Appendix: Details of Optimization

To minimize (10), we differentiate $\ell(\boldsymbol{\beta}, \boldsymbol{\gamma})$ with respect to $(\boldsymbol{\beta}, \boldsymbol{\gamma})$. Then $(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\gamma}})$ solves the following estimating equations:

$$\begin{aligned}
\sum_{t=1}^{T}\left[\left\{\frac{z_R(t)}{r(t)}-1\right\}\frac{\partial r(t)}{\partial \boldsymbol{\beta}}+\left\{\frac{z_I(t)}{i(t)}-1\right\}\frac{\partial i(t)}{\partial \boldsymbol{\beta}}\right] &= 0,\\
\sum_{t=1}^{T}\left[\left\{\frac{z_R(t)}{r(t)}-1\right\}\frac{\partial r(t)}{\partial \boldsymbol{\gamma}}+\left\{\frac{z_I(t)}{i(t)}-1\right\}\frac{\partial i(t)}{\partial \boldsymbol{\gamma}}\right] &= 0,
\end{aligned}$$

where the involved partial derivatives can be computed recursively. Specifically, taking partial derivatives on the both sides of (7) yields that, for $t = 0, \ldots, T-1$,

$$\begin{aligned}
\frac{\partial i(t+1)}{\partial \boldsymbol{\beta}} &= [\{\beta(t)+1\}\{1-\gamma(t)\}-2\beta(t)i(t)]\frac{\partial i(t)}{\partial \boldsymbol{\beta}}+i(t)\{1-\gamma(t)-i(t)\}\frac{\partial \beta(t)}{\partial \boldsymbol{\beta}},\\
\frac{\partial i(t+1)}{\partial \boldsymbol{\gamma}} &= [\{\beta(t)+1\}\{1-\gamma(t)\}-2\beta(t)i(t)]\frac{\partial i(t)}{\partial \boldsymbol{\gamma}}-i(t)\{\beta(t)+1\}\frac{\partial \gamma(t)}{\partial \boldsymbol{\gamma}},\\
\frac{\partial r(t+1)}{\partial \boldsymbol{\beta}} &= \frac{\partial r(t)}{\partial \boldsymbol{\beta}}+\gamma(t)\frac{\partial i(t)}{\partial \boldsymbol{\beta}},\\
\frac{\partial r(t+1)}{\partial \boldsymbol{\gamma}} &= \frac{\partial r(t)}{\partial \boldsymbol{\gamma}}+\gamma(t)\frac{\partial i(t)}{\partial \boldsymbol{\gamma}}+i(t)\frac{\partial \gamma(t)}{\partial \boldsymbol{\gamma}}.
\end{aligned}$$

Here, $\frac{\partial}{\partial \boldsymbol{\beta}}\beta(t) = \beta(t) \times \mathbf{B}(t)$, and $\frac{\partial}{\partial \boldsymbol{\gamma}}\gamma(t) = \gamma(t) \times \mathbf{B}(t)$, and $\frac{\partial}{\partial \boldsymbol{\beta}}i(0) = \frac{\partial}{\partial \boldsymbol{\gamma}}i(0) = \frac{\partial}{\partial \boldsymbol{\beta}}r(0) = \frac{\partial}{\partial \boldsymbol{\gamma}}r(0) = 0$ by using the initial conditions.