Supplemental Material for sMRT: Multi-Resident Tracking in Smart Homes with Sensor Vectorization

1 Nomenclature

q	Number of sensors in the smart home
s_i	i^{th} sensor in the smart home
$\mathbf{z}_i$	The vector representation of i-th sensor in the measurement space
c	Window size for calculating sensor vectors
m	The dimension of measurement space
r	Resident identifier or track identifier
$\mathbf{x}$	The state vector of a single resident
X, X_k	Finite set of resident state vectors (at time step k)
$\mathcal{X}$	Single resident state vector space
$\mathbf{z}$	The vector representation of a sensor measurement
Z, Z_k	Finite set of sensor observation (at time step k)
n_Z	The number of active sensors in the sensor observation Z
$N_r^{(k)}$	The number of residents who enter the smart home at time step k
$R_i^{(k)}$	The i^{th} resident who enter the smart home at time k
$D_k(\mathbf{x})$	The multi-resident PHD at time step k
$w_k^{(i)}$	The weight of the i^{th} Gaussian component in the multi-resident PHD $D_k(\mathbf{x})$
$\mathbf{m}_k^{(i)}$	The mean vector of the i^{th} Gaussian component in the multi-resident PHD $D_k(\mathbf{x})$
$\mathbf{P}_k^{(i)}$	The covariance matrix of the i^{th} Gaussian component in the multi-resident PHD $D_k(\mathbf{x})$
J	The number of Gaussian components in a Gaussian mixture.
$\mathbf{F}$	The linear multiplier of constant velocity dynamic model
$\mathbf{Q}$	The covariance matrix of the linear Gaussian dynamic model
$\mathbf{H}$	The linear multiplier of the measurement model
$\mathbf{R}$	The covariance matrix of the linear Gaussian measurement model
$\int_S f(X)\delta X$	Set integral on region S
$\mathbf{I}_m$	Identity matrix of size m
λ_c	Poisson event rate of the clutter process
$c(\mathbf{z})$	Spatial distribution of the false alarms
x, x_k	$m \times 1$ vector representing the location of a point target in the measurement space (at time step k)
v, v_k	$m \times 1$ vector representing the velocity of a point target
N_R	Number of residents in the multi-resident smart home dataset
D	Number of sensor events in the multi-resident smart home dataset

2 Constant Velocity Model

Let us consider a point target maneuvering in a m-dimensional measurement space $\mathcal{Z}$. The state vector of each target is a $(2m+1)\times 1$ vector $\mathbf{x} = \begin{bmatrix} x^T & v^T & r \end{bmatrix}^T$, where x is an $m \times 1$ vector representing the location of the target in the measurement space $\mathcal{Z}$, v is an $m \times 1$ vector representing the velocity of the resident in the space, and r is an integer representing the target identifier. Based on the constant velocity assumption, the velocity of the target remains constant, and the location of the target at the next time step $k+1$, x_{k+1}, can be calculated according to (1). In (1), t is the virtual time span of each step.

$$x_{k+1} = x_k + v_k t \tag{1}$$

Thus, in matrix format, the target state $\mathbf{x}_{k+1}$ can be calculated as a linear function of the target state at previous time step $\mathbf{x}_k$, as shown in

$$\mathbf{x}_{k+1} = \begin{bmatrix} x_{k+1} \\ v_{k+1} \\ r \end{bmatrix} = \begin{bmatrix} \mathbf{I}_m & \mathbf{I}_m t & 0 \\ 0 & \mathbf{I}_m & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ v_k \\ r \end{bmatrix} = \mathbf{F} \cdot \mathbf{x}_k. \tag{2}$$

However, in real-world applications, due to various environmental noise, an additive error term is introduced to model such errors. With a $m \times 1$ vector $\mathbf{w}$ denoting the error of actual target velocity in the space with respect to the assumed constant velocity in the previous target state, the state update function (4) can be derived by solving the physical dynamic equation as shown in (3).

$$\frac{\partial x}{\partial t} = \begin{bmatrix} 0 & \mathbf{I}_m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{I}_m \end{bmatrix} \mathbf{w} \tag{3}$$

$$\mathbf{x}_{k+1} = \mathbf{F} \cdot \mathbf{x}_k + \mathbf{G} \cdot \mathbf{w} \tag{4}$$

In (4), $\mathbf{F}$ represents the linear motion multiplier, as shown in (5), and $\mathbf{G}$ represents the linear error multiplier, as shown in (6).

$$\mathbf{F} = \begin{bmatrix} \mathbf{I}_m & \mathbf{I}_m t & 0 \\ 0 & \mathbf{I}_m & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$\mathbf{G} = \begin{bmatrix} \frac{1}{2} t^2 \mathbf{I}_m \\ \mathbf{I}_m \end{bmatrix} \tag{6}$$

3 GM-PHD Filter

We start by considering a single-target scenario, where one target is present in the space. At each time step, a single sensor observation is acquired. The dynamic model of the target is presented in the form of a conditional probability density $f(\mathbf{x}|\mathbf{x}')$, which predicts the next state of the target, $\mathbf{x}$, based on its current state, $\mathbf{x}'$. The measurement model is also formulated as a conditional probability density, $f(\mathbf{z}|\mathbf{x})$, representing the likelihood of a sensor observation $\mathbf{z}$ given the target state $\mathbf{x}$. Provided with the probability distribution of target state $f_{k-1}(\mathbf{x})$ at time $k-1$, the target state probability distribution at time step k, $f_k(\mathbf{x})$, can be calculated using probability theory's chain rule and Bayes' rule according to Equations 7 and 8.

$$f_{k|k-1}(\mathbf{x}) = \int f(\mathbf{x}|\mathbf{x}') f_{k-1}(\mathbf{x}') d\mathbf{x}' \tag{7}$$

$$f_k(\mathbf{x}) = \frac{f(\mathbf{z}|\mathbf{x}) f_{k|k-1}(\mathbf{x})}{\int f(\mathbf{z}|\mathbf{x}') f_{k|k-1}(\mathbf{x}') d\mathbf{x}'} \tag{8}$$

Equation 7 represents the predictor that generates the target state probability density, $f_{k|k-1}(\mathbf{x})$, based on the target state at time step $k-1$ and the dynamic model. Equation 8, representing the corrector, updates the probability density generated by the predictor so that the likelihood of the sensor observation at time step k is maximized. If the dynamic model and the measurement model are both linear and take the form of Gaussian distributions, the single target Bayes filter can be reduced to the traditional Kalman filter [1]. Otherwise, sequential Monte Carlo sampling methods, also known as particle filters, can be used to numerically solve both equations [2].

The situation gets more complicated when there are multiple targets to track and the number of targets is unknown. In this case, the states of all targets in the space are modeled as a set $X = \mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathcal{F}(\mathcal{X})$ where $\mathcal{F}(\mathcal{X})$ represents the collection of all finite subsets of the state space $\mathcal{X}$. Since the number of elements n in the set X is a random variable in $\mathbb{Z}_0^+$, the set X is called a random finite set (RFS). Building on the finite set statistics (FISST) [3], Mahler [1] has shwon that a multi-target probability density function $f(X)$ can be defined for the RFS X and propagated similarly to the single-target Bayes filter shown in Figure 1. The equations for the predictor and the corrector filter are shown in Equations 9 and 10, respectively.

$$f_{k|k-1}(X) = \int f(X|X') f_{k-1}(X') dX' \tag{9}$$

$$f_k(X) = \frac{f(Z_k|X) f_{k|k-1}(X)}{\int f(Z_k|X') f_{k|k-1}(X') \delta X'} \tag{10}$$

Equation 9 mirrors (7) by replacing the single target state variable $\mathbf{x}$ with the multi-target state set X, and the vector integral with a set integral. The multi-target dynamic model, represented by the conditional probability $f(X|X')$, predicts the state distribution of all targets given the states of all targets at previous steps. In addition to the dynamics of persisting targets (i.e., the existing residents still in the space), the multi-target dynamic model must also consider the birth of a new target (i.e., a resident entering the space or becoming active) and the death of an existing target (i.e., a resident leaving the space or becoming inactive). The term $Z_k = \{\mathbf{z}_1^{(k)}, \ldots, \mathbf{z}_{n_Z^{(k)}}^{(k)}\} \in \mathcal{F}(\mathcal{Z})$ in (10) is a set of $n_Z^{(k)}$ sensor observations taken at time step k. The multi-target measurement model, $f_k(Z|X)$, represents the probability density of a set of sensor observations, Z, when the states of all residents in the space is characterized by the set X. The multi-target measurement model encapsulates the information about the likelihood of the sensor observations being triggered by the existing targets in the space (i.e., an existing resident triggering a sensor), the possibility that an existing target fails to be detected by any sensor (i.e., an "active" resident moving to a location not covered by any sensor), and the cases where some sensor observations belong to the clutter process due to sensor failures or communication errors.

The major challenge of the multi-target Bayes filter is the exponential growth of the computational complexity during the numerical calculation of the set integrals and multi-target probability density in Equations 9 and 10 when the cardinality of X increases. To reduce the computational complexity, Mahler [4] proposed a PHD filter that propagates the first-order moment, or probability hypothesis density (PHD), instead. Thus, both the predictor and the corrector equation can be solved in polynomial time using sequential Monte Carlo sampling methods. To further simplify the computation of the PHD filter, Vo and Ma [5] proposed the GM-PHD filter, which is a closed-form solution of the PHD filter where each PHD is represented using Gaussian mixtures.

The PHD of an RFS X, denoted $D(\mathbf{x})$ ($\mathbf{x} \in \mathcal{X}$), parameterized by a multi-target probability density function $f(X)$, is a probability density function whose integral on any region S of the state space $\mathcal{X}$ ($S \subseteq \mathcal{X}$) equals the expected number of targets, $N(S)$, in the region S [1]. Mathematically, the PHD is defined as shown in (11), or equivalently in (12).

$$\int_S D(\mathbf{x}) d\mathbf{x} = N(S) = \int |X \cap S| \cdot f(X) \delta X \tag{11}$$

$$D(\mathbf{x}) = \int f(\{\mathbf{x}\} \cup W) \delta W \tag{12}$$

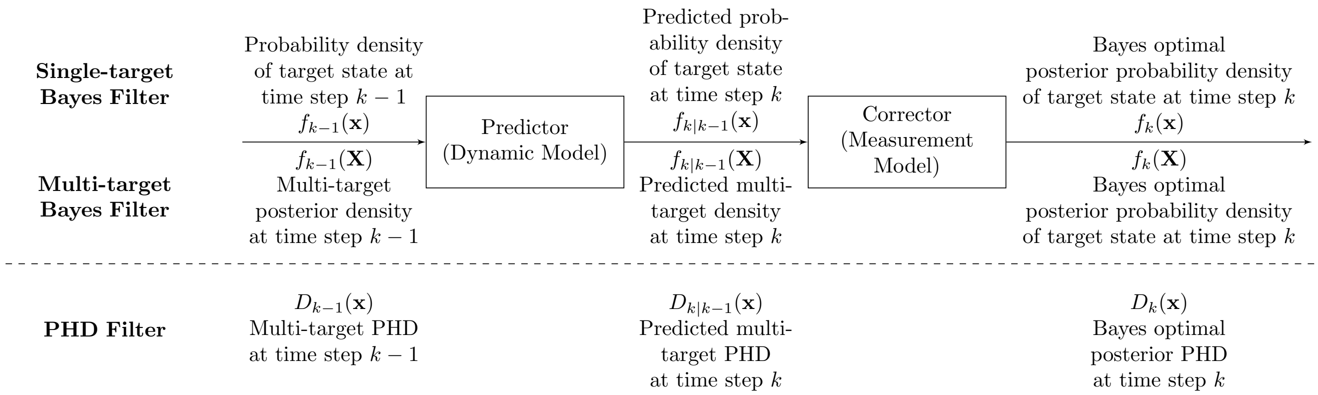

Fig. 1. The propagation pipeline of single-target Bayes filter, multi-target Bayes filter and PHD filter.

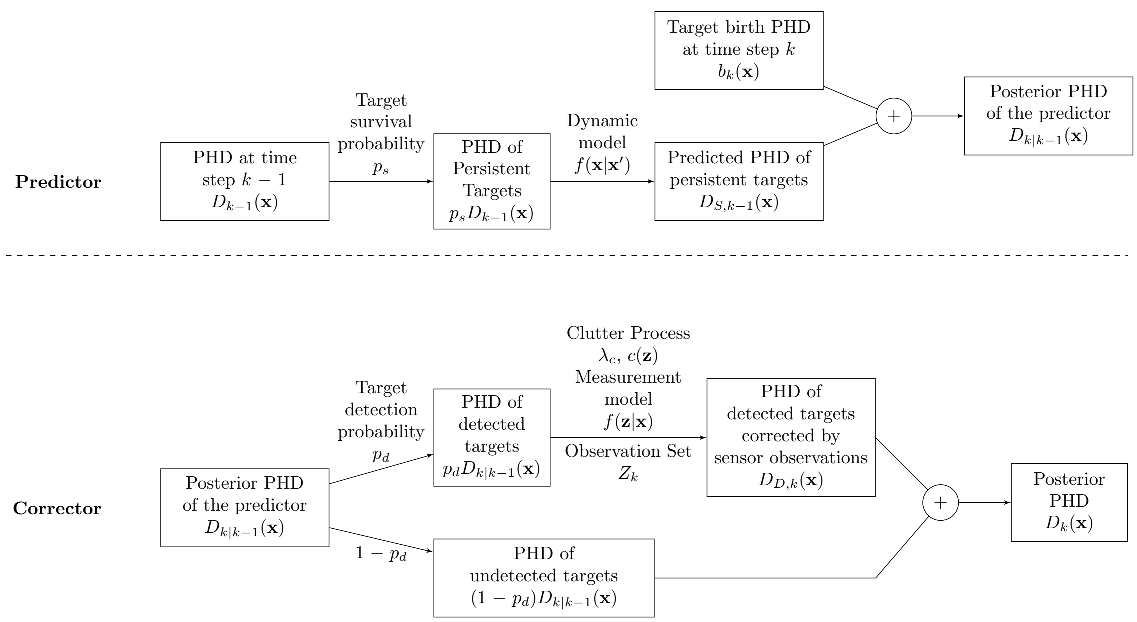

Fig. 2. The propagation of multi-target PHD in a GM-PHD filter implemented in sMRT to solve the multi-resident tracking problem.

Figure 2 illustrates the PHD filter propagation pipeline adapted to the context of multi-resident tracking. The PHD propagation equation of the predictor and corrector can be derived if the following assumptions are true.

- Each target (resident) evolves and generates sensor observations independently of the others. Thus, we can predict the state of each resident independently using the dynamic model characterized by the conditional probability distribution $f(\mathbf{x}|\mathbf{x}')$. Similarly, we can also estimate the likelihood of a sensor observation triggered by some resident in the space independently using the measurement model characterized by the conditional probability distribution $f(\mathbf{z}|\mathbf{x})$.
- The clutter process is a *Poisson point process* (PPP) and is independent of target-originated measurements. The number of sensor observations that are not associated with any resident in the smart home at any given time step follows a Poisson distribution with parameter λ_c, while each false alarm follows a spatial distribution $c(\mathbf{z})$ ($\mathbf{z} \in \mathcal{Z}$).
- The distribution of resident states governed by PHD $D_k(\mathbf{x})$ at any time step k is Poisson.
- At every time step, an existing resident may leave the home or become "inactive". The probability of an existing resident still being "active" in the following time step is characterized by the target survival probability p_s. Similarly, we can use the target detection probability p_d to characterize an "active" resident who fails to be detected by any sensor. In order to derive the PHD filter equations, both the survival probability p_s and the detection probability p_d are constant and independent of resident states.
- At time step k, a PHD, $b_k(\mathbf{x})$, representing the target birth process (i.e., a resident being "active" again or a new resident entering the home) is injected into the predictor. The integral of the target birth PHD, according to the definition of PHD in (11), equals the expected number of new residents at the corresponding time step.

With the above assumptions, the equations for the predictor and corrector in the PHD classifier can be derived using (9) and (10), respectively. The posterior PHD of the predictor, as shown in (13), is the summation of the target birth PHD, $b_k(\mathbf{x})$, and the predicted PHD, $D_{s,k-1}(\mathbf{x})$, of all persisting residents. The term $D_{s,k-1}(\mathbf{x})$ can be calculated using (14). The posterior PHD of the corrector, as shown in (15), is composed of two terms. The first term, $(1-p_d)D_{k|k-1}(\mathbf{x})$, corresponds to the PHD of residents that are not detected by any sensor. The second term, $D_{d,k}(\mathbf{x})$, is the PHD of the detected residents that are corrected by the sensor observations $Z^{(k)}$ at time step k. The term $D_{d,k}(\mathbf{x})$ can be calculated according to (16).

$$D_{k|k-1}(\mathbf{x}) = b_k(\mathbf{x}) + D_{s,k-1}(\mathbf{x}) \tag{13}$$

$$D_{s,k-1}(\mathbf{x}) = \int p_s f(\mathbf{x}|\mathbf{x}') D_{k-1}(\mathbf{x}') d\mathbf{x}' \tag{14}$$

$$D_k(\mathbf{x}) = (1-p_d) D_{k|k-1}(\mathbf{x}) + D_{d,k}(\mathbf{x}) \tag{15}$$

$$D_{d,k}(\mathbf{x}) = \sum_{\mathbf{z}\in Z_k} \frac{p_d f(\mathbf{z}|\mathbf{x}) D_{k|k-1}(\mathbf{x})}{\lambda_c c(z) + \int p_d f(\mathbf{z}|\mathbf{x}') D_{k|k-1}(\mathbf{x}') d\mathbf{x}'} \tag{16}$$

In addition to the assumptions of the PHD filter, the GM-PHD filter further makes the following assumptions.

- The dynamic model and the measurement model can be represented as a linear Gaussian model as shown in Equations 17 and 18.

$$f(\mathbf{x}|\mathbf{x}') = \mathcal{N}(\mathbf{x}; \mathbf{F}\mathbf{x}, \mathbf{Q}) \tag{17}$$

$$f(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mathbf{z}; \mathbf{H}\mathbf{x}, \mathbf{R}) \tag{18}$$

- The intensity of the target birth PHD, $b_k(\mathbf{x})$, can be represented in the form of a Gaussian mixture as shown in (19), where $J_{b,k}$ is the number of Gaussian components in the target birth PHD, and $w_{b,k}^{(i)}$, $\mathbf{m}_{b,k}^{(i)}$ and $\mathbf{P}_{b,k}^{(i)}$ are the weight, mean vector and covariance matrix of the i^{th} Gaussian component in the target birth PHD.

$$b_k(\mathbf{x}) = \sum_{i=1}^{J_{b,k}} w_{b,k}^{(i)} \mathcal{N}\left(\mathbf{x}; \mathbf{m}_{b,k}^{(i)}, \mathbf{P}_{b,k}^{(i)}\right) \tag{19}$$

Suppose that the PHD of resident states at time step $k-1$ is a Gaussian mixture as shown in

$$D_{k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; \mathbf{m}_{k-1}^{(i)}, \mathbf{P}_{k-1}^{(i)}\right). \tag{20}$$

By substituting (17), (19) and (20) into (13), the posterior PHD of the predictor can be represented in the form of a Gaussian mixture as shown in (18).

$$\begin{aligned} D_{k|k-1}(\mathbf{x}) =& \sum_{i=1}^{J_{b,k}} w_{b,k}^{(i)} \mathcal{N}\left(\mathbf{x}; \mathbf{m}_{b,k}^{(i)}, \mathbf{P}_{b,k}^{(i)}\right) \\ &+ p_d \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; \mathbf{F}\mathbf{m}_{k-1}^{(i)}, (\mathbf{Q} + \mathbf{F}\mathbf{P}_{k-1}^{(i)}\mathbf{F}^T)\right) \end{aligned} \tag{21}$$

For simplicity, we rewrite the posterior PHD of the predictor as shown in (22).

$$D_{k|k-1}(\mathbf{x}) = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; \mathbf{m}_{k|k-1}^{(i)}, \mathbf{P}_{k|k-1}^{(i)}\right). \tag{22}$$

By substituting (18) and (22) into (15), the posterior PHD of all residents at time k can be calculated as in

$$\begin{aligned} D_k(\mathbf{x}) =& (1-p_d) \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}\left(\mathbf{x}; \mathbf{m}_{k|k-1}^{(i)}, \mathbf{P}_{k|k-1}^{(i)}\right) \\ &+ \sum_{\mathbf{z}\in Z_k} \sum_{j=1}^{J_{k|k-1}} w_k^{(j)}(\mathbf{z}) \mathcal{N}\left(\mathbf{x}; \mathbf{m}_k^{(j)}(\mathbf{z}), \mathbf{P}_k^{(j)}\right). \end{aligned} \tag{23}$$

In (23),

$$w_k^{(j)}(\mathbf{z}) = \frac{p_d w_{k|k-1}^{(j)} q_k^{(j)}(\mathbf{z})}{\lambda_c c(\mathbf{z}) + p_d \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} q_k^{(i)}(\mathbf{z})} \tag{24}$$

$$q_k^{(j)}(\mathbf{z}) = \mathcal{N}\left(\mathbf{z}; \mathbf{H}\mathbf{m}_{k|k-1}^{(j)}, \mathbf{R} + \mathbf{H}\mathbf{P}_{k|k-1}^{(j)}\mathbf{H}^T\right) \tag{25}$$

$$\mathbf{m}_k^{(j)}(z) = \mathbf{m}_{k|k-1}^{(j)} + \mathbf{K}_k^{(j)}(\mathbf{z} - \mathbf{H}\mathbf{m}_{k|k-1}^{(j)}) \tag{26}$$

$$\mathbf{P}_k^{(j)} = (\mathbf{I} - \mathbf{K}_k^{(j)}\mathbf{H})\mathbf{P}_{k|k-1}^{(j)} \tag{27}$$

$$\mathbf{K}_k^{(j)} = \mathbf{P}_{k|k-1}^{(j)}\mathbf{H}^T(\mathbf{H}\mathbf{P}_{k|k-1}^{(j)}\mathbf{H}^T + \mathbf{R})^{-1} \tag{28}$$

According to the predictor and the corrector of the GM-PHD filter as shown in (21) and (23), the number of Gaussian components in the PHD grows from J_{k-1} at time step $k-1$ to $(J_{b,k} + J_{k-1})|Z^{(k)}|$ at time step k. To balance computational complexity with accuracy, a maximum number of J_{max} Gaussian components with the highest weights are kept and propagated through time.

References

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