Supplementary materials to

Environmental variability, reliability of information and the timing of migration

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Supplementary Material S1: Dynamic programming equations

Terminal reward

Set $V(i,k,N) = 0$ for all $i = -Q, -(Q-1), \cdots, 0,1,2, \cdots, Q$ and all $k = 0, 1, \cdots, K-1$.

Set $V(i,K,n) = R(i\Delta, n\Delta)$ for all $i = -Q, -(Q-1), \cdots, 0,1,2, \cdots, Q$ and all $n = 0,1, \cdots, N$.

Backwards step

Let $0 \le n \le N-1$. Suppose that $V(i,k,n+1)$ is given for all $i = -Q, -(Q-1), \cdots, 0,1,2, \cdots, Q$ and all $k = 0, 1, \cdots, K$.

for each $i = -Q, -(Q-1), \cdots, 0,1,2, \cdots, Q$ and each $k = 0,1, \cdots, K-1$:

Let $H_{stay} = [1 - M((n+i)\Delta, k)\Delta] V(i,k,n+1)$.

Let $H_{leave} = \sum_{j=-Q}^{Q} a_{i,j}(k+1) V(j, k+1, n+1)$. (***)

Set $V(i,k,n) = \max\{H_{stay}, H_{leave}\}$.

Set $\text{Action}(i,k,n) = 1$ if $H_{\text{stay}} \ge H_{\text{leave}}$, else set $\text{Action}(i,k,n) = 0$.

[Note migration takes one time-unit in the above, but delta is small.]

The leaving strategy

If the optimal strategy is of the obvious simple form, then

$L(i\Delta, k) = \Delta \sum_{n=0}^{N} \text{action}(i,k,n)$.

Numerical implementation

We draw a distinction between grid time and real time. Specifically, the time-grid step is Δ, which will typically be small. The time grid takes integer values of n = 0, 1, 2, …, N. These values correspond to calendar times t = 0, Δ, 2Δ, …, NΔ. The advancement of spring relative to the mean is assumed to lie on a grid i = -Q, -(Q-1), …, 0, 1, 2, …, Q. Grid time q corresponds to real time $q\Delta$.

The transition matrices

Define the function *fnorm* by

$$fnorm(x,m,v) = \exp\{-(x-m)^2/2v\}.$$

<u>For each k in the range k = 1,2, ... K,</u>

Set $v = \sigma^2(k)\left(1-(\rho(k))^2\right)$.

For each i in the range $i = -Q, -(Q-1), \cdots, 0,1,2, \cdots, Q$ set $m = i\Delta\rho(k)\sigma(k)/\sigma(k-1)$

{ For each j are in the range $j = -Q, -(Q-1), \cdots, 0,1,2, \cdots, Q$ set $x = j\Delta$.

Set $\tilde{a}_{i,j}(k) = fnorm(x,m,v)$.}

Set $sum = \sum_{j=-Q}^{Q} \tilde{a}_{i,j}(k)$.

For each j in the range $j = -Q, -(Q-1), \cdots, 0, 1, 2, \cdots, Q$ set $a_{i,j}(k) = \tilde{a}_{i,j}(k)/sum$.

[We can interpret these variables as follows:

$a_{i,j}(k) = P(Y(k) = \Delta j \vee Y(k-1) = \Delta i)$,

and $A(k) \equiv \left(a_{i,j}(k)\right)$ is the transition matrix related the (grid values of) advancement of spring at location *k*-1 to that at *k*.]

Figure S1 – Relation between landscape *ρ* and *ρ(k)*

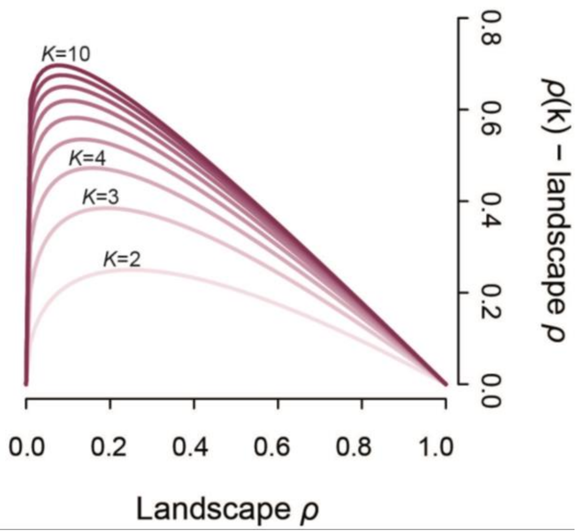

Fig. S1. As the landscape *ρ* is the product of all *ρ(k)*'s, changing the number of sites, *K*, while keeping the landscape *ρ* constant, changes the correlation between successive sites, *ρ*(*k*). If the landscape *ρ* is either very high or very low, the correlations between successive sites, *ρ(k)*, hardly differ from landscape *ρ*, while the greatest differences appear for low to intermediate *ρ* and these differences increase further with more sites (darker lines).

Figure S2 – Reproductive values for varying number of sites and sigma

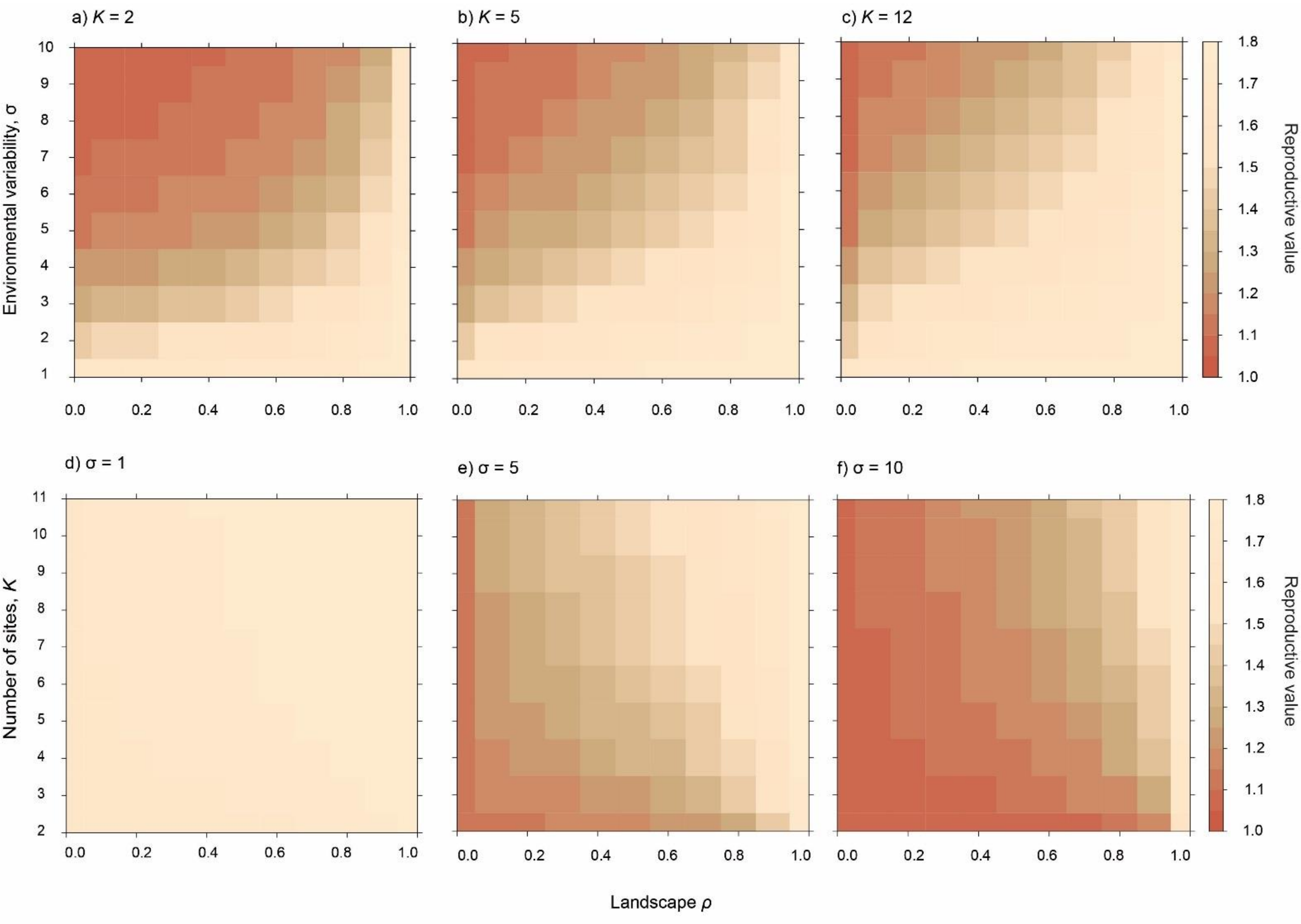

Fig. S2. Reproductive values over a gradient of overall predictability (landscape ρ) and environmental variability (upper panel) and varying number of sites (lower panel), respectively.

Adding intermediate sites does not yield any fitness-benefit in almost constant environment (d) while it clearly increases reproductive values when environments become more variable (e-f).