# Electronic Supplementary Material for:

## Self-employment over the Business Cycle in the USA: A Decomposition

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# Appendix A: Nonlinear Decomposition

The logit model of the probability of entry into self-employment can be written as:

$$Y = F(X\beta) \tag{A.1}$$

where $Y$ is the vector of predicted entry probabilities, $X$ the matrix of independent variables, $\beta$ the coefficient vector, and $F$ the cumulative logistic distribution function. A nonlinear decomposition of the mean difference in entry into self-employment by period can be written as:

$$\overline{Y}_R - \overline{Y}_C = \left[\overline{F(X_R\beta_R)} - \overline{F(X_C\beta_R)}\right] + \left[\overline{F(X_C\beta_R)} - \overline{F(X_C\beta_C)}\right] \tag{A.2}$$

where index $R$ stands for the observations during the Great Recession and index $C$ for the observations in the comparison period. In Equation A.2 the first summand is the contribution of the distribution of the variables to the overall difference in the entry rate, i.e. the explained part, whereas the second summand is the contribution of differences in the coefficients (including the constant), i.e. the unexplained part.[1] Following the approach of Yun (2004), for a detailed decomposition which assesses the contributions of each single variable (or group of variables) separately in this non-linear setting, two approximations are necessary. First, I consider predictions at the mean values of the explanatory variables:

$$\overline{Y}_R - \overline{Y}_C = \left[F(\overline{X}_R\beta_R) - F(\overline{X}_C\beta_R)\right] + \left[F(\overline{X}_C\beta_R) - F(\overline{X}_C\beta_C)\right] + R_A, \tag{A.3}$$

[1] More precisely, as mentioned in Section 3 of the paper, we use the coefficient estimates from a pooled estimation for the decomposition of the contributions of the observed characteristics to the differential; see Jann (2008) for the technical details.

where

$$R_A = \left[\overline{F(X_R\beta_R)} - \overline{F(X_C\beta_R)}\right] + \left[\overline{F(X_C\beta_R)} - \overline{F(X_C\beta_C)}\right] - \left[F(\overline{X}_R\beta_R) - F(\overline{X}_C\beta_R)\right] - \left[F(\overline{X}_C\beta_R) - F(\overline{X}_C\beta_C)\right]. \quad \text{(A.4)}$$

Second, a first order Taylor expansion around the mean characteristics is used. Hence, I can rewrite Equation A.3 as follows:

$$\begin{aligned} \overline{Y}_R - \overline{Y}_C &= \left[(\overline{X}_R - \overline{X}_C)\beta_R\right] f(\overline{X}_R\beta_R) \\ &+ \overline{X}_C(\beta_R - \beta_C) f(\overline{X}_C\beta_C) + R_A + R_T, \end{aligned} \quad \text{(A.5)}$$

where $f(\cdot)$ is the first order derivative of $F(\cdot)$ and $R_T$ is the approximation error. Using Equation A.5, a detailed decomposition of Equation A.2 can be written as

$$\overline{Y}_R - \overline{Y}_C = \sum_{i=1}^{K} W^i_{\Delta X}\left[\overline{F(X_R\beta_R)} - \overline{F(X_C\beta_R)}\right] + \sum_{i=1}^{K} W^i_{\Delta\beta}\left[\overline{F(X_C\beta_R)} - \overline{F(X_C\beta_C)}\right], \quad \text{(A.6)}$$

i.e., the detailed decomposition includes weights for the contributions of the characteristics ($W^i_{\Delta X}$) and for the contributions of the coefficients ($W^i_{\Delta\beta}$), with

$$W^i_{\Delta X} = \frac{(\overline{X}^i_R - \overline{X}^i_C)\beta^i_R}{(\overline{X}_R - \overline{X}_C)\beta_R} \quad \text{and} \quad W^i_{\Delta\beta} = \frac{\overline{X}^i_C(\beta^i_R - \beta^i_C)}{\overline{X}_C(\beta_R - \beta_C)}$$

for variable $i$ in the set of $K$ explanatory variables (Yun, 2004).

# Appendix B: Supplementary Tables

Table B.1: Descriptive statistics for non-binary variables

| Variable | Age | Number of children |
|---|---|---|
| Mean | 40.98 | 0.963 |
| Std. deviation | 11.88 | 1.188 |
| Median | 41 | 1 |
| Minimum | 21 | 0 |
| Maximum | 64 | 9 |
| Skewness | 0.048 | 1.228 |
| Kurtosis | 1.886 | 4.532 |
| Observations | 1,598,341 | 1,598,341 |

*Source:* Own calculations based on the Current Population Survey.

Table B.2: Nonlinear decomposition of entry rate into self-employment using the method of Fairlie (2005)

| | Total entrepreneurship | | Unincorp. entrepreneurship | | Incorp. entrepreneurship | |
|---|---|---|---|---|---|---|
| | GR versus before | GR vs. after | GR vs. before | GR vs. after | GR vs. before | GR vs. after |
| Overall difference in entry rates into self-employment (in percentage points) | | | | | | |
| Great Rec. | 0.6122 | 0.6122 | 0.4847 | 0.4847 | 0.1274 | 0.1274 |
| Comp. period | 0.5414 | 0.5680 | 0.4296 | 0.4387 | 0.1118 | 0.1293 |
| Difference | 0.0708 | 0.0442 | 0.0552 | 0.0461 | 0.0156 | -0.0018 |
| Explained | 0.0678 | 0.0124 | 0.0599 | 0.0139 | 0.0075 | -0.0014 |
| Unexplained | 0.0030 | 0.0318 | -0.0047 | 0.0322 | 0.0081 | -0.0005 |
| Difference explained by individual characteristics | | | | | | |
| Unemployed | 0.222*** | 0.0985*** | 0.199*** | 0.0597*** | 0.0140*** | 0.00696** |
| | (0.0156) | (0.0132) | (0.0160) | (0.0139) | (0.00352) | (0.00322) |
| Non-particip. | -0.167*** | -0.0900*** | -0.152*** | -0.0519*** | -0.00629 | -0.00521 |
| | (0.0150) | (0.0129) | (0.0154) | (0.0130) | (0.00511) | (0.00453) |
| Male | 0.0212*** | 0.00441** | 0.0201*** | 0.00456*** | 0.00211 | 0.000575 |
| | (0.00301) | (0.00208) | (0.00273) | (0.00154) | (0.00235) | (0.00216) |
| Education | -0.00356*** | -0.00226** | -0.00127 | 0.000770 | -0.00205* | -0.00301*** |
| | (0.00113) | (0.00109) | (0.000800) | (0.00119) | (0.00121) | (0.00111) |
| Race | -0.000911 | 0.00234** | -0.00196** | 0.00255** | -0.000157 | 0.000352 |
| | (0.000843) | (0.000930) | (0.000920) | (0.00121) | (0.000278) | (0.000325) |
| Married | -0.00101 | 0.000492 | 0.0000887 | -0.000519 | -0.000397 | 0.00142* |
| | (0.00168) | (0.00133) | (0.00154) | (0.00144) | (0.000692) | (0.000812) |
| No. children | 0.000128 | 0.00000169 | 0.000283 | 0.00000277 | -0.00000963 | 0.00000479 |
| | (0.00102) | (0.000879) | (0.000921) | (0.000730) | (0.000479) | (0.000404) |
| Metropolitan | -0.000550 | 0.00118*** | -0.00174** | 0.00135** | -0.0000743 | -0.000356 |
| | (0.000425) | (0.000455) | (0.000718) | (0.000654) | (0.000212) | (0.000258) |
| Age | -0.000817 | 0.00132 | -0.000520 | 0.000426 | 0.000486 | -0.00151 |
| | (0.00203) | (0.00203) | (0.00194) | (0.00226) | (0.00139) | (0.00136) |
| Region | -0.00209** | -0.00361*** | -0.00271*** | -0.00303*** | -0.000105 | -0.000602* |
| | (0.000925) | (0.000944) | (0.00104) | (0.00106) | (0.000321) | (0.000352) |
| N | 1,079,986 | 1,059,872 | 1,079,986 | 1,059,872 | 1,079,986 | 1,059,872 |
| N: GR | 541,517 | 541,517 | 541,517 | 541,517 | 541,517 | 541,517 |
| N: Comp. per. | 538,469 | 518,355 | 538,469 | 518,355 | 538,469 | 518,355 |

*Notes:* Nonlinear Oaxaca decomposition of the monthly entry rate into self-employment based on logit estimations using the method suggested by Fairlie (2005). The order of the variables is randomized in each of the 100 replications. All figures are in percentage points. Standard errors in parentheses. *,**,***: Significant at the 10%/5%/1%-levels. *Source:* Own calculations based on the Current Population Survey 2007-2014.

Table B.3: Nonlinear decomposition of entry rate into self-employment with two-year comparison periods

| | Total entrepreneurship | | Unincorp. entrepreneurship | | Incorp. entrepreneurship | |
|---|---|---|---|---|---|---|
| | GR versus before | GR vs. after | GR vs. before | GR vs. after | GR vs. before | GR vs. after |
| Overall difference in entry rates into self-employment (in percentage points) | | | | | | |
| Great Rec. | 0.612*** | 0.612*** | 0.485*** | 0.485*** | 0.127*** | 0.127*** |
| | (0.0106) | (0.0106) | (0.00941) | (0.00941) | (0.00485) | (0.00485) |
| Comp. period | 0.545*** | 0.578*** | 0.435*** | 0.449*** | 0.110*** | 0.129*** |
| | (0.00706) | (0.00743) | (0.00631) | (0.00656) | (0.00320) | (0.00352) |
| Difference | 0.0667*** | 0.0338*** | 0.0497*** | 0.0353*** | 0.0170*** | -0.00148 |
| | (0.0127) | (0.0129) | (0.0113) | (0.0115) | (0.00581) | (0.00599) |
| Explained | 0.0742*** | 0.0215*** | 0.0647*** | 0.0220*** | 0.00885*** | -0.000521 |
| | (0.00219) | (0.00163) | (0.00201) | (0.00148) | (0.000762) | (0.000581) |
| Unexplained | -0.00749 | 0.0123 | -0.0150 | 0.0133 | 0.00814 | -0.000962 |
| | (0.0119) | (0.0127) | (0.0105) | (0.0112) | (0.00559) | (0.00604) |
| Difference explained by individual characteristics | | | | | | |
| Unemployed | 0.0712*** | 0.0504*** | 0.0635*** | 0.0399*** | 0.00732*** | 0.00140 |
| | (0.00215) | (0.00468) | (0.00202) | (0.00294) | (0.000665) | (0.000897) |
| Non-particip. | -0.000602 | -0.0283*** | -0.000520 | -0.0216*** | -0.0000788 | -0.00105 |
| | (0.00124) | (0.00359) | (0.00107) | (0.00232) | (0.000162) | (0.000731) |
| Male | 0.00158** | -0.00308*** | 0.00124** | -0.00200*** | 0.000284** | -0.000184 |
| | (0.000626) | (0.000919) | (0.000493) | (0.000573) | (0.000114) | (0.000140) |
| Education | 0.00153*** | -0.00498*** | -0.0000741 | 0.00136 | 0.00144*** | -0.00134 |
| | (0.000309) | (0.00142) | (0.000261) | (0.000933) | (0.000158) | (0.000936) |
| Race | -0.000872*** | 0.00739*** | -0.000803*** | 0.00624*** | -0.0000817** | 0.000145 |
| | (0.000211) | (0.00100) | (0.000194) | (0.000776) | (0.0000416) | (0.000111) |
| Married | -0.000211 | 0.00264** | 0.000235 | -0.00106 | -0.000340*** | 0.000806 |
| | (0.000183) | (0.00105) | (0.000170) | (0.000944) | (0.0000793) | (0.000531) |
| No. children | 0.0000435 | -0.000117 | 0.0000579 | -0.0000963 | -0.00000280 | -0.0000108 |
| | (0.0000570) | (0.000176) | (0.0000564) | (0.000145) | (0.0000174) | (0.0000170) |
| Metropolitan | -0.000401*** | 0.00264*** | -0.000473*** | 0.00290*** | 0.0000505* | -0.000124 |
| | (0.000144) | (0.000559) | (0.000165) | (0.000464) | (0.0000264) | (0.000103) |
| Age | 0.00278*** | 0.00424*** | 0.00227*** | 0.00312*** | 0.000434** | 0.000246 |
| | (0.000709) | (0.00131) | (0.000595) | (0.00102) | (0.000174) | (0.000165) |
| Region | -0.000845*** | -0.00939*** | -0.000649*** | -0.00676*** | -0.000163*** | -0.000409 |
| | (0.000226) | (0.00174) | (0.000199) | (0.00127) | (0.0000519) | (0.000321) |
| N | 1,620,093 | 1,577,181 | 1,620,093 | 1,577,181 | 1,620,093 | 1,577,181 |
| N: GR | 541,517 | 541,517 | 541,517 | 541,517 | 541,517 | 541,517 |
| N: Comp. per. | 1,078,576 | 1,035,664 | 1,078,576 | 1,035,664 | 1,078,576 | 1,035,664 |

*Notes:* Nonlinear Oaxaca decomposition of the monthly entry rate into self-employment based on logit estimations. Here, the period before the GR is 04/2006-03/2008, during the GR 09/2008-08/2009, and after the GR 05/2013-04/2015. All figures are in percentage points. Robust standard errors in parentheses. *,**,***: Significant at the 10%/5%/1%-levels. *Source:* Own calculations based on the Current Population Survey 2006-2015.

Table B.4: Nonlinear decomposition of entry rate into self-employment with modified comparison periods

| | Total entrepreneurship | | Unincorp. entrepreneurship | | Incorp. entrepreneurship | |
|---|---|---|---|---|---|---|
| | GR versus before | GR vs. after | GR vs. before | GR vs. after | GR vs. before | GR vs. after |
| Overall difference in entry rates into self-employment (in percentage points) | | | | | | |
| Great Rec. | 0.593*** | 0.593*** | 0.468*** | 0.468*** | 0.124*** | 0.124*** |
| | (0.00851) | (0.00851) | (0.00757) | (0.00757) | (0.00392) | (0.00392) |
| Comp. period | 0.543*** | 0.590*** | 0.432*** | 0.467*** | 0.111*** | 0.123*** |
| | (0.0115) | (0.00476) | (0.0103) | (0.00424) | (0.00525) | (0.00218) |
| Difference | 0.0495*** | 0.00262 | 0.0365*** | 0.00140 | 0.0130** | 0.00122 |
| | (0.0143) | (0.00975) | (0.0128) | (0.00867) | (0.00655) | (0.00448) |
| Explained | 0.0461*** | | 0.0410*** | | 0.00498*** | |
| | (0.00166) | | (0.00150) | | (0.000536) | |
| Unexplained | 0.00342 | | -0.00445 | | 0.00801 | |
| | (0.0140) | | (0.0124) | | (0.00647) | |
| Difference explained by individual characteristics | | | | | | |
| Unemployed | 0.0454*** | | 0.0412*** | | 0.00425*** | |
| | (0.00171) | | (0.00161) | | (0.000471) | |
| Non-particip. | -0.00146 | | -0.00129 | | -0.000174 | |
| | (0.00143) | | (0.00127) | | (0.000170) | |
| Male | 0.000937 | | 0.000733 | | 0.000164 | |
| | (0.000750) | | (0.000587) | | (0.000132) | |
| Education | 0.000493** | | -0.000429** | | 0.000749*** | |
| | (0.000234) | | (0.000190) | | (0.000137) | |
| Race | -0.000112 | | -0.000103 | | -0.0000110 | |
| | (0.000205) | | (0.000187) | | (0.0000203) | |
| Married | -0.000117 | | 0.000140 | | -0.000179*** | |
| | (0.000121) | | (0.000116) | | (0.0000629) | |
| No. children | 0.00000234 | | 0.0000302 | | -0.0000116 | |
| | (0.0000503) | | (0.0000519) | | (0.0000160) | |
| Metropolitan | -0.0000827 | | -0.000101 | | 0.0000112 | |
| | (0.000152) | | (0.000186) | | (0.0000210) | |
| Age | 0.00188*** | | 0.00150** | | 0.000310** | |
| | (0.000725) | | (0.000596) | | (0.000145) | |
| Region | -0.000883*** | | -0.000733*** | | -0.000128** | |
| | (0.000302) | | (0.000270) | | (0.0000584) | |
| N | 1,213,146 | 3,387,098 | 1,213,146 | 3,387,098 | 1,213,146 | 3,387,098 |
| N: GR | 809,330 | 809,330 | 809,330 | 809,330 | 809,330 | 809,330 |
| N: Comp. per. | 403,816 | 2,577,768 | 403,816 | 2,577,768 | 403,816 | 2,577,768 |

*Notes:* Nonlinear Oaxaca decomposition of the monthly entry rate into self-employment based on logit estimations. Here, the period before the GR is 04/2007-12/2007, during the GR 01/2008-06/2009, and after the GR 07/2009-04/2014. The differences between the entry rates during and after the Great Recession are not decomposed because they are close to zero and statistically insignificant using these definitions of the periods. All figures are in percentage points. Robust standard errors in parentheses. *,**,***: Significant at the 10%/5%/1%-levels. *Source:* Own calculations based on the Current Population Survey 2007-2014.

# References

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