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Supplementary Information for

Dynamics of electrohydraulic soft actuators

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Estimation of the RC time constant of the actuators

In this section, we estimate the time to charge the electrodes of the actuator. We model the actuator as an RC circuit (Fig. S1*A*). The RC time constant of the actuator can be calculated from the resistance R_e of the electrodes and the capacitance C_a of the actuator. The electrodes are connected to the voltage source and to ground near the sides of the actuator (Fig. S1*B*), so that that the resistance of the electrodes can be approximated as

$$R_{\mathrm{e}} = 2\rho \frac{w}{h_{\mathrm{e}} L_{\mathrm{e}}}, \quad \textbf{[S1]}$$

where ρ is the resistivity of the electrode material, w is the width of the actuator, h_e the thickness of the electrodes, and $L_e = L/2$ the length of the electrodes (Fig. S1*B*). The capacitance is largest when the electrodes are completely zipped. For that case, the capacitance of the zipped region of the electrodes can be calculated as

$$C_{\mathrm{a}} = \varepsilon_0 \varepsilon_{\mathrm{r}} \frac{w L_{\mathrm{e}}}{2h}, \quad \textbf{[S2]}$$

where ε_0 is the vacuum permittivity, ε_r the relative permittivity of the material of the shell, and h the thickness of the shell. Combining Eqs. **S1** and **S2** leads to the RC time constant t_{RC}:

$$t_{\mathrm{RC}} = R_{\mathrm{e}} C_{\mathrm{a}} = \rho \varepsilon_0 \varepsilon_{\mathrm{r}} \frac{w^2}{h h_{\mathrm{e}}}, \quad \textbf{[S3]}$$

The largest RC time constant that occurs in this article is for the actuators made from L0WS (ε_r = 3.5, h = 20.3 μm) with width w = 12 cm. The resistivity of the electrode material is $\rho < 10^{-3}$ Ωm (specification sheet of the manufacturer). The thickness of the electrodes is approximately $h_e \approx 10$ μm (measured with a micrometer). With those values we calculate an RC time constant $t_{RC} \approx 2$ μs, which is three orders of magnitude smaller than the fastest observed time scales of actuation. The influence of the charging time of the actuators on the dynamics can therefore be neglected.

Scaling analysis

It is useful to determine the dimensionless groups of parameters which govern the dynamics of Peano-HASEL actuators. When neglecting the mass of the actuator, the elasticity of the shell, and the electric field inside the liquid-filled region of the shell, the dimensionless transition time t_n of the actuator in response to a step-voltage signal ($t_n = t_R$ or $t_n = t_F$ depending on whether the voltage is turned on or off) can be determined from the following equation:

$$F(\Phi, \varepsilon_0 \varepsilon_{\mathrm{r}}, h, w, L, L_{\mathrm{e}}, V, \mu, M, g, t_{\mathrm{n}}) = 0, \quad \textbf{[S4]}$$

where Φ [V] is the applied voltage, ε_0 [mkg/s^{2}V^2] the vacuum permittivity, ε_r [-] and h [m] the relative permittivity and thickness of the material of the shell, respectively, w [m] and L [m] the dimensions of the shell, L_e [m] the length of the electrodes, V [m^3] and μ [kg/sm] the volume and the viscosity of the liquid dielectric, respectively, M [kg] the mass of the hanging weight, and g [m^2/s] the gravitational acceleration (Fig. S2*A* and *B*). Using dimensional analysis Eq. **S4** can be rewritten in a dimensionless form:

$$t_{\mathrm{n}} \left(\frac{g}{L}\right)^{1/2} = G\left(\frac{\Phi^2 \varepsilon_0 \varepsilon_{\mathrm{r}} w}{Mgh}, \frac{\mu w L^{1/2}}{M g^{1/2}}, \frac{V}{wL^2}, \frac{L_{\mathrm{e}}}{L}, \frac{h}{L}, \frac{w}{L}\right) \quad \textbf{[S5]}$$

The parameters V/wL^2 and L_e/L describe the ratio of the volume of the liquid dielectric to the dimensions of the shell and the electrode coverage ratio, respectively. We keep both parameters constant in this article and do not further investigate their influence on the dynamics. The parameters h/L and w/L describe the influence of the bending stiffness and the side constraints on the behavior of the actuator (1). We have previously shown that both parameters can be neglected for the actuator dimensions $L > w >> h$ (as used in this article) (1). It is important to note that this analysis made no assumption about the magnitude of the actuation strain (such as linearization for small strains). The scaling analysis is therefore also valid for large strains (if none of the other assumptions are violated).

Using the simplifications described in the preceding paragraph, Eq. **S5** reduces to

$$t_\mathrm{n}\left(\frac{g}{L}\right)^{1/2}=G\left(\frac{\Phi^2\varepsilon_0\varepsilon_\mathrm{r}w}{Mgh},\frac{\mu wL^{1/2}}{Mg^{1/2}}\right). \quad \textbf{[S6]}$$

The dimensionless parameter $\Phi^2\varepsilon_0\varepsilon_r w/Mgh$ describes the ratio of the electrostatic forces and the weight of the load. We have previously shown that this parameter governs the equilibrium strain (1) and expect it to be an important parameter for the dynamics of Peano-HASEL actuators.

The meaning of the parameters $(L/g)^{1/2}$ and $\mu wL^{1/2}/Mg^{1/2}$ can be explained by analyzing the two limiting cases $\mu \to 0$ and $\mu \to \infty$. In the case $\mu \to 0$, the viscosity of the liquid dielectric becomes negligible and the parameter μ disappears as an independent parameter from Eq. **S4** reducing the number of independent dimensionless parameters by one. Repeating the scaling analysis under this assumption leads to

$$t_\mathrm{n}\left(\frac{g}{L}\right)^{1/2}=H\left(\frac{\Phi^2\varepsilon_0\varepsilon_\mathrm{r}w}{Mgh}\right). \quad \textbf{[S7]}$$

From Eq. **S7** follows that when the parameter $\Phi^2\varepsilon_0\varepsilon_r w/hMg$ is constant, the rise and fall times are proportional to the timescale $\tau_i = (L/g)^{1/2}$. In the limit $\mu \to \infty$ inertial effects can be neglected and the actuator contracts in a quasistatic motion. With this assumption the parameters M and g enter as the product Mg into Eq. **S4**, which also reduces the number of independent dimensionless parameters by one. Repeating the scaling analysis under this assumption leads to

$$t_\mathrm{n}\frac{Mg}{\mu wL}=I\left(\frac{\Phi^2\varepsilon_0\varepsilon_\mathrm{r}w}{Mgh}\right). \quad \textbf{[S8]}$$

From Eq. **S8** follows that when the parameter $\Phi^2\varepsilon_0\varepsilon_r w/Mgh$ is constant, the rise and fall times are proportional to the timescale $\tau_v = \mu wL/ Mg$. In Eq. S6, the parameter $\mu wL^{1/2}/Mg^{1/2}$ is the ratio of the two timescales τ_v/τ_i. The value of τ_v/τ_i governs if an actuator lies in the inertial or the viscous regime (see main text).

Dynamic equations of motion

We assume that the polymer membrane in the liquid-filled region of the shell takes the shape of cylinder sections and parameterize its geometry with half the angle α, at which the cylinder sections intersect (Fig. S2*B*). We treat the liquid dielectric as incompressible. Using the length of the shell L, the width of the shell w, and the volume of the liquid dielectric V, the initial angle α_0 can be calculated from

$$L=\sqrt{\frac{2V\alpha_0^2/w}{\alpha_0-\sin(\alpha_0)\cos(\alpha_0)}}. \quad \textbf{[S9]}$$

and the initial length l_0 of the actuator as

$$l_0=\frac{L\sin(\alpha_0)}{\alpha_0}. \quad \textbf{[S10]}$$

In the zipped state, the zipping length z is defined by

$$z=L-\sqrt{\frac{2V\alpha^2/w}{\alpha-\sin(\alpha)\cos(\alpha)}}, \quad \textbf{[S11]}$$

and the stroke of the actuator becomes

$$x = l_0 - z - (L - z)\frac{\sin(\alpha)}{\alpha}. \quad \textbf{[S12]}$$

The equations of motion of the system can be calculated using the Lagrangian equations of the second kind (2):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial A}{\partial \dot{\alpha}}\right) - \frac{\partial A}{\partial \alpha} = q, \quad \textbf{[S13]}$$

where *A* = *T*-*P* (*T* = kinetic energy, *P* = potential energy) is the Lagrangian of the system and *q* the generalized force due to viscous dissipation.

When neglecting the electric energy stored in the electric field inside the fluid-filled region of the shell, the capacitance of the actuator can be described as a parallel plate capacitor with capacitance equal to the capacitance of the zipped area of the electrodes. With this assumption, the potential energy of the system becomes

$$P = Mgx + \frac{hQ^2}{\varepsilon_0 \varepsilon_r wz} - \Phi Q, \quad \textbf{[S14]}$$

where *M* is the mass of the weight, *g* the gravitational acceleration, ε_0 the vacuum permittivity, ε_r the relative permittivity of the shell material, *h* the thickness of the shell, Φ the applied voltage, and *Q* the charge on the electrode. The first term on the right hand side of Eq. **S14** describes the potential energy of the weight, the second equation the electric energy stored in the zipped region of the electrodes, and the third term the change in energy of the voltage source when the charge *Q* flows onto the actuator. Using the relationship between voltage and charge on a capacitor ($Q = C\Phi$, where *C* is the capacitance), we can simplify Eq. **S14** to

$$P = Mgx - \frac{\varepsilon_0 \varepsilon_r wz\Phi^2}{4h}. \quad \textbf{[S15]}$$

Since we neglect the mass of the actuator, only the kinetic energy of the weight contributes to the kinetic energy of the system:

$$T = \frac{1}{2}M\dot{x}^2, \quad \textbf{[S16]}$$

where $\dot{x}$ is the velocity of the weight (Fig. S2*B*).

To obtain a model for viscous dissipation, we approximate the flow of the liquid dielectric with the Poiseuille flow (3) between two parallel plates (Fig. S2*C*). As the length of the plates we use the length of the fluid-filled region of the shell

$$l = (L - z)\frac{sin(\alpha)}{\alpha}. \quad \textbf{[S17]}$$

As the distance between the plates, we use the average thickness of the fluid-filled region of the shell. It can be calculated using the radius of the shell $r = (L-z)/2\alpha$ as

$$d = \frac{2}{l}\int_{-\alpha}^{\alpha} r(cos(\theta) - \cos(\alpha))rcos(\theta)\mathrm{d}\theta = \frac{2r^2}{l}\left(\alpha - \frac{1}{2}\sin(2\alpha)\right). \quad \textbf{[S18]}$$

We assume that the average flow velocity of the fluid is the speed $\dot{z}$ at which the electrodes zip. The pressure drop along a plate section of length *l* is (3)

$$\Delta p = -\frac{12\mu l\dot{z}}{d^2}. \quad \textbf{[S19]}$$

Using Eqs **S17**-**S19**, the generalized force q due to viscous dissipation can be calculated as (2)

$$q = \eta\Delta pwd\frac{\partial z}{\partial\alpha} = -\eta 24\mu w\frac{sin(\alpha)^2}{\alpha-0.5\sin(2\alpha)}\left(\frac{\partial z}{\partial\alpha}\right)^2\dot{\alpha}, \quad \textbf{[S20]}$$

where $\dot{\alpha}$ is the rate of change of α with time, and η is a fitting factor to account for the simplifications made to obtain the equation for q.

Combining Eqs. **S13**, **S15**, **S16**, and **S20** leads to the equation of motion for the actuator:

$$\ddot{\alpha} = \frac{\varepsilon w\Phi^2}{4Mh}\frac{\cos(\alpha)}{1-\cos(\alpha)}\left(\frac{\partial x}{\partial\alpha}\right)^{-1} - g\left(\frac{\partial x}{\partial\alpha}\right)^{-1} - \left(\frac{\partial^2 x}{\partial\alpha^2}\right)\left(\frac{\partial x}{\partial\alpha}\right)^{-1}\dot{\alpha}^2 - \frac{24\eta\mu w}{M}\frac{sin(\alpha)^2}{\alpha-0.5\sin(2\alpha)}\frac{\cos(\alpha)^2}{\left(1-\cos(\alpha)\right)^2}\dot{\alpha} \quad \textbf{[S21]}$$

Equation **S21** can be integrated numerically using Eq. **S11** and

$$\frac{\partial \mathrm{x}}{\partial\mathrm{\alpha}} = (L-z)\left(\frac{\sin(\alpha)}{\alpha^2} - \frac{\cos(\alpha)}{\alpha}\right) + \frac{\partial z}{\partial\alpha}\left(\frac{\sin(\alpha)}{\alpha} - 1\right), \quad \textbf{[S22a]}$$

$$\frac{\partial \mathrm{z}}{\partial\mathrm{\alpha}} = -\frac{2V}{w(L-z)}\frac{\alpha^2\cos(\alpha)^2 - \alpha\sin(\alpha)\cos(\alpha)}{(\alpha-\sin(\alpha)\cos(\alpha))^2}, \quad \textbf{[S22b]}$$

$$\frac{\partial^2 \mathrm{x}}{\partial\mathrm{\alpha}^2} = (L-z)\left(\frac{\sin(\alpha)}{\alpha} + \frac{2\cos(\alpha)}{\alpha^2} - \frac{2\sin(\alpha)}{\alpha^3}\right) - 2\frac{\partial z}{\partial\alpha}\left(\frac{\sin(\alpha)}{\alpha^2} - \frac{\cos(\alpha)}{\alpha}\right) + \frac{\partial^2 z}{\partial\alpha^2}\left(\frac{\sin(\alpha)}{\alpha} - 1\right), \quad \textbf{[S22c]}$$

and

$$\frac{\partial^2 \mathrm{z}}{\partial\mathrm{\alpha}^2} = -\frac{2V}{w(L-z)}\frac{2\alpha\sin(\alpha)^3\cos(\alpha) + \alpha^2\sin(\alpha)^4 - 2\alpha^3\sin(\alpha)\cos(\alpha)}{(\alpha-\sin(\alpha)\cos(\alpha))^3}. \quad \textbf{[S22d]}$$

Transition between viscous and inertial regime

We determine the transition between the inertial to viscous regimes (Fig. 5) by calculating at which value of $\mu wL^{1/2}/Mg^{1/2}$ a simulation in which inertia is neglected leads to the same transition time as a simulation in which viscosity is neglected.

When viscosity is neglected, Eq. **S21** reduces to

$$\ddot{\alpha} = \frac{\varepsilon w\Phi^2}{4Mh}\frac{\cos(\alpha)}{1-\cos(\alpha)}\left(\frac{\partial x}{\partial\alpha}\right)^{-1} - g\left(\frac{\partial x}{\partial\alpha}\right)^{-1} - \left(\frac{\partial^2 x}{\partial\alpha^2}\right)\left(\frac{\partial x}{\partial\alpha}\right)^{-1}\dot{\alpha}^2. \quad \textbf{[S23]}$$

When inertial effects are neglected, Eq. **S21** reduces to

$$\dot{\alpha} = \frac{\frac{\varepsilon w\Phi^2}{4h} - Mg\frac{1-\cos(\alpha)}{cos(\alpha)}}{\eta 24\mu w\frac{sin(\alpha)^2}{\alpha-0.5\sin(2\alpha)}\frac{\partial z}{\partial\alpha}}. \quad \textbf{[S24]}$$

Frequency dependence of the actuation strain for a periodic excitation signal

The scaling analysis for a periodic excitation voltage amplitude Φ and frequency f is similar to that for a step-voltage. Here we present the scaling analysis for the extreme cases $\mu \to 0$ and $\mu \to \infty$. In the case of $\mu \to 0$, the amplitude of the actuation strain e_a can be determined from

$$F(\Phi, f, \varepsilon_0\varepsilon_\mathrm{r}, h, w, L, L_\mathrm{e}, V, M, g, e_a) = 0, \quad \textbf{[S25]}$$

where Φ [V] and f [1/s] are the amplitude and frequency of the excitation voltage, respectively, ε_0 [mkg/s^{2}V^2] the vacuum permittivity, ε_r [-] and h [m] the relative permittivity and thickness of the material of the shell, respectively, w [m] and L [m] the dimensions of the shell, L_e [m] the length of the electrodes, V [m^3] the volume of the liquid dielectric, M [kg] the mass of the hanging weight, g [m^2/s] the gravitational acceleration, and e_a [-] the amplitude of the actuation strain. Using dimensional analysis Eq. **S25** can be rewritten in a dimensionless form:

$$e_a = G\left(\frac{\Phi^2 \varepsilon_0 \varepsilon_r w}{Mgh}, f\sqrt{\frac{L}{g}}, \frac{V}{wL^2}, \frac{L_e}{L}, \frac{h}{L}, \frac{w}{L}\right). \quad \textbf{[S26]}$$

As discussed above (see *Scaling Analysis*), the last four parameters in G can be neglected, which leads to

$$e_a = G\left(\frac{\Phi^2 \varepsilon_0 \varepsilon_r w}{Mgh}, f\sqrt{\frac{L}{g}}\right). \quad \textbf{[S27]}$$

In the case of $\mu \to \infty$ the starting function for the dimensional analysis is

$$F(\Phi, f, \varepsilon_0\varepsilon_r, h, w, L, L_e, V, \mu, Mg, e_a) = 0, \quad \textbf{[S28]}$$

where Φ [V] and f [1/s] are the amplitude and frequency of the excitation voltage, respectively, ε_0 [mkg/s^{2}V^2] the vacuum permittivity, ε_r [-] and h [m] the relative permittivity and thickness of the material of the shell, respectively, w [m] and L [m] the dimensions of the shell, L_e [m] the length of the electrodes, V [m^3] and μ [kg/sm] the volume and the viscosity of the liquid dielectric, respectively, Mg [mkg/s^2] the weight of the attached mass, and e_a [-] the amplitude of the actuation strain. Using dimensional analysis and the simplifications described above, Eq. **S28** can be rewritten in a dimensionless form:

$$e_a = H\left(\frac{\Phi^2 \varepsilon_0 \varepsilon_r w}{Mgh}, f\frac{\mu w L}{Mg}\right). \quad \textbf{[S29]}$$

Equation **S27** and **S29** show that for a periodic signal the inertial timescale $\tau_i = (L/g)^{1/2}$ and the viscous timescale $\tau_v = \mu wL/Mg$, which govern the transition times, also govern the frequency dependence of the actuation strain of Peano-HASEL actuators in the two regimes. Important dynamic characteristics of the actuators such as the roll-off frequency therefore also scale with these timescales. It is important to note that in this analysis no assumption of the shape of the excitation signal (*e.g.*, sinusoidal, rectangular, triangular) was made. For different excitation signals, the functions G and H may be different, but the scaling behavior still follows Eq. **S27** and **S29** (Adding a static offset to the voltage signal adds an additional term to G and H, but it does not change the overall scaling behavior)**.**

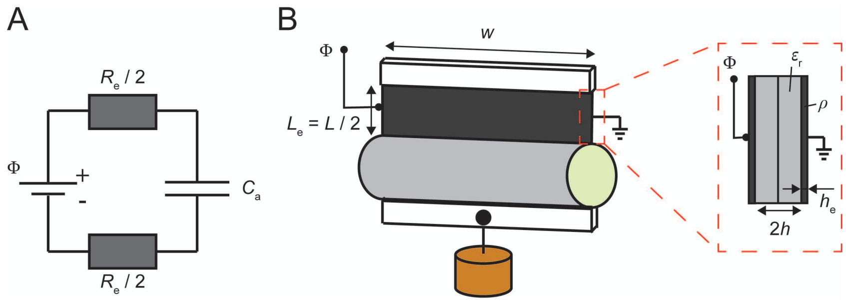

Fig. S1. Estimation of the RC time constant of Peano-HASEL actuators. (*A*) Equivalent circuit of a Peano-HASEL actuator. The actuator is modeled as two resistors $R_e/2$ representing the resistance of the electrodes and a capacitor C_a representing the capacitance of the zipped region of the electrodes. (*B*) Definition of the parameters used to estimate the RC time of Peano-HASEL actuators.

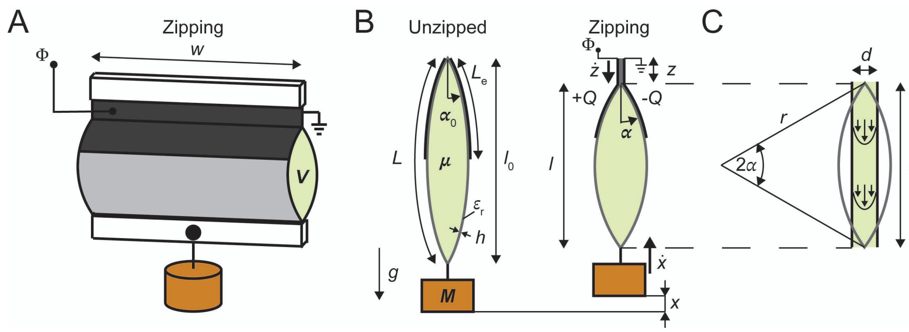

Fig. S2. Model of Peano-HASEL actuators. (*A*) and (*B*) Definition of the parameters used in the scaling analysis and in the theoretical model. (*C*) The fluid flow is approximated as the Poiseuille flow between two parallel plates. The length of the plates is equal to the length of the unzipped region of the shell. The distance between the plates is equal to the average thickness of the unzipped region of the shell. The average velocity of the fluid flow is assumed to be equal to the zipping speed.

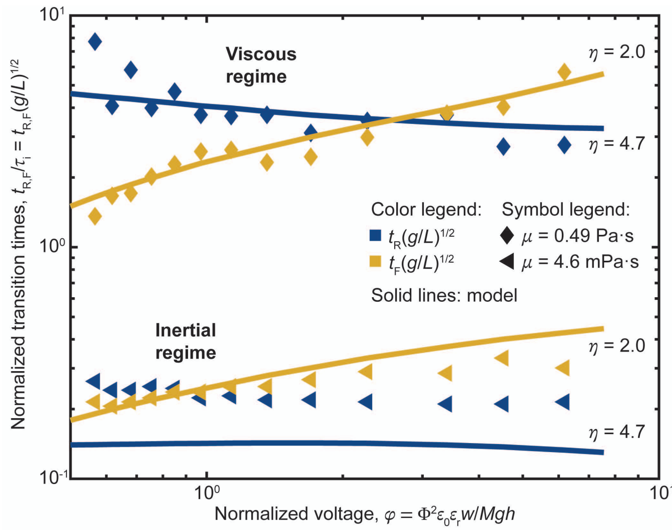

Fig. S3. Comparison between experimental results and model predictions when the parameter η is chosen separately for the rise and the fall time. Comparison of the measured normalized rise times (blue) and fall times (yellow) as functions of φ for an actuator in the viscous regime (diamonds) and an actuator in the inertial regime (triangles) with model predictions. For the calculation of the rise time $\eta = 4.7$; for the calculation of the fall time $\eta = 2.0$. Experimental parameters: w = 6 cm, L = 2 cm, h = 20.3 mm, $\varepsilon_r = 3.5$, Φ = 6 kV.

Movie S1 (separate file). Electrostatic zipping of Peano-HASEL actuators in the inertial and viscous regimes. Peano-HASEL actuators were made from L0WS with dimensions w = 12 cm, L = 4 cm and filled with liquid dielectric of viscosities μ = 4.6 mPa·s (left) and μ = 0.96 Pa·s (right). Both actuators were suspended from the top frame and a 600 g weight was attached to the top frame. Both actuators were excited with a square wave signal of 6 kV at 0.1 Hz with reversing polarity. The video is replayed in real time.

Movie S2 (separate file). High-speed video of zipping in the viscous regime. A Peano-HASEL actuator was made from L0WS with dimensions w = 12 cm, L = 4 cm and filled with liquid dielectric of viscosity μ = 0.96 Pa·s. The actuator was suspended from the top frame and a 600 g weight was attached to the top frame. The actuator was excited with a square wave signal of 6 kV at 0.1 Hz with reversing polarity. The video is replayed at 0.1 speed.

Movie S3 (separate file). High-speed video of zipping in the inertial regime. A Peano-HASEL actuator was made from L0WS with dimensions w = 12 cm, L = 4 cm and filled with liquid dielectric of viscosity μ = 4.6 mPa·s. The actuator was suspended from the top frame and a 600 g weight was attached to the top frame. The actuator was excited with a square wave signal of 6 kV at 0.5 Hz with reversing polarity. The video is replayed at 0.01 speed.

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