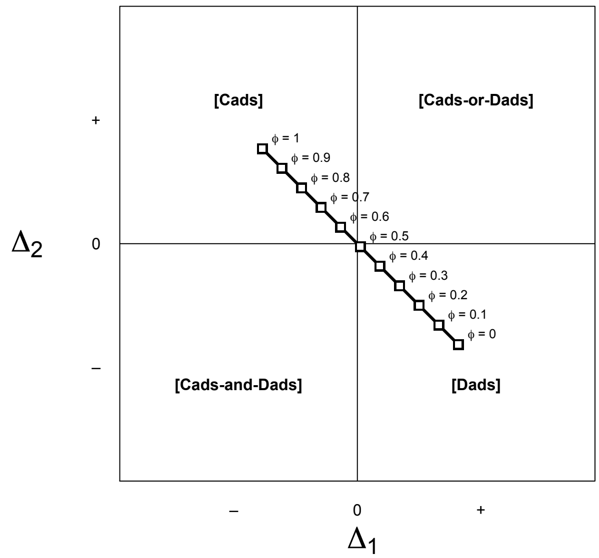

Fig. S5. *Ceteris paribus*, the evolutionary game class depends on female openness to extra-pair copulations, ϕ (see Proposition 6). $A = 20$, $x = 5$, $\mu = \kappa = 1/4$, $\delta = 1$, $Y_D = Y_C = 2.5$.

Proposition 7 *Consider any Dad type (δ, Y_D) for which $\delta > 0$. Suppose that the male-male complementarity in food production, i.e., the value of μ, increases. If $Y_D > Y_C$, then:*
*(7.i) if the evolutionary game is initially in class [**Cads**], it remains in this class for a small enough increase in μ, it switches to class [**Cads-or-Dads**] for an intermediate increase in μ, and to class [**Dads**] for a large enough increase in μ;*
*(7.ii) if the evolutionary game is initially in class [**Cads-or-Dads**], it remains in this class for a small enough increase in μ, and it switches to class [**Dads**] for a large enough increase in μ;*
*(7.iii) if the evolutionary game is initially in class [**Dads**], it remains so for any increase in μ.*

Proposition 8 *Consider any Dad type (δ, Y_D) for which $\delta > 0$. Suppose that the male-male complementarity in food production, i.e., the value of μ, increases. If $Y_C > Y_D$, then:*
*(8.i) if the evolutionary game is initially in class [**Cads**], it remains in this class for a small enough increase in μ, it switches to class [**Cads-and-Dads**] for an intermediate increase in μ, and it may switch to class [**Dads**] for a large enough increase in μ;*
*(8.ii) if the evolutionary game is initially in class [**Cads-or-Dads**], it either remains in this class for any increase in μ, or it switches to class [**Dads**] for a large enough increase in μ;*
*(8.iii) if the evolutionary game is initially in class [**Dads**], it remains so for any increase in μ for sufficiently small values of ϕ, while it switches to class [**Cads-and-Dads**] for a large enough increase in μ and a large enough value of ϕ.*

Proof: Given that $Y_D > 1$ and $Y_C > 1$, for any given δ an increase in μ induces an increase in both y_{DD} and y_{DC}. This in turn implies that Δ_2 is decreasing in μ. As for $s_{DD} - s_{DC}$, we obtain:

$$\begin{aligned}\frac{\partial\left(\tilde{s}_{DD}(\delta,\mu,\kappa)-\tilde{s}_{DC}(\delta,\mu,\kappa)\right)}{\partial\mu} &= \left[(1-\kappa)\cdot\delta\cdot\frac{Y_D^2-2Y_D}{2}+\kappa\cdot x\cdot\delta\cdot\frac{Y_D^2-2Y_D}{2}\right]/A \\ &- \left[(1-\kappa)\cdot\delta\cdot\frac{Y_DY_C-Y_D-Y_C}{2}+\kappa\cdot x\cdot\delta\cdot\frac{Y_DY_C-Y_D-Y_C}{2}\right]/A.\end{aligned} \quad [21]$$

Simplification of this expression reveals that it has the same sign as:

$$(1-\kappa)\cdot\delta\cdot(Y_D^2-2Y_D-Y_DY_C+Y_D+Y_C)+\kappa\cdot x\cdot\delta\cdot(Y_D^2-2Y_D-Y_DY_C+Y_D+Y_C). \quad [22]$$

Since $\kappa \in [0,1]$ and $\delta > 0$, this has the same as

$$Y_D^2-2Y_D-Y_DY_C+Y_D+Y_C=(Y_D-1)(Y_D-Y_C). \quad [23]$$

Since $Y_D > 1$ this implies that the expression in Eq. (21) has the same sign as $Y_D - Y_C$.

Recalling the proof of Propositions 4 and 5, we note that an increase in μ has the same qualitative effect on Δ_1, Δ_2, and $\Delta_1 + \Delta_2$ as an increase in δ, and we can thus refer to the reasoning at the end of that proof to conclude the proof. **Q.E.D.**

An increase in μ facilitates the propagation of Dads when $Y_D > Y_C$. By contrast, it may hinder it if $Y_C > Y_D$ and initially the game class is [**Dads**]). This is intuitive, if $Y_D > Y_C$ then an increase in male-male complementarity in food production means that the additional benefit that a Dad stands to gain from being matched with another Dad rather than with a Cad increases. By contrast, if $Y_C > Y_D$, the opposite is true: the mutualism between Dads and Cads can then become strong enough for Cads to see their share increase, if initially there were almost no Cads around (i.e., if the initial game class is [**Dads**]).

Turning finally to κ, the parameter that measures the degree of female-male complementarity for the survival probability of offspring, we find:

Proposition 9 *Consider any Dad type $(\delta, Y_D) \in (0,1] \times [2,+\infty)$. Suppose that the female-male complementarity, i.e., the value of κ, increases. If $Y_D > Y_C$, then:*
*(9.i) if the evolutionary game is initially in class [**Cads**], it remains in this class for a small enough increase in κ, it switches to class [**Cads-or-Dads**] for an intermediate increase in κ, and to class [**Dads**] for a large enough increase in κ;*
*(9.ii) if the evolutionary game is initially in class [**Cads-or-Dads**], it remains in this class for a small enough increase in κ, and it switches to class [**Dads**] for a large enough increase in κ;*
*(9.iii) if the evolutionary game is initially in class [**Dads**], it remains so for any increase in κ.*

Proposition 10 *Consider any Dad type (δ, Y_D) for which $\delta > 0$. Suppose that the female-male complementarity, i.e., the value of κ, increases. If $Y_C > Y_D$, then:*
*(10.i) if the evolutionary game is initially in class [**Cads**], it remains in this class for a small enough increase in κ, it switches to class [**Cads-and-Dads**] for an intermediate increase in κ, and it may switch to class [**Dads**] for a large enough increase in κ;*
*(10.ii) if the evolutionary game is initially in class [**Cads-or-Dads**], it either remains in this class for any increase in κ, or it switches to class [**Dads**] for a large enough increase in κ;*
*(10.iii) if the evolutionary game is initially in class [**Dads**], it either remains so for any increase in κ, or it switches to class [**Cads-and-Dads**] for a large enough increase in κ.*

Proof: Fix all the parameters but κ, and study how Δ_1, Δ_2, and $\Delta_1 + \Delta_2$ vary with κ. Recall first from Lemma 2 that $\delta > 0$ implies that the survival probability $s(x, y)$ is strictly increasing in κ. Since s_C is a constant, this implies that $\Delta_2 = s_C - (1-\phi) \cdot s_{DC}$ is strictly decreasing in κ for any $\delta > 0$.

Next, we analyze how $\Delta_1 + \Delta_2 = s_{DD} - s_{DC}$ varies with κ:

$$\begin{aligned}\frac{\partial\left[\tilde{s}_{DD}(\delta,\mu,\kappa) - \tilde{s}_{DC}(\delta,\mu,\kappa)\right]}{\partial\kappa} = & \left[-\left(x + \delta \cdot \frac{(1-\mu)\, 2Y_D + \mu \cdot Y_D^2}{2}\right) + x \cdot \left(\delta \cdot \frac{(1-\mu)\, 2Y_D + \mu \cdot Y_D^2}{2} + 1\right)\right]/A \\ & - \left[-\left(x + \delta \cdot \frac{(1-\mu)\,(Y_D + Y_C) + \mu \cdot (Y_D Y_C)}{2}\right) + x \cdot \left(\delta \cdot \frac{(1-\mu)\,(Y_D+Y_C) + \mu \cdot (Y_D Y_C)}{2} + 1\right)\right]/A.\end{aligned} \quad [24]$$

Simplification reveals that this expression has the same sign as

$$(x-1) \cdot \delta \cdot (1-\mu) \cdot (Y_D - Y_C) + (x-1) \cdot \mu \cdot Y_D \cdot (Y_D - Y_C).$$

Since $x > 1$, this implies that if $Y_D > Y_C$, then an increase in κ induces an increase in $\Delta_1 + \Delta_2 = s_{DD} - s_{DC}$, while if $Y_C > Y_D$, then an increase in κ induces a decrease in $\Delta_1 + \Delta_2 = s_{DC} - s_{DD}$.

Together with the observation made at the beginning of the proof that Δ_2 is strictly decreasing in κ, this means that an increase in κ induces qualitatively similar changes in the vector (Δ_1, Δ_2), as does an increase in δ (see the grey shade areas in Figure S4). **Q.E.D.**