Supporting information for "Topological portraits of multiscale coordination dynamics" by Mengsen Zhang, William D. Kalies, J. A. Scott Kelso, and Emmanuelle Tognoli

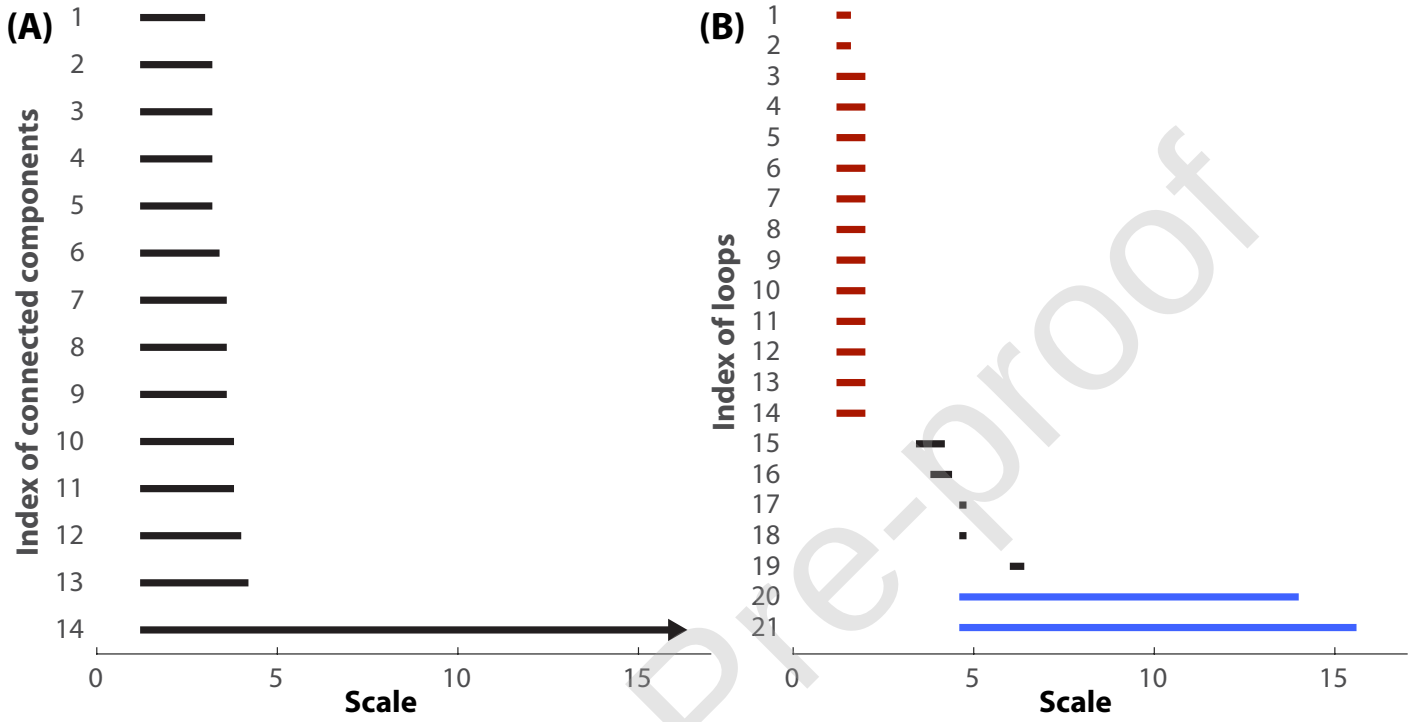

Fig S1. Persistent homology of a big letter B of many small letter A's shown as barcodes. The 0^{th} and 1^{st} persistent homology characterize the scale-dependency of connected components (A) and loops (B) respectively, computed from an image of a big letter B of many small letter A's (Fig 4 left box; the image used for computation has 61×100 pixels). In (A), each bar represents a particular connected component, whose left end indicates at which scale this connected component emerges, and the right end at which scale it disappears (by merging into a larger component). There are clearly 14 connected components at finer scales (scale, i.e. radius of the disk centered at each pixel, below 5) and 1 at grosser scales (right arrow in bar 14 indicates that this component never disappears with increasing scale). In (B), each bar represents a loop. There are clearly 14 loops at finer scales (below 3; red bars) and two major loops at grosser scales (blue bars; some more transient features also appear in the transition between the 14-loop and the 2-loop configuration, which are also features emerged during scaling of the image; colors are only for highlights, not computed). Together, these two multiscale topological portraits (A-B) faithfully capture the coexistence of two descriptions of the image (many A's or one B) and their separation in scale.

S1 Text Simulations of metastable coordination Examples of metastable dynamics shown in Fig 5 are simulated following the equation

$$\dot{\varphi}_i = \omega_i - a\sum_{j=1}^{N}\sin(\varphi_i - \varphi_j) - b\sum_{j=1}^{N}\sin 2(\varphi_i - \varphi_j) \tag{S1}$$

where φ_i, $\dot{\varphi}_i$, and ω_i are the phase, the time derivative of phase, and the natural frequency of the i^{th} oscillator in an ensemble of N, respectively, and $a, b \geq 0$ are the

first-order and second-order coupling strengths between oscillators. This system [65] has been shown to capture key experimental observations of coordination between multiple people as observed in [66]. For the example shown in Fig 5 A-B, $\omega_{1,2} = 0, 0.6$ Hz and $a = b = 1$ with zero initial phases. For the example shown in Fig 5 C-D, the average natural frequency $\bar{\omega} = 1.5$ Hz, the natural frequency difference between two adjacent oscillators $\omega_{i+1} - \omega_i = 0.075$ Hz, and overall coupling strength $a = b = 0.15$ with random initial conditions.

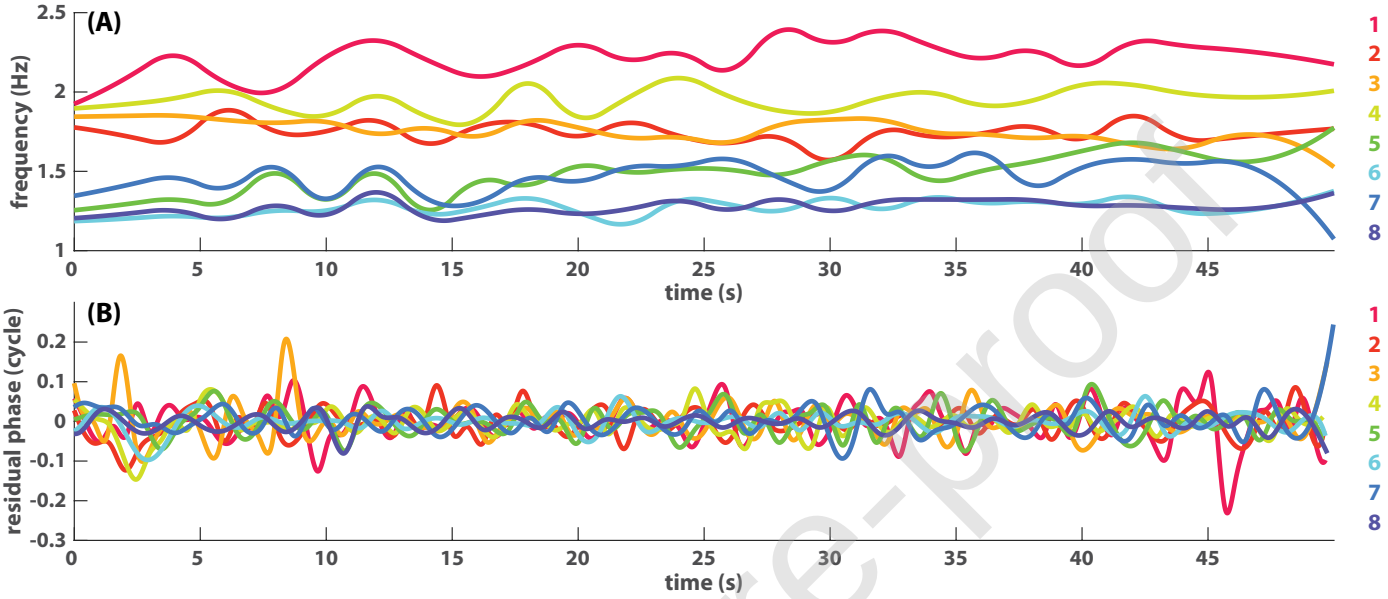

Fig S2. Decomposition of absolute phase dynamics for the eight-agent example (Fig 2). The absolute phase of each agent is decomposed into to a slowly varying frequency component (A) and a fast varying residual phase (B).