Derivation of $var(\hat{\gamma}_2)$ for the cohort design without attrition.

Supplement to *The cluster randomized crossover trial: the effects of attrition in the AB/BA design and how to account for it in sample size calculations*.

The variance of the mean outcome within cluster *j* in time period $h = 1$ is given by

$$var(\bar{y}_{1.j}) = \frac{m\, var(y_{1ij})}{m^2} + \frac{m(m-1) cov(y_{1ij}, y_{1i'j})}{m^2} = \frac{\tilde{\sigma}_C^2 + \tilde{\sigma}_{CP}^2 + \tilde{\sigma}_I^2 + \tilde{\sigma}_M^2}{m} + \frac{(m-1)(\tilde{\sigma}_C^2 + \tilde{\sigma}_{CP}^2)}{m}.$$

The same formula holds for $var(\bar{y}_{2.j})$.

The covariance of these two cluster-period means within cluster *j* is equal to

$$cov(\bar{y}_{1.j}, \bar{y}_{2.j}) = \frac{m\, cov(y_{1ij}, y_{2ij})}{m^2} + \frac{m(m-1)\, cov(y_{1ij}, y_{2i'j})}{m^2} = \frac{(\tilde{\sigma}_C^2 + \tilde{\sigma}_I^2)}{m} + \frac{(m-1)\tilde{\sigma}_C^2}{m}.$$

The difference in cluster-period means in cluster *j* has variance

$$var(\bar{y}_{1.j} - \bar{y}_{2.j}) = var(\bar{y}_{1.j}) + var(\bar{y}_{2.j}) - 2cov(\bar{y}_{1.j}, \bar{y}_{2.j}) =$$

$$2\frac{\tilde{\sigma}_C^2 + \tilde{\sigma}_{CP}^2 + \tilde{\sigma}_I^2 + \tilde{\sigma}_M^2}{m} + 2\frac{(m-1)(\tilde{\sigma}_C^2 + \tilde{\sigma}_{CP}^2)}{m} - 2\frac{(\tilde{\sigma}_C^2 + \tilde{\sigma}_I^2)}{m} - 2\frac{(m-1)\tilde{\sigma}_C^2}{m}$$

$$= \frac{2}{m}(\tilde{\sigma}_C^2 + \tilde{\sigma}_{CP}^2 + \tilde{\sigma}_I^2 + \tilde{\sigma}_M^2 + (m-1)(\tilde{\sigma}_C^2 + \tilde{\sigma}_{CP}^2) - (\tilde{\sigma}_C^2 + \tilde{\sigma}_I^2) - (m-1)\tilde{\sigma}_C^2)$$

$$= \frac{2}{m}(\tilde{\sigma}_M^2 + m\tilde{\sigma}_{CP}^2).$$

The treatment effect γ_1 is then estimated by taking the mean of the difference in means across all *k* clusters. This estimator has variance:

$$var(\hat{\gamma}_2) = \frac{2}{mk}(\tilde{\sigma}_M^2 + m\tilde{\sigma}_{CP}^2).$$

This variance can be reformulated in terms of correlations by making the following substitutions:

$\tilde{\sigma}_{CP}^2 = \sigma_T^2(\rho - \eta)$

$\tilde{\sigma}_M^2 = \sigma_T^2 - \tilde{\sigma}_C^2 - \tilde{\sigma}_{CP}^2 - \tilde{\sigma}_I^2 = \sigma_T^2\big(1 - \eta - (\rho - \eta) - (\xi - \eta)\big) = \sigma_T^2(1 - \rho - \xi + \eta).$

It then follows

$$var(\hat{\gamma}_2) = \frac{2\sigma_T^2(1 - \xi + (m-1)(\rho - \eta))}{mk}.$$