

Tsetse population dynamics

We model changes in the numbers of adult tsetse and pupae over time and space on a lattice using two recursion equations. Movement of adult tsetse is assumed to be random and is approximated by a diffusion process, as previously described by Hargrove (2003).

Changes in the number of adults (A) at time t in cell i, j are described by:

$$A_{(i,j,t+1)} = (A_{(i,j,t)} + aC)(1 - \mu_B) + \beta P_{(i,j,t)} \quad (1)$$

where $C = A_{(i,j-1,t)} + A_{(i-1,j,t)} + A_{(i,j+1,t)} + A_{(i+1,j,t)} - 4A_{(i,j,t)}$ and $P_{(i,j,t)}$ is the number of pupae at time t in cell i, j . Each day, adults move to an adjacent cell with probability a and β represents the proportion of the pupal population emerging as adults on any given day. A proportion of adults (μ_B) then die due to natural, density-independent causes.

Changes in numbers of pupae are described by:

$$P_{(i,j,t+1)} = P_{(i,j,t)}(1 - \beta)(1 - \mu_P)(1 - P_{(i,j,t)}(1 - \beta P_{(i,j,t)})(1 - \mu_P)\delta) + lA_{(i,j,t)}/2 \quad (2)$$

where l is the proportion of females larvipositing each day and μ_P is the proportion of pupae which die each day due to density-independent processes. In addition pupal deaths result from density-dependent mortality, implemented using the coefficient δ .

We include additional adult tsetse mortality (μ_F) for tsetse in cells modelled as ‘farming areas’ where μ_F is a matrix with increased mortality (> 0) in cells $i, f_1 \geq j \leq f_2$ and zero otherwise so that Eq. 1 therefore becomes:

$$A_{(i,j,t+1)} = (A_{(i,j,t)} + aC)(1 - \mu_B)(1 - \mu_F) + \beta P_{(i,j,t)} \quad (3)$$

Tsetse population and trypanosome transmission dynamics

We extend the model in Eq. 2 and 3 to include trypanosome transmission. We use a single-vector, single-host system.

Our model assumptions generally follow those described by Rogers (1998) and Hargrove et al. (2012), except that we allow for a probability > 0 that non-teneral tsetse – flies that have taken at least one bloodmeal – can become infected with *T. brucei*, and for a decreased probability of infection with *T. congolense* for non-teneral flies, based on laboratory studies including Welburn and Maudlin (1992) and Kubi et al. (2006).

Each day, a proportion β_H of total hosts (N_H) in each cell produce offspring, which are susceptible (S_H). Susceptible hosts become exposed (E_H) with probability λ_H . Exposed hosts become infectious with probability σ_H and infected individuals (I_H) recover with probability φ . Recovered hosts (R_H) lose immunity and become susceptible with probability γ . All hosts die with probability μ_H .

Hosts:

$$S_{H(i,j,t+1)} = S_{H(i,j,t)}(1 - \lambda_H)(1 - \mu_H) + \gamma R_{H(i,j,t)}(1 - \mu_H) + \beta_H N_{H(i,j,t)} \quad (4)$$

$$E_{H(i,j,t+1)} = E_{H(i,j,t)}(1 - \sigma_H)(1 - \mu_H) + \lambda_H S_{H(i,j,t)}(1 - \mu_H) \quad (5)$$

$$I_{H(i,j,t+1)} = I_{H(i,j,t)}(1 - \varphi)(1 - \mu_H) + \sigma_H E_{H(i,j,t)}(1 - \mu_H) \quad (6)$$

$$R_{H(i,j,t+1)} = R_{H(i,j,t)}(1 - \gamma)(1 - \mu_H) + \varphi I_{H(i,j,t)}(1 - \mu_H) \quad (7)$$

where $\lambda_H = 1 - (1 - 1/N_H)^{\alpha I_V p_H}$ where α the daily probability of tsetse biting and I_V the number of infectious tsetse.

Vectors:

In addition to tsetse reproduction, death and movement as described above, each day teneral (unfed) tsetse leave the susceptible teneral (S_V) class with probability α – the probability of taking a blood meal. A proportion λ_{VS} of these flies become exposed (E_{1V}), depending on the proportion of infected hosts and the probability p_S of becoming infected given an bite on an infected host, and the rest become susceptible non-teneral G_V . Susceptible non-teneral flies have probability p_G of becoming infected given a bite on an infected host. Infected flies move through three ‘exposed’ classes each with probability $3\sigma_V$ before developing a mature infection (I_V).

$$S_{V(i,j,t+1)} = (S_{V(i,j,t)} + aC_S)(1 - \alpha)(1 - \mu_B)(1 - \mu_F) + \beta P_{(i,j,t)} \quad (8)$$

$$E_{1V(i,j,t+1)} = (E_{1V(i,j,t)} + aC_{E1})(1 - 3\sigma_V)(1 - \mu_B)(1 - \mu_F) \\ + \lambda_{VS}(S_{V(i,j,t)} + aC_S) + \lambda_{VG}(G_{V(i,j,t)} + aC_G) \quad (9)$$

$$E_{2V(i,j,t+1)} = (E_{2V(i,j,t)} + aC_{E2})(1 - 3\sigma_V)(1 - \mu_B)(1 - \mu_F) + 3\sigma_V(E_{1V(i,j,t)} + aC_{E1}) \quad (10)$$

$$E_{3V(i,j,t+1)} = (E_{3V(i,j,t)} + aC_{E3})(1 - \mu_B)(1 - \mu_F)(1 - 3\sigma_V) + 3\sigma_V(E_{2V(i,j,t)} + aC_{E2}) \quad (11)$$

$$I_{V(i,j,t+1)} = (I_{V(i,j,t)} + aC_I)(1 - \mu_B)(1 - \mu_F) + 3\sigma_V(E_{3V(i,j,t)} + aC_{E3}) \quad (12)$$

$$G_{V(i,j,t+1)} = (G_{V(i,j,t)} + aC_G)(1 - \lambda_{VG})(1 - \mu_B)(1 - \mu_F) + (1 - \lambda_{VS})(S_{V(i,j,t)} + aC_S) \quad (13)$$

$$P_{(i,j,t+1)} = (P_{(i,j,t)} - \beta P_{(i,j,t)})(1 - \mu_P)(1 - (P_{(i,j,t)} - \beta P_{(i,j,t)})(1 - \mu_P)\delta) + lN_{V(i,j,t)}/2 \quad (14)$$

where $\lambda_{VS} = \alpha p_S I_H / N_H$ and $\lambda_{VG} = \alpha p_G I_H / N_H$.

References

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