SUPPLEMENTARY INFORMATION for:

Controllable skyrmion chirality in ferroelectrics

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Phase-field simulations.

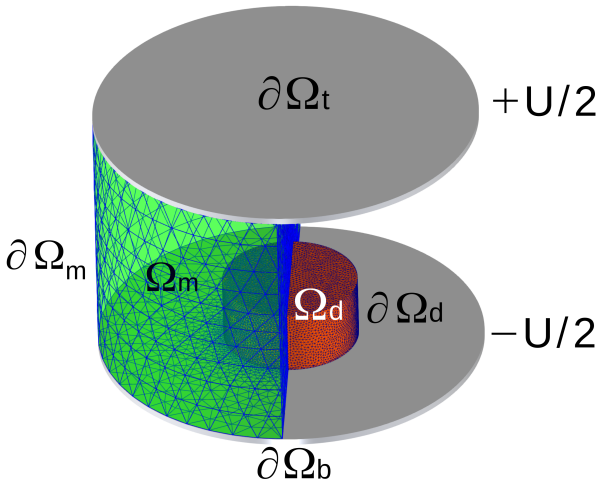
The phase-field simulations have been carried out in the framework of open source FERRET package [1], developed for the open source multi-physics simulation environment MOOSE [2]. After variation of free-energy functional F (equation (2) in paper) with respect to polarization components P_i ($i = 1, 2, 3$)

$$-\gamma \frac{\partial P_i}{\partial t} = \frac{\delta F}{\delta P_i}, \tag{1}$$

we obtain actual equations to expose to finite element computation scheme. The relaxation parameter γ setting the fictitious time scale for the computational procedure, was taken equal to unity since this fictitious time scale does not influence the free energy minima distribution in our problem.

Supplementary Fig. S1 depicts the conceptual simulation space exposing the details of the generated finite-element grid. The computational domain Ω is a cylindrical medium Ω_m embracing the ferroelectric nanodot Ω_d. The total surface of Ω_m, i.e. that of Ω is defined as $\partial\Omega_m \cup \partial\Omega_t \cup \partial\Omega_b$, where $\partial\Omega_m$ is side surface of Ω_m and $\partial\Omega_t$, $\partial\Omega_b$ are the top and bottom surfaces of Ω_m, respectively. The total surface of the nanodot is denoted as $\partial\Omega_d$. An open source 3D finite element mesh generator gmsh [3] was used to generate the finite element meshes representing the computational domain. We used the non-structured meshes based on the tetrahedral elements. The variable density of elements lowering from the surface of the nanodot towards the surface of the medium results in the faster and less in volume consuming memory (both the storage and the random-access) consuming computations and does not reduce the precision of the results.

The variational formulation of problem was transformed into the weak form [4] for which the unknowns polarization P and the electric potential φ are functions of the $H^1(\Omega)$ space. The solution for the discreticized variational problem was sought in the form of P_1 Lagrange finite elements. The voltage applied to the electrodes was introduced into the problem as boundary conditions of the Dirichlet type at the $\partial\Omega_t$ and $\partial\Omega_b$ imposed on unknown φ. The Newton-Raphson method coupled with the generalized minimal residual method (GMRES) and block Jacobi preconditioner [5, 6] was used to solve the resulting system of the non-linear algebraic equations. When solving the partial differential equations which depend on time, one needs to choose the appropriate time discretization technique and the initial conditions for the polarization distribution. We used the 2-step backward differentiation formula to implement the time-stepping scheme with the random distribution of individual polarization components varying at each node of the finite-element mesh from -10^{-5} to $10^{-5}\,\mathrm{C/m^2}$ at the initial time step in the case of the simu-



Supplementary Figure S1. Simulation setup.

lation at the point 0 and the distribution acquired as the previous solution in all other points of the hysteresis curve. The criteria for the terminating the solution process at each point is $|(|\mathcal{F}_n| - |\mathcal{F}_{n-1}|)/|\mathcal{F}_n|| < 10^{-6}$, where $\mathcal{F}$ is the full energy of system defined by the integral of Eq. (2) of the main text divided by Ω_d.

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