Supplementary Material

Developing a COVID-19 mortality risk prediction model when individual level data is not available

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Supplementary Table 1: Population table for baseline model

Population characteristics for the three population sets used to build and validate the baseline model.

Abbreviations: SD, Standard Deviation;

Supplementary Table 2: Corrections table for the "multi-calibration" recalibration procedure

Total number of corrections performed and cumulative magnitude of the correction for each subgroup of age and sex.

Supplementary Table 3: Sensitivity analysis results for the discrimination performance considering the composite outcome (severe cases and mortality)

Abbreviations: AUROC, Area Under the Receiver Operating Characteristic curve; PPV, Positive Predictive Value

Caption: A list of the variables used in the model, including their type, units, time frame of extraction, definitions, how they were selected and potential for missingness.

All variables were extracted at or before February 1st, 2020, prior to the onset of the COVID-19 pandemic in Israel.

Abbreviations: COPD, Chronic Obstructive Pulmonary Disease; g, gram; mcg, microgram; mEq, mili-equivalent; ml, milliliter; dl, deciliter; L, liter; PCR, Polymerase Chain Reaction; ICD9, International Classification of Disease, 9th revision; ATC4, Anatomical Therapeutic Chemical (ATC) Classification System, 4th level;

Supplementary Figure 1: Odds ratio as a function of variable values for the baseline predictor

A plot of the odds ratios for different values of the predictors in the baseline model, using SHAP values¹. A smoothed red line is fit to the curves and a horizontal grey line is drawn at odds ratio = 1.

All figures use a random sample of patients from this same population, n=10,000 unique patients.

Abbreviations: SHAP, SHapley Additive exPlanations; COPD, Chronic Obstructive Pulmonary Disease; HDL, High Density Lipoprotein;

Supplementary Figure 2: Cumulative Distribution Function of time-to-death among CHS COVID-19 patients

Empirical cumulative distribution function for time-to-death of all COVID-19 patients in CHS' database. The black line shows the cumulative incidence as calculated after accounting for censoring and the "competing risk" of cure, derived using the Aalen-Johansen estimator². The vertical dashed line denotes the follow-up time used in the study, 91 days.

The figure uses all COVID-19 patients among the Clalit Health Services insured population until the extraction date (July 16th, 2020), n=16,049 unique patients. Abbreviations: CDF, Cumulative Distribution Function; CHS, Clalit Health Services; COVID-19, Corona Virus Disease 2019;

Supplementary Methods 1: Transformation of SHAP values to odds ratios

In Shapley analysis for binary outcomes, the log-odds of the outcome probability is mathematically expressed as a summation of the SHAP values of each of the covariates. The exponent of a single SHAP value (similar to logistic regression), is thus the ratio of the odds of the outcome given that SHAP value divided by the baseline odds of the outcome (using only the intercept), i.e. the odds ratio.

For example, in a model with a single predictor, with x_i marking the SHAP value of that predictor:

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$$
\begin{cases} \log\left(\frac{p_0}{1-p_0}\right) = x_0 \\ \log\left(\frac{p_1}{1-p_1}\right) = x_0 + x_1 \end{cases} =>
$$

\n•
$$
\log\left(\frac{\frac{p_1}{1-p_1}}{\frac{p_0}{1-p_0}}\right) = \log\left(\frac{p_1}{1-p_1}\right) - \log\left(\frac{p_0}{1-p_0}\right) = x_0 + x_1 - x_0 = x_1 =>
$$

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$$
\frac{\frac{p_1}{1-p_1}}{\frac{p_0}{1-p_0}} = e^{x_1}
$$

Supplementary Methods 2: Pseudo-code of the "multi-calibration" algorithm

- 1. Input: An initial predictor p' , a collection of subpopulations C , a training set $D=\{(x_i,y_i)_{i=1}^m\}$ and a violation parameters $\alpha>0$
- **2. Output:** a post-processed predictor **p** satisfying (C, α) -multiaccuracy on **D**
- **3.** $p \leftarrow p'$ 4. $done \leftarrow False$ 5. while $\neg done$ do 5.1. $done \leftarrow True$
5.2. foreach $S \in C$ do 5.2. **foreach** ∈ **do** // iterate over subpopulations 5.2.1. $\Delta_{\mathcal{S}} = \frac{1}{|S_{\Omega}|}$ $\frac{1}{|S \cap D|} (\sum_{x_i \in S} y_i - \sum_{x_i \in S} p_i)$ // magnitude of violation on S 5.2.2.**If** $|\Delta_{S}| > \alpha$ then 5.2.2.1.1. $p \leftarrow p + \Delta_S \mathbf{1}_S$
5.2.2.1.2. $done \leftarrow False$ // update predictor by "nudging" S $done \leftarrow False$ 5.2.3.**end if** 5.3. **end foreach**
- 6. **end while**
- 7. **return p**

Detailed pseudo-code of the multi-calibration algorithm, as described in Hebert-Johnson et al.³

In this study, the algorithm accepts as input the predictions (p') from the baseline model and the collection of subpopulations (C) defined by intersections of age and sex groups. It returns predictions recalibrated to COVID-19 outcome rates, which replace the (usually empirically derived) outcomes y_i .

Supplementary Methods 3: Technical description of adjustment procedure

Purpose: to adjust predictions to external one-way conditional probabilities.

Inputs:

- Baseline predictions for another outcome on the entire population of interest, $P(Y_{old}|\textbf{X})$, conditioned on as many independent variables as wanted.
- Published one-way conditional probabilities for the outcome of interest, from an external population, conditional on n variables. $P_{external}(Y_{new}|X_i)$, $I = 1..n$
- \bullet The co-occurrence matrix $\bm{M}_{n\times n}$, where $\bm{M}_{i,j}=P_{external}(X_j|X_i)$ in the population on which the one-way conditional probabilities were published
- Rates of the different population characteristics in the local population, $P_{local}(X_i)$, $i = 1..n$

Process:

- By solving a system of linear equations, $P_{external}(Y|.) = Mw$, the *n* coefficients for each variable (W_i) are calculated.
- These coefficients are used to derive $P_{local}(Y|.)$ by multiplying them with the local population characteristics.

Steps:

1. Solving for the coefficients of the linear probability model

This is done by deriving the coefficients for each independent variable in a linear probability model. That is, the model is $P(Y) = W_1P(X_1) + \cdots + W_nP(X_n)$, and the coefficients are solved for by solving a system of $i=1..n$ equations: $P_{external}(Y|X_i) = W_i P_{external}(X_1|X_i) + \cdots + W_n P_{external}(X_n|X_i)$

Where $P(X_i|X_j)$ is the i,j entry of the \boldsymbol{M} matrix.

2. Adjust external one-way conditionals to the local population

With the coefficients in hand, rates for the local population are calculated using the local probabilities of the independent variables, for $i = 1..n$ $P_{local}(Y|X_i) = W_i P_{local}(X_1|X_i) + \cdots + W_n P_{local}(X_n|X_i)$

3. Recalibrate the predictions for the baseline outcome to the outcome rates of the new outcome With the calculated local outcome rates in hand, the baseline predictions are adjusted using the multi-accuracy algorithm depicted in Supplementary Methods 2.

When the process terminates, we have $P(Y_{new} | \boldsymbol{X})$ for the entire population of interest.

References

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